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GianCarlo Ghirardi

Renata Grassi

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Tullio Weber

INTERNATIONAL ATOMIC ENERGY AGENCY

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QUANTUM MECHANICS PARADOXES AT THE Φ-FACTORY *

GianCarlo Ghirardi, Renata Grassi and Tullio Weber

International Centre for Theoretical Physics, Trieste, Italy.

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ABSTRACT

We discuss the possibility of performing experiments allowing one to test quantum mechanics versus any local realistic model within the context of the physics at the Φ-factory. After having sketched the main features of the physical process under consideration and having focused the locality requirements for it, we derive Bell's inequalitiy for the two-meson system. Comparison with quantum predictions shows that the inequality is not violated for any choice of the parameters characterizing the measurement process. Contrary to the case of spin variables, there is then no way to exclude, by experiments at the Φ-factory, the possibility of a local realistic description of the process. A recent suggestion about a test of quantum predictions versus the assumption of a spontaneous factorization mechanism, as well as the claimed validity of an inequality which is different from Bell's one, are also discussed. The general conclusion is that the Φ-factory facility does not seem to open new ways of testing quantum mechanics versus alternative general schemes of the type which are usually regarded as worth considering in the debate about locality and quantum mechanics. The concluding Section is devoted to making clear our position with respect to the problems discussed in this paper. It is pointed out that, in our opinion, the existing experimental evidence makes already clear that one has to accept the "mysterious" features of microscopic systems. The really crucial problem is that of investigating whether one can restore a coherent worldview which generally conforms with our experience at the macroscopic level, by keeping all highly successful predictions of quantum theory at the microscopic one.

1. GENERAL CONSIDERATIONS.

1.1. Introduction

Why is it interesting to investigate the possibility of experimental tests about "quantum mechanical paradoxical situations" in the specific context of the physics at the Φ-Factory? The reason, when reduced to its essence, can be expressed in the following terms. As is well known, quantum mechanics does not allow one, in general, to attribute individual objective properties to the constituents of a composite system. This fact has been widely discussed with reference to observables like the spin components, and has been considered quite peculiar and embarrassing. However some recent clear-cut experiments[1] have shown that nature, as far as spin correlations are concerned, is actually that peculiar. There still is, however, a rooted reluctance to accept the fact that objective properties do not exist when they refer to the very nature of the constituents, e.g., to their being a K^0 or a $\overline{K^0}$ meson. This is why it seems appropriate to make a conceptual analysis of this point and to investigate the possibility of performing correlation experiments on states originating from the decay of a Φ-meson, testing the predictions of quantum theory against those deriving from other possible conceptual frameworks which might seem more "acceptable" since they allow one to attribute definite physical properties, at the individual level, to the constituents of the composite system.

1.2. Properties.

As is well known, quantum mechanics is a theory which allows one to make only probabilistic conditional predictions about the outcomes of prospective and, in general, incompatible measurements of physical observables. One is then led to face the problem of the attribution of "objective (i.e. independent from any observer) physical properties" to physical systems. There is a standard way to do this, which was proposed in the celebrated EPR-paper [2]. Denoting by $P(A=\alpha|\Psi)$, the probability of getting the result α in a measurement of the observable A when the system is in the state $|\Psi\rangle$, we say that a physical system possesses the property (or element of physical reality: EPR) $A=\alpha$, iff $P(A=\alpha|\Psi)=1$.

1.3. Entanglement

For the sake of simplicity, to discuss this point we consider the spin states of a composite system S1+S2 of two spin-1/2 particles. Suppose their state is the factorized state

$$|n1\rangle|m2\rangle, \qquad (1.1)$$

where the labels 1 and 2 refer to the constituents and **n** (**m**) specifies that the state is an eigenstate of $\sigma\cdot\mathbf{n}$ ($\sigma\cdot\mathbf{m}$), belonging to the eigenvalue +1. For state (1.1), according to the previous prescription, one is allowed to assert that particle 1 has the property that its spin is along **n**, and particle 2 has the property that its spin is along **m**. However, not all states of S1+S2 are factorized. The most commonly considered entangled state is the singlet state:

$$|Singlet\rangle=(1/\sqrt{2})\,[|1+\rangle|2-\rangle-|1-\rangle|2+\rangle] \qquad (1.2)$$

It is worth remarking that, for such a state, even though the system S1+S2 possesses properties (in the considered case $S^2=S_z=0$), the individual constituents do not have any property at all. In particular, since there is no observable (spin component) referring to S1 or to S2 for which the probability of an outcome equals 1, one is not allowed to state, or even to think, that the spin of particle 1 (or of particle 2) is "along a direction", even though the two constituents are far apart and noninteracting.

1.4. Wave Packet Reduction.

The theory must embody a principle leading to the disentanglement of entangled states. The postulate of wave packet reduction (WPR) plays such a role and, in the case of entangled states describing far apart and noninteracting systems, leads to factorized states. For the above considered state (1.2), measuring, e.g., $\sigma_z^{(1)}$ and finding the result -1, WPR leads to the factorized state $|\Psi\rangle=|1-\rangle|2+\rangle$ for which $P(\sigma_z^{(2)}=+1|\Psi)=1$, so that one can state that a property has emerged in the far-apart region where S2 is, as a consequence of a measurement on S1.

It has to be remarked that this is one of the ways in which the fundamental nonlocality of the theory shows up. However, it has to be stressed that the nonlocal features are such [3] that they allow *the peaceful coexistence of quantum mechanics and relativity*,[4] since they do not permit faster than light signalling.

1.5. The EPR Argument.

Immediately after the measurement, as we have seen, one can predict with certainty the outcome of a prospective measurement of the z-component of the spin of S2, and, as a consequence, there is a definite physical property (EPR) for it. If one adds the EPR-locality requirement, i.e. that objective properties of a system cannot be instantaneously created at a distance, one is led to conclude that such an EPR existed even before the measurement took place. The quantum description of the physical situation by the singlet state, however, does not contain any formal element referring to this objective individual property of S2 before the measurement. As a consequence, Einstein Podolsky and Rosen were led to the conclusion that quantum mechanics is an incomplete theory .

It has to be remarked that J.S. Bell [5], with his deep analysis, has proved, completely in general, that the above mentioned fact is really not a drawback of the theory, but a peculiar feature of nature itself at the microscopic level; in fact, if one assumes locality and that the quantum correlations are "true", then objective properties of the constituents cannot (even be thought to) exist before the measurement.

To conclude this Subsection we consider it important to stress that the occurrence of the 100% correlations involved in an EPR-type set up, and which have been referred to as [6] "paradoxes" or "non intuitive quantum mechanical expectations",

does not allow to draw any definite conclusion about the problem of local realism. In fact, as is well known, it is trivial to build up local hidden variable theories which account for these correlations. Testing quantum mechanics versus local realism requires, due to Bell's theorem, the consideration of the quantum predictions concerning correlation measurements involving non commuting observables of the subsystems of the composite system (or, alternatively [7], involving more than two far away systems in appropriate quantum states).

1.6. Bell's Inequality.

Let us consider a local hidden variable theory and let us denote by $A(\mathbf{a},\lambda)$ and $B(\mathbf{b},\lambda)$ the definite values of two quantum observables $A(\mathbf{a})$ and $B(\mathbf{b})$, respectively, given the value λ of the hidden variables. Suppose that $|A|, |B|\leq 1$. For a two particle system we can identify, e.g., $A(\mathbf{a})$ with $\sigma^{(1)}\cdot\mathbf{a}$ and $B(\mathbf{b})$ with $\sigma^{(2)}\cdot\mathbf{b}$, respectively. Let us consider the quantity:

$$M(\mathbf{a},\mathbf{b})=\int d\lambda\rho(\lambda)A(\mathbf{a},\lambda)B(\mathbf{b},\lambda) \tag{1.3}$$

which corresponds to the quantum mechanical mean value $\langle A(\mathbf{a})\cdot B(\mathbf{b})\rangle$. One has

$$\begin{aligned} M(\mathbf{a},\mathbf{b})-M(\mathbf{a},\mathbf{b}') &= \int d\lambda\rho(\lambda)[A(\mathbf{a},\lambda)B(\mathbf{b},\lambda)-A(\mathbf{a},\lambda)B(\mathbf{b}',\lambda)] \\ &= \int d\lambda\rho(\lambda)A(\mathbf{a},\lambda)B(\mathbf{b},\lambda)[1\pm A(\mathbf{a}',\lambda)B(\mathbf{b}',\lambda)] \\ &\quad -\int d\lambda\rho(\lambda)A(\mathbf{a},\lambda)B(\mathbf{b}',\lambda)[1\pm A(\mathbf{a}',\lambda)B(\mathbf{b},\lambda)] \end{aligned} \tag{1.4}$$

There follows:

$$|M(\mathbf{a},\mathbf{b})-M(\mathbf{a},\mathbf{b}')|\leq\int d\lambda\rho(\lambda)[1\pm A(\mathbf{a}',\lambda)B(\mathbf{b}',\lambda)] + \int d\lambda\rho(\lambda)[1\pm A(\mathbf{a}',\lambda)B(\mathbf{b},\lambda)] \tag{1.5}$$

or

$$|M(\mathbf{a},\mathbf{b})-M(\mathbf{a},\mathbf{b}')|\leq 2\pm[M(\mathbf{a}',\mathbf{b}')+M(\mathbf{a}',\mathbf{b})] \tag{1.6}$$

i.e.

$$|M(\mathbf{a},\mathbf{b})-M(\mathbf{a},\mathbf{b}')|+|M(\mathbf{a}',\mathbf{b}')+M(\mathbf{a}',\mathbf{b})|\leq 2 \tag{1.7}$$

As is well known, when one attributes to the quantities $M(\mathbf{a},\mathbf{b})$ the values implied by quantum mechanics, the inequality (1.7) is remarkably violated for appropriate choices of the directions appearing in it.

1.7 The Spontaneous Factorization Hypothesis.

It is important to remark that, in the case of a statistical mixture of factorized states with weights p_i and states $|1i\rangle|2i\rangle$, the quantum mechanical mean value $\langle A(\mathbf{a})\cdot B(\mathbf{b})\rangle$ becomes

$$\langle A(\mathbf{a})\cdot B(\mathbf{b})\rangle = \Sigma_i p_i \langle 1i|A(\mathbf{a})|1i\rangle\langle 2i|B(\mathbf{b})|2i\rangle \tag{1.8}$$

which has the same formal structure as (1.3). One can therefore derive again a Bell inequality for it. As a consequence, the hypothesis [8)] of a spontaneous factorization of state vectors of a composite system when the constituents are far apart, can, in principle,

be experimentally tested against quantum predictions (see, however, the detailed discussion of Section 4.1).

2. LOCALITY AND THE K^0, $\overline{K^0}$ MESONS.

2.1. Dynamics of Mesons

We consider the strong decay of the $\Phi(1020)$, $J^{PC}=1^{--}$, vector meson into a pair of neutral pseudoscalar mesons K^0, $\overline{K^0}$. Due to C-conservation in strong interactions the initial state of the decay products is

$$|\Psi(t=0)\rangle=\frac{1}{\sqrt{2}}[|K^0\rangle_l|\overline{K^0}\rangle_r-|\overline{K^0}\rangle_l|K^0\rangle_r] \tag{2.1}$$

where l and r refer to the particles propagating to the left and to the right, respectively. It has to be remarked that since the meson masses are about 500 MeV, the process is nonrelativistic.

The dynamics of the K-mesons is governed by weak interactions which are responsible for the strangeness oscillations as well as for the decays of such systems. The strangeness oscillations can be phenomenologically described within the space spanned by the $|K^0\rangle$ and $|\overline{K^0}\rangle$ states by a non-Hermitian Hamiltonian H_W, which also takes into account the loss of probability into the decay channels. The eigenstates of H_W are appropriate linear combinations of $|K^0\rangle$ and $|\overline{K^0}\rangle$:

$$|K_S\rangle= p|K^0\rangle-q|\overline{K^0}\rangle$$

$$|K_L\rangle= p'|K^0\rangle+q'|\overline{K^0}\rangle \tag{2.2}$$

Let us denote by

$$\lambda_S=m_S-(i/2)\gamma_S,\ \ \lambda_L=m_L-(i/2)\gamma_L \tag{2.3}$$

the eigenvalues of H_W associated to the states (2.2).

One can choose the phases of $|K_S>$, $|K_L>$, $|K^0>$ and $|\overline{K^0}>$ in such a way that p, q, and p' are real and positive. We remark that CPT invariance implies p=p', q=q', while the further requirement of CP invariance would imply $p=q=(1/\sqrt{2})$. In this case, using the convention that $CP|K^0>=-|\overline{K^0}>$ and $CP|\overline{K^0}>=-|K^0>$, the states $|K_S>$ and $|K_L>$ are eigenstates of CP belonging to the eigenvalues +1 and -1, respectively and are therefore orthogonal.

2.2. Local Properties of the K^0, $\overline{K^0}$ system.

The complete evolution of states (2.2) is described by a unitary operator U(t,0) whose effect can be written as

$$U(t,0)|K_{S,L}>=e^{-i\lambda_{S,L}t}|K_{S,L}>+|\Omega_{S,L}(t)> \tag{2.4}$$

where $|\Omega_{S,L}(t)>$ describes the decay products.

Independently of any invariance requirement the initial state (2.1) turns out to be

$$|\Psi(t=0)>=\frac{1}{(p'q+q'p)\sqrt{2}}[|K_S>_l|K_L>_r-|K_L>_l|K_S>_r] \tag{2.5}$$

The state at time t is then obtained from (2.5) by applying to it a unitary operator which is the direct product

$$U(t,0)=U_l(t,0)U_r(t,0) \tag{2.6}$$

of the operators $U_l(t,0)$ and $U_r(t,0)$ acting on the space of the l and r mesons in accordance with (2.4). One then sees that the probability of finding, in two measurements

performed at the same time, mesons with the same strangeness, i.e two K^0 or two $\overline{K^0}$ mesons, is equal to zero, independently of any specific invariance assumption.

One can then argue as follows: suppose one performs at time t a measurement aimed to detect the presence of a $\overline{K^0}$ meson at r, and suppose one finds it. Then one can assert that the subsystem at l has the objective property of not being a $\overline{K^0}$. Any local theory would then imply that this EPR at left must be present independently of the measurement at the right having been performed or not. This fact gives rise to "difficulties" for the quantum scheme which are the analogue of those discussed in Sect. 1.4 for spin variables. One is then led to investigate whether one can find for the considered process a local deterministic completion of quantum mechanics, i.e. a theory in which the outcomes of measurements at any time t (in particular of those ascertaining the very nature of the particles which are present) are determined and correspond to possessed properties of the systems at the considered times.

2.3. The Quantum Probabilities.

We derive now the basic expressions which will be useful in what follows and which will allow the evaluation of the probabilities, according to standard quantum mechanics, of finding a K^0 or a $\overline{K^0}$ meson in two measurements performed at two different times t_r and t_l ($t_l > t_r$) at positions x_r and x_l, respectively. We denote by $P_i(k)$, where i=l,r; k=K^0, $\overline{K^0}$, the projection operators on the l,r-meson states, so that, e.g. $P_r(\overline{K^0})=|\overline{K^0}>_{rr}<\overline{K^0}|$. As usual, we also denote by $Q_i(k) = 1-P_i(k)$, the operators projecting on the manifolds orthogonal to those associated to $P_i(k)$. Obviously all the now considered operators commute among themselves.

According to eq.(2.6), starting from the initial state (2.5) one gets, at time t_r the state

$$|\Psi(t=t_r)>=U_l(t_r,0)U_r(t_r,0)|\Psi(t=0)> \tag{2.7}$$

Measuring $P_r(k)$ yields reduction to the state

$$|\tilde{\Psi}(t=t_r)\rangle = \frac{P_r(k)|\Psi(t=t_r)\rangle}{\|P_r(k)|\Psi(t=t_r)\rangle\|} \qquad (2.8)$$

with probability $\|P_r(k)\,|\Psi(t=t_r)\rangle\|^2$. One then evaluates the state at time t_l to which the state (2.8) has evolved. The probability of finding a meson state **k** in the measurement at the right and a meson state k' in the one at the left is therefore given by the squared norm of the state

$$|\tilde{\Psi}(t_l,t_r)\rangle = P_l(k')U_l(t_l,t_r)U_r(t_l,t_r)P_r(k)|\Psi(t=t_r)\rangle \qquad (2.9)$$

Such a squared norm, taking into account the unitarity and composition laws of the operators U, as well as the fact that operators referring to the different (r and l) Hilbert spaces commute, coincides with the squared norm of the state

$$|\Psi(t_l,t_r)\rangle = P_l(k')P_r(k)U_l(t_l,0)U_r(t_r,0)|\Psi(t=0)\rangle \qquad (2.10)$$

In what follows we will be led to consider also the probabilities of not finding a specific particle in a measurement. Obviously this requires the consideration of the projection operators $Q_i(k)$ defined above. Evaluation of the corresponding probabilities for such processes can be done by using again formula (2.10) with the operators Q replacing, where required, the operators P.

It is physically interesting (see the discussion of the next section) to consider the joint probabilities that in each of the two measurements at (x_r,t_r) and (x_l,t_l) a $\overline{K^0}$ meson is detected (Y) or not detected (N). We denote such probabilities by $P(Y,t_l;Y,t_r)$, $P(Y,t_l;N,t_r)$, $P(N,t_l;Y,t_r)$ and $P(N,t_l;N,t_r)$, with obvious meaning of the symbols. Typically, e.g.

$$P(Y,t_l;N,t_r) = \|P_l(\overline{K^0})Q_r(\overline{K^0})U_l(t_l,0)U_r(t_r,0)|\Psi(t=0)\rangle\|^2 \qquad (2.11)$$

It has to be remarked that in the evaluation of such probabilities the scalar product of the decay states $|\Omega_L(t)\rangle$ and $|\Omega_S(t)\rangle$ appears. As is well known, such states are not orthogonal when CP-invariance does not hold. One has:

$$\langle\Omega_L(t)|\Omega_S(t)\rangle = \langle K_L|K_S\rangle[1-e^{i\Delta m t}e^{-(\gamma_L+\gamma_S)t/2}] \qquad (2.12)$$

The modulus ε of the quantity $\langle K_L|K_S\rangle$ can be taken as a measure of CP-violation and it turns out to be smaller than 10^{-2}. As a consequence the computations of the following section, as well as the conclusions we will draw in it, are easily seen not to be affected in any significant way by making the approximation $\varepsilon=0$.

3. BELL'S INEQUALITIES FOR THE K^0, $\overline{K^0}$ SYSTEM

3.1. Introductory Considerations.

The problem of testing the predictions of quantum mechanics against those of any local deterministic hidden variable theory presents some analogies but also significant practical and conceptual differences with respect to the corresponding problem in the case of spin variables. The differences derive from two specific features. First, while in the spin case one can devise a test to check whether a spin 1/2-particle is or it is not in any chosen spin state $a|z+\rangle+b|z-\rangle$ (which corresponds to the spin being "up" in an appropriate direction), there is no analogous way to test whether the system is in the linear superposition $a|K^0\rangle+b|\overline{K^0}\rangle$. Actually, what one can do is to identify the strangeness of the mesons at a given time. In particular, since in the literature reference is always made to detection of $\overline{K^0}$ mesons, we will take into account probabilities of the kind of the one considered in (2.11).

The second important difference derives from the fact that, while in the spin case the direct product space $H^{(1)}_{spin} \otimes H^{(2)}_{spin}$ is sufficient to account for all spin properties of the system, in the case under consideration the state vector acquires by evolution components on the manifold of the decay products, orthogonal to the $H^{(l)}_K \otimes H^{(r)}_K$ space. As a consequence, the norm of the component on such a space decreases with time and this fact, through a subtle interplay of the damping and of the strangeness oscillations, makes it more difficult or even impossible to test the theory against local hidden variable models.

On the other hand, there are analogies between the two cases which deserve to be stressed. The most direct way to illustrate this point is to consider only the strangeness oscillations, disregarding the meson decays. Such oscillations can be described by a Hermitian Hamiltonian which is obtained from the H_W of section 2 by putting $\gamma_S=\gamma_L=0$. In this approximation the quantum probabilities considered in the previous subsection become

$$P(Y,t_l;Y,t_r)=P(N,t_l;N,t_r)=\frac{1}{4}[1-\cos\theta_{lr}]$$
$$P(Y,t_l;N,t_r)=P(N,t_l;Y,t_r)=\frac{1}{4}[1+\cos\theta_{lr}] \qquad (3.1)$$

where

$$\theta_{lr}=(m_L-m_S)(t_l-t_r) \qquad (3.2)$$

We remark that, in the case of the singlet state, the probability of finding, in a simultaneous spin measurement of the two particles, spins "up" (↑) or "down" (↓) along two chosen directions **a** and **b** is given by

$$P(\mathbf{a},\uparrow;\mathbf{b},\uparrow)=P(\mathbf{a},\downarrow;\mathbf{b},\downarrow)=\frac{1}{4}[1-\cos\theta_{ab}]$$
$$P(\mathbf{a},\uparrow;\mathbf{b},\downarrow)=P(\mathbf{a},\downarrow;\mathbf{b},\uparrow)=\frac{1}{4}[1+\cos\theta_{ab}] \qquad (3.3)$$

Comparing these equations one sees that the analogue of a spin correlation measurement along two arbitrary directions at the same time is the detection of $\overline{K^0}$ mesons at appropriate different times.

Obviously eq.(3.1) does not describe the actual situation; the true probabilities are damped due to the presence of the decay channels.

3.2. Implications of the Locality Requirement

As discussed in Section 1.6, in the case of spin variables one can derive the inequality (1.7) for the averaged values (1.3) of spin correlations along arbitrary directions **a** and **b**. The analogue of the free choice of the spin directions is, in the present case, the free choice of the times at which measurements aimed to detect $\overline{K^0}$ mesons are performed at the left and at the right, respectively. Let us consider four times, the first two, t_1 and t_2, referring to measurements performed on the particle at the left (at points x_1 and x_2, respectively) the other two, t_3 and t_4, referring to measurements performed on the particle at the right (at points x_3 and x_4, respectively). The times are chosen in such a way that each event at the left, e.g. (x_1,t_1), is space-like with respect to each event at the right, e.g. (x_3,t_3), and so on.

The locality assumption requires then that the results at one side be completely independent of the time at which the measurement at the other side is performed. To define the appropriate correlation functions to be used in Bell's inequality, one considers an observable $O(t_i)$, which assumes the value +1 if in the measurement at (x_i,t_i) a $\overline{K^0}$ meson is detected and the value -1 in the opposite case. In terms of such an observable we can define the correlation function $O(t_i,t_j)$, i=1,2, j=3,4, which takes the value +1 both when two or when no $\overline{K^0}$ mesons have been found in the two measurements at (x_i,t_i) and at (x_j,t_j), and the value -1 if only one meson has been found, no matter at which side. The locality assumption implies then that $O(t_i,t_j)$, in a specific individual experiment, equals the product of $O(t_i)$ and $O(t_j)$:

$$O(t_i,t_j)=O(t_i)\cdot O(t_j) \tag{3.4}$$

From this equation one derives immediately

$$|O(t_1,t_3)-O(t_1,t_4)|+|O(t_2,t_3)+O(t_2,t_4)|=2 \tag{3.5}$$

Let us consider now a sequence of N identical measurements, and let us denote by O_n the value taken by O in the n-th experiment. The average

$$M(t_i,t_j)=\frac{1}{N}\sum_1^N O_n(t_i,t_j) \tag{3.6}$$

satisfies then

$$|M(t_1,t_3)-M(t_1,t_4)|+|M(t_2,t_3)+M(t_2,t_4)|\leq \frac{1}{N}\sum_1^N \{|O_n(t_1,t_3)-O_n(t_1,t_4)|+|O_n(t_2,t_3)+O_n(t_2,t_4)|\}=2 \tag{3.7}$$

Equation (3.7) is the Bell inequality for the considered case*. It has to be remarked that, even though it has been derived having in mind a deterministic local hidden variable model, it can also be proved to follow from different assumptions. For example one can derive it [11,12] by counterfactual arguments making use only of locality requirements about the records of macroscopic apparatuses.

3.3. The Quantum Case

We are now in the position of comparing the predictions of quantum mechanics with those of local theories, and to investigate whether one can, as in the spin case, perform clear-cut experiments which prove that it is impossible to account for the Φ-decay into K mesons in terms of a local theory. To this purpose one has to substitute in eq.(3.7) the quantum expressions for the quantities $M(t_i,t_j)$ and to study whether one can violate eq.(3.7) by an appropriate choice of the times t_1, t_2, t_3 and t_4 appearing in it. To this purpose we observe that $O(t_i,t_j)$, in the quantum case, is an observable whose value is determined by the outcome of the measurement aimed to detect a $\overline{K^0}$ meson at time t_i at left and at time t_j at right. In the case in which two or no such mesons are detected, $O(t_i,t_j)$ takes the value +1, in the opposite case it takes the value -1. The quantum mean value $M(t_i,t_j)$ of such an observable is then the sum of the statistical frequencies of the results YY, NN minus the sum of the statistical frequencies of the results YN and NY. As the number of measurements becomes very large the statistical frequencies become the quantum probabilities, so that

$$M(t_i,t_j)=P(Y,t_i;Y,t_j)+P(N,t_i;N,t_j)-P(Y,t_i;N,t_j)-P(N,t_i;Y,t_j) \tag{3.8}$$

* In a recent paper[9] it has been claimed that one could build a local hidden variable theory reproducing exactly the quantum mechanical predictions for the Φ-decay process. If this would be true the derivation of eq.(3.7) would have been useless. However, as remarked in ref.(10), the procedure of ref. (9) requires the consideration of negative probabilities.

One can simplify the computation by remarking that the four probabilities appearing in eq.(3.8) sum up to 1, so that one has

$$M(t_i,t_j)=1-2\{P(Y,t_i;N,t_j)+P(N,t_i;Y,t_j)\} \tag{3.9}$$

One has then simply to evaluate the probabilities appearing in eq.(3.9) by using the procedure we have outlined in Section 2.3. A trivial but cumbersome calculation gives:

$$\begin{aligned} P(Y,t_i;N,t_j)=&\frac{1}{4(1+\varepsilon)}[e^{-\gamma_L t_i}+e^{-\gamma_S t_i}]+\frac{\varepsilon}{2(1+\varepsilon)}e^{-(\gamma_L+\gamma_S)t_i/2}\cos(\Delta m t_i) \\ &-\frac{1-\varepsilon}{8(1+\varepsilon)}\{e^{-(\gamma_L t_i+\gamma_S t_j)}+e^{-(\gamma_S t_i+\gamma_L t_j)}-2e^{-(\gamma_S+\gamma_L)(t_i+t_j)/2}\cos[\Delta m(t_i-t_j)]\} \end{aligned} \tag{3.10}$$

The probability $P(N,t_i;Y,t_j)$ is obtained from (3.10) by interchanging t_i with t_j. Substitution of (3.9) and (3.10) into (3.7) allows one to check whether one can get a violation of Bell's inequality in the case under consideration. As already remarked at the end of Section 2, the contribution to $M(t_i,t_j)$ deriving from CP nonconservation is extremely small, and turns out to be totally insignificant for what concerns the possibility of putting into evidence a violation of Bell's inequality. We are then allowed to assume $\varepsilon=0$ in the above formulae. From eq.(3.10) one sees that, if $\gamma_S t_k$ (k=i,j) is appreciably larger than 1, the terms describing the strangeness oscillations disappear. As a consequence the times which have to be considered must satisfy $\gamma_S t_k \leq 1$.Since $\gamma_S \cong 582\ \gamma_L$, one can then put in expression (3.10) $\gamma_L t_k \cong 0$. With these approximations, substitution of (3.10) and (3.9) in (3.7) gives:

$$\begin{aligned} &|e^{-\gamma_S(t_1+t_3)/2}\cos[\Delta m(t_1-t_3)]-e^{-\gamma_S(t_1+t_4)/2}\cos[\Delta m(t_1-t_4)]| \\ &+|e^{-\gamma_S(t_2+t_3)/2}\cos[\Delta m(t_2-t_3)]+e^{-\gamma_S(t_2+t_4)/2}\cos[\Delta m(t_2-t_4)]|\leq 2 \end{aligned} \tag{3.11}$$

A detailed numerical study of eq.(3.11) shows that, due to the specific values of the parameters γ_S and Δm, there is no possibility of choosing the four times appearing in it in such a way that the inequality be violated# . One way of making plausible this fact, which can be proved to hold completely in general, is the following. Let us choose values for the four times appearing in eq.(3.11) such that, if the exponential factors appearing in it would all be equal to 1 one would get the maximum possible violation of the inequality, which, as well known, occurs when the left hand side takes the value $2\sqrt{2}$. The considered precise choice of the times makes then definite the values of the exponentials. In such a case one gets for the left hand side a value which is much smaller than 2. The same kind of considerations show that (3.11) could be violated if $|\Delta m|$ would be appreciably larger than $\gamma_S + \gamma_L$. So, it is a fact about the specific properties of the K-meson system that inequality (3.11) turns out not to be violated by any choice of the times appearing in it. The experimental exclusion of a local deterministic account of the process under consideration cannot therefore be obtained.

4. OTHER SUGGESTIONS FOR TESTING QUANTUM MECHANICS

4.1. Possibility of Testing the Spontaneous Factorization Hypothesis

As we remarked in Section 1.7, the hypothesis that the state vector of a composite system spontaneously factorizes can, in principle, be experimentally tested against quantum predictions. However, some important remarks concerning such an assumption are necessary.

It is useful to remark that eq. (3.11) holds only under the specific assumption $\gamma_L t_k \cong 0$. If one does not take this into account one could be mislead by the fact that the left hand side of the above equation vanishes for large times. This cannot be, since for such times $P(N,t_i;N,t_j)$ tends to 1 and, as a consequence, the left hand side of (3.7) actually tends to 2. However, it is easy to see that such a limit is reached from below. This could have been expected: the strangeness oscillations are the specific features of the process which might lead (recall the analogy with the spin case) to a violation of Bell's inequality. When they are drastically suppressed the inequality is satisfied.

i)- As it has been shown long ago [13], such a hypothesis cannot be simply fitted within the quantum formalism. In fact one can prove with great generality that such an assumption entails that different (from the point of view of their composition in pure subensembles) statistical ensembles which correspond to the same statistical operator (and as such are physically indistinguishable within the quantum formalism) would evolve, as a consequence of the spontaneous factorization mechanism, into statistical ensembles described by different statistical operators, and as such they would become physically distinguishable. Thus, one cannot even describe the process in the statistical operator formalism. This argument is, in our opinion, sufficient to make the assumption of spontaneous factorization untenable.

ii)- Even if one disregards the above important remark, the simple assumption of spontaneous factorization, if one does not make a specific choice for the factorization mechanism, leads only to Bell's inequality, and therefore to test it against quantum mechanics is equivalent to test the general requirement of locality we discussed in the previous Section.

Obviously, the situation is radically different if one makes precise the factorization mechanism. To understand this one can make reference to the spin singlet case repeatedly considered above. If one assumes that such a state spontaneously factorizes e.g. to the statistical mixture with equal weights of the factorized states $|1+\rangle|2-\rangle$ and $|1-\rangle|2+\rangle$ then, obviously, measuring the spin components of the two particles along the x-axis will give in 50% of the cases the same result, an occurrence which is strictly forbidden by quantum mechanics.

A particular mechanism of this type for the $K^0, \overline{K^0}$ entangled state (2.1) has been discussed by Six [14]. He considers the assumption that the state (2.1) spontaneously factorizes, in a very short time after the Φ-decay, to the equal weights mixture of the states $|K_L\rangle_l |K_S\rangle_r$ and $|K_S\rangle_l |K_L\rangle_r$. In such a case it is easy to see that a measurement of the type of those considered in the previous Section, aimed to detect $\overline{K^0}$ -mesons at

times t_l and t_r, allows one, in principle, to discriminate between quantum mechanics and the dynamics leading to the spontaneous factorization.

To see this one has simply to follow the lines of Section 2.3. The quantum mechanical probability $P(Y,t_l;Y,t_r)$, when the approximation of CP invariance is made, turns out to be

$$P(Y,t_l;Y,t_r)=\frac{1}{8}\{e^{-\gamma_L t_l-\gamma_S t_r}+e^{-\gamma_S t_l-\gamma_L t_r}-2e^{-(\gamma_S+\gamma_L)(t_l+t_r)}\cos[\Delta m(t_l-t_r)]\} \qquad (4.1)$$

while the corresponding quantity using the above factorization hypothesis is

$$P_F(Y,t_l;Y,t_r)=\frac{1}{8}\{e^{-\gamma_L t_l-\gamma_S t_r}+e^{-\gamma_S t_l-\gamma_L t_r}\} \qquad (4.2)$$

The two expressions differ by the interference term containing the damped oscillations. As discussed in ref.(15), such a difference seems to be experimentally detectable provided one has a machine with a luminosity of the order of 10^{32} cm^{-2} sec^{-1}.

However, we want to stress that even though we have considered it appropriate to discuss the above matter for completeness, in view of the remarks under i), we do not believe it worthwhile to follow this line of thought. Before the spontaneous factorization hypothesis could be taken so seriously that it could be thought worthwhile to subject it to experimental tests, one should be able to give to it an acceptable formal and conceptual status. An attempt in this direction can be found in ref.(16).

4.2. On an Alleged Possibility of Testing Local Theories.

It has been claimed [17] that the requirements of local realism plus some essential invariance conditions allow the derivation of an inequality for the probability of finding two $\overline{K^0}$ mesons in the two measurements at times t_l and t_r which can be violated by quantum mechanics. This claim has to be discussed since, if correct, it could allow a refutation of local theories by experimental tests which differ from those considered in connection with Bell's inequality.

The argument of ref.(17) goes as follows. A general local realistic model is considered, the only restrictions which are imposed on it being:

i)- That it reproduces the quantum correlations for the state that evolves from the state (2.1) and, in particular, that it forbids two $\overline{K^0}$ mesons from being found in simultaneous measurements, i.e. at $t_l = t_r$.

ii)- That it satisfies CP-invariance, which in turn implies, due to the CPT theorem, T-invariance.

From these assumptions the author derives the following inequality for the above-mentioned probability:

$$P(Y,t_l;Y,t_r) \leq \frac{1}{8}\{e^{-\gamma_L t_l - \gamma_S t_r} + e^{-\gamma_S t_l - \gamma_L t_r}\} \tag{4.3}$$

Such an inequality has to be confronted with the quantum expression (4.1) for the same process; one sees that, for appropriate choices for the times t_l and t_r the probability (4.1) turns out to be greater than the limit (4.3).

It is easy to prove that the above argument is not correct, due to an inappropriate way of dealing with the requirement ii). We recall that in any realist model the mesons must be assumed to possess at all times the objective property of having a definite strangeness, i.e., of being either K^0 or $\overline{K^0}$. The author also assumes that the mesons decays are stochastic processes which are independent of the strangeness oscillations and remarks that the locality requirement, together with assumption i), imply that the strangeness oscillations are governed by a deterministic mechanism. To discuss the author's derivation of eq. (4.3) we make reference to the system of a single meson. According to the previous analysis, there must then be a hidden variable λ which characterizes completely its strangeness oscillations.

Within the quantum formalism the T-invariance requirement implies:

$$|\langle \overline{K^0}|U(t,0)|K^0\rangle|^2 = |\langle K^0|U(t,0)|\overline{K^0}\rangle|^2 \tag{4.4}$$

whose physical meaning is obvious: for a meson which is initially in the state $|K^0\rangle$ the probability that it is found in the state $|\overline{K^0}\rangle$ at time t must equal the probability that a meson which initially is in the state $|\overline{K^0}\rangle$ be found in the state $|K^0\rangle$ at the same time. We note that eq.(4.4) refers to probabilities which can be experimentally tested.

We discuss now the local realistic model and we consider an ensemble of systems corresponding to the quantum state $|K^0\rangle$ at time t=0. At time t the ensemble will turn out to be the union of two "hidden" subensembles $E(K^0,t)$ and $E(\overline{K^0},t)$ which contain only K^0 and $\overline{K^0}$ mesons, respectively. (We disregard the decays of the mesons since we have assumed, with the author, that they are indipendent of the strangeness oscillations). Let us denote by $\omega(t)$ the weight associated to $E(\overline{K^0},t)$. The author of ref. (17) makes an assumption which amounts to stating that $\omega(t)$ evolves according to

$$\frac{d\omega(t)}{dt} = -\gamma(t)\omega(t) + \delta(t)[1-\omega(t)] \tag{4.5}$$

and he claims that T-reversal invariance implies $\gamma(t)=\delta(t)$. This claim derives from a too restrictive way of satisfying the requirement (4.4); actually it amounts to requiring that the fraction $\gamma(t)dt$ of systems which, belonging to $E(\overline{K^0},t)$ jump to $E(K^0,t)$ in the time interval dt equals the fraction $\delta(t)dt$ of systems which, belonging to $E(K^0,t)$, jump, in the same time interval dt, to $E(\overline{K^0},t)$. However, $\gamma(t)$ and $\delta(t)$ are "hidden" rates, and as such they are not testable. The more general way to satisfy the probability condition (4.4) is to assume that the hidden variable probability densities associated to the initial states $|K^0\rangle$ and $|\overline{K^0}\rangle$ be equal:

$$\rho(|K^0\rangle,\lambda)=\rho(|\overline{K^0}\rangle,\lambda) \quad (4.6)$$

In a sense the author has transferred in a too straightforward way a requirement of the quantum theory for probabilities to a corresponding requirement for the hidden variable theory, in the same way in which von Neumann, in his proof of the impossibility of a deterministic completion of quantum mechanics, had transferred the requirement about mean values of sum of observables to the values determined by the assignment of the hidden variables themselves. This is not logically necessary. As a consequence one cannot legitimately use the argument of the author leading to the inequality (4.1).

5. COMMENTS AND CONCLUSIONS.

We have considered in this paper whether it is possible to perform experiments which would allow a clear-cut refutation of any local realistic completion of the quantum mechanical description of Φ-decay processes. Such a program would correspond to the one which has been developed in connection with spin properties of the constituents of a composite system in an entangled state and which has led to the conclusion that in such a case such a completion is actually impossible.

In the case under consideration we have seen that, due to the differences with the spin case, in particular due to the fact that we cannot devise apparatuses which would correspond to measurements of all observables in the K^0, $\overline{K^0}$ space, to the fact that mesons undergo decay processes in very short times and finally, to the specific physical properties of the considered systems, i.e. their mass differences and their lifetimes, it turns out to be impossible to perform *experimenta crucis* which would set up the question. One could then be tempted to consider this fact as indicating that, when fundamental properties of physical systems, such as their being particles of a definite

type, are involved, one is allowed to mantain that such properties are indeed possessed by the systems, independently of any "act of measurement" aimed to detect them.

We consider it appropriate to state clearly that this is not our position. In fact we want to stress that the case of Φ-decays is only one, among the innumerable ones occurring in nature in which superpositions of states occur for which the various terms of the linear combination refer to particles of different types. This is obvious if one takes into account that almost all particles are unstable and that quantum mechanics describes the dynamics of such systems in terms of the linear superposition of the unstable state and its decay products. The above conclusion would be legitimate only if one could prove that in all processes involving unstable particles a local completion of quantum mechanics could be found.

We can briefly summarize our point of view as follows. As we have seen, in the case under consideration there is no direct way to discriminate between quantum mechanics and local realistic models. But we keep the conviction that, as it has been proved for other variables like the spin ones, the quantum assumption that the wave function gives a complete description of physical processes is still the appropriate one for such microscopic systems. This amounts to say that we accept that the constituents of state (2.1) cannot be considered as having definite EPR's referring to them being K^0 or $\overline{K^0}$ mesons up to the moment in which a "measurement like procedure" takes place. Our position corresponds exactly to the one taken by J. Bell [18]: *"a fundamental physical theory would allow electrons to enjoy the cloudiness of waves, while allowing tables and chairs, and ourselves, and black marks on photographs, to be rather definitely in one place rather than another, and to be described in "classical terms"*. We stress that the expression "the cloudiness of waves" involves not being in a definite position, not having a definite spin component and then also (in our opinion) not being a K^0 or a $\overline{K^0}$.

To make even more clear what is our point of view we consider it appropriate to make reference to the clear attitude taken by D. Mermin[19] and to stress what we share of it and what we think should be modified. In presenting his beautiful and elegant derivation of Bell's inequality, Mermin starts by giving two quotations:

We often discussed his notions on objective reality. I recall that during one walk Einstein suddenly stopped, turned to me and asked whether I really believed that the moon exists only when I look at it. A. Pais[20].

*....one should no more rack one's brain about the problem of whether something one cannot know anything about exists all the same, than about the ancient question of how many angels are able to sit on the point of a needle. But it seems to me that Einstein's questions are ultimately always of this kind.*W. Pauli[21].

Mermin then adds his comments about the above statements: *Pauli and Einstein were both wrong. The questions with which Einstein attacked the quantum theory do have answers; but they are not the answers Einstein expected them to have. We now know that the moon is demonstrably not there when nobody looks.* Mermin was obviously making reference to the conceptual analysis of an EPR-Bohm situation with the singlet state. His sentence should then read: *We now know that in an entangled state of two electrons the individual electrons do not have any definite spin property when nobody (directly or indirectly) looks at them.* We share this point of view and we would add that, in spite of the fact that , as discussed in this paper, one cannot have in the case of the K-mesons such a conclusive experimental proof as the one we have for spin variables, the mesons in the entangled state do not have the property of being K^0 or $\overline{K^0}$ *when nobody looks*.

However, we do not share Mermin's position when one passes from the microscopic domain to the macroscopic one, in particular when microscopic processes trigger macroscopic changes (this is what we meant above by the expression "measurement like processes"). In particular we still consider unacceptable, with Einstein and Bell, that *the moon is not there when nobody looks*. This problem, i.e. the one of the

possible occurrence of linear superpositions of macroscopically distinguishable states is, in our as well as in many others opinion, the real crucial point of quantum mechanics.

We conclude by mentioning that in recent times, through the consideration of specific phenomenological models[22], it has been proved that one can devise theories which, even though allowing all peculiar, unreasonable and fuzzy behaviours and modes of being for microscopic systems that quantum mechanics has compelled us to accept, nevertheless *forbid the moon to be not there when nobody looks.*

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