INSTITUTE OF PLASMA PHYSICS
CZECHOSLOVAK ACADEMY OF SCIENCES

APPROXIMATE ANALYTIC THEORY OF THE MULTIJUNCTION GRILL

O. Huťťák, J. Preinhaelter

IPPCZ - 309

March 1991

RESEARCH REPORT

POD VODÁRENSKOU VĚŽÍ 4, 18069 PRAGUE 8
CZECHOSLOVAKIA

APPROXIMATE ANALYTIC THEORY OF THE MULTIJUNCTION GRILL

O. HUŘTÁK and J. PREINHAELTER

Institute of Plasma Physics

Czechoslovak Academy of Sciences

Pod vodárenskou věží 4, 182 11, Prague, Czechoslovakia

IPPCZ 309 MARCH 1991

Abstract

Approximate analytic theory of the general multijunction grill is developed. Omitting the evanescent modes in the subsidiary waveguides both at the junction and at the grill mouth and neglecting multiple wave reflection between the same places we derive simple formulae for the reflection coefficient, the amplitudes of the incident and reflected waves and the spectral power density. All these quantities are expressed through the basic grill parameters (the electric length of the structure and phase shift between adjacent waveguides) and two sets of the reflection coefficients describing wave reflections in the subsidiary waveguides at the junction and at the plasma. The approximate expressions for these coefficients are also given. The results are compared with numerical solution of two specific examples and they prove to be useful for the optimization and design of the multijunction grills. For the JET structure it is shown that, in the case of the dense plasma, many results can be obtained from the simple formulae for two-waveguide multijunction grill.

1. Introduction

At present much attention is paid to the investigation of supplementary non-inductive current drive in tokamaks. Usually the lower hybrid waves launched by phased waveguide structures called grills are used for this purpose.

The original theory of the wave radiation from the grills was given by BRAMBILLA (1976) and it is well suited for structures consisting of the independently fed waveguides. Later, a technically simpler launching structure named the multijunction grill was suggested by NGUYEN and MOREAU (1982). It is now frequently used as a building element of the large antenna arrays for the big contemporary tokamaks (GORMEZANO, 1985). The same authors gave the first theoretical description of these structures (MOREAU and NGUYEN, 1983-4). Later, PREINHAELTER (1989, 1990) used an improved version of the theory of the multijunction grill to optimize these structures for the lower current drive. The approximate analytical approach turned out also useful for practical explanation of the effect of the length and phasing of the multijunction grill on the power spectra, on the waveguide overloading and the current drive efficiency (see also HUŘŤÁK, 1990)

The present paper gives an outline of the analytical solution of the problem of the wave radiation from the multijunction grill into the plasma (Section 2). It generalizes previous results for an arbitrary N-waveguide multijunction grill and gives also some results for big arrays composed of them.

Similarly as in the numerical solution of this problem, we treat both waveguide discontinuities, i.e. the junction and grill mouth, separately. Neglecting the evanescent modes in the subsidiary waveguides (the main waveguide is supposed to be split into N subsidiary waveguides at the junction) we obtain the relations between the incident and reflected waves involving only two sets of the reflection coefficients: ρ_{ik} at the junction and R_{ik} at the grill mouth.

If we set the thickness of the dividers between the subsidiary waveguides equal to zero we get simple formulae for ρ_{ik}. For R_{ik} we can derive semi-analytical expressions (the coupling between the modes is given by an integral which can be evaluated only numerically) and it is better to determine them directly from the Brambilla's theory of the conventional grill. The tractable formulae for the amplitudes of the incident and reflected waves at the grill mouth can be derived only if we neglect the multiple reflections between junction and the grill mouth. The case N=2, however, can be solved without this limitation.

If the incident waves are predominantly reflected back to the same waveguide at the grill mouth (i.e. R_{ii}, i=1,2,...N are dominant) it is possible to neglect the coupling between the separate multijunction sections in large arrays and express the spectral power density in a simple form.

The preceding results are applied to two examples (Section 3). We use them to optimize the three waveguide multijunction grill which is mounted on the small tokamak CASTOR (BADALEC et al, 1988) and also to obtain an analytic expression for the spectral power density of the array installed at JET (LITAUDON and MOREAU, 1990).

The first example has a rather methodological importance because we mainly estimate the applicability limits of the analytic solution comparing it with the "exact" many-modes numerical solution.

The second example is more practical and it reveals that the JET arrays can be under certain conditions interpreted as 16 two-waveguide multijunction grills.

2. Analytic theory of multijunction grill

First we give the analytic solution of the problem of the wave diffraction on the split of the main waveguide into the N subsidiary waveguides. As it was stated, we suppose this discontinuity is at a sufficient distance from another one and the thickness of the dividers is equal to zero. If we also neglect the evanescent modes in the subsidiary waveguides, than, from the continuity conditions for the tangential components of the electric and magnetic field at the split (or at the junction from the subsidiary waveguides point of view), we obtain the following set of N equations for the amplitude B' of the reflected wave in the main waveguide (TE_{10} mode) and (N-1) amplitudes a_n of the evanescent modes in the same waveguide[1]

---

1 The schematic sketch of the multijunction grill, the derivation of the equations and the notation of the electromagnetic fields can be found in PREINHAELTER (1989).

$$B' + \sum_{n=1}^{N-1} C_n(p)a_n = B'_p \ , \ p=1,2,\ldots,N. \qquad (1)$$

The coefficients $C_n(p)$ can be expressed as

$$C_n(p)=\left[1-\frac{ik_x}{\gamma_n}\right]\frac{N}{n\pi}\sin\frac{n\pi}{2N}\cos\left[\frac{n\pi}{2N}(2p-1)\right], \qquad (2)$$

where $k_x=(k_v^2-(\pi/a)^2)^{\frac{1}{2}}$, $k_v=\omega/c$, $\gamma_n=[(n\pi/b)^2+(\pi/a)^2-k_v^2]^{\frac{1}{2}}$, a and b are height and width of the main waveguide, respectively. We also suppose that the width of the subsidiary waveguides $b_p=b/N$.

From the same continuity conditions we acquire the expression for the amplitudes A'_p of waves travelling in the separate subsidiary waveguides from the junction to the grill mouth

$$A'_p = A' + \sum_{n=1}^{N-1} C^*_n(p)a_n \ , \quad p=1,2,\ldots,N \qquad (3)$$

Taking into account that $\sum_{p=1}^{N} C_n(p)= 0$ we obtain from (1) for B' following expression

$$B' = \frac{1}{N}\sum_{p=1}^{N} B'_p \ . \qquad (4)$$

The reflected wave is formed by superposition of the waves reflected from a plasma into the separate subsidiary waveguides (we neglect the reflection from the faces of the dividers). This superposition is usually destructive and, therefore, the reflection coefficient of the multijunction grill is very small.

To determine a_n we can use several symmetries of the coefficients $C_n(p)$, e.g. $C_n(p) = (-1)^n C_n(N-p-1)$. The system

(1) can than be reduced to several independent sets which can be easily solved. Substituting α_n into (3) and regrouping the terms we have

$$A'_p = A' + \sum_{j=1}^{N} \rho_{pj} B'_j \, , \quad p=1,2,\ldots,N. \tag{5}$$

The amplitude reflection coefficients ρ_{pj} display two important symmetry relations

$$\begin{aligned} \rho_{pj} &= \rho_{jp}, \\ \rho_{pj} &= \rho_{(N-p-1)(N-j-1)}. \end{aligned} \tag{6}$$

It is possible to find out other simple linear relations between them thus the total number of independent coefficients is equal $N-1$. E.g. for $N=2$ we have

$$\rho_{11} = \rho_{22} = -\rho_{12} = -\rho_{21} = \rho = \frac{1}{2} \, \frac{\left(1 + \dfrac{ik_x}{\gamma_1} \right)}{\left(1 - \dfrac{ik_x}{\gamma_1} \right)} \, . \tag{7}$$

Thus it is clear that the waves reflected from the junction both back into the same waveguide and into the adjacent waveguide have identical amplitudes ($|\rho_{ii}|=\frac{1}{2}$) and are in antiphase.

The reflection coefficients ρ_{pj} for the case $N=3$ are given in Appendix and the case $N=4$ was studied by PREINHAELTER (1989). While deriving these expressions for $N\geq 3$, it is worth using some system of algebraic manipulation as e.g. REDUCE (HEARN, 1985).

We assume that the electric field at the grill mouth is expressed as

$$E_z = \sum_{p=1}^{N} \theta_p(z)\exp(i\phi_p - \omega t)\left[A_p e^{ik_v x} + B_p e^{-ik_v x} + \text{evan. modes}\right], \quad (8)$$

where A_p and B_p are the amplitudes (complex) of incident and reflected waves in the p-th waveguide at the grill mouth, respectively. In the p-th waveguide mouth, $\theta_p(z)=1$ and elsewhere $\theta_p(z)=0$. We choose the same coordinate system as in PREINHAELTER (1989). The phase $\phi_p = \phi_0 + (p-1)\Delta\phi$, where $\Delta\phi$ is the phase shift between adjacent waveguides and ϕ_0 corresponds to the electric length of the first subsidiary waveguide in the multijunction grill.

To determine the relations between the amplitudes of the incident and reflected waves at the grill mouth we apply the theory of the standard grill (BRAMBILLA, 1976). It gives

$$B_p = \sum_{k=1}^{N} R_{pk} \exp[i(k-p)\Delta\phi] A_k \ , \quad (9)$$

where R_{pk} is the amplitude reflection coefficient of the wave incident upon a plasma in the k-th waveguide and reflected back to the p-th waveguide. The coefficients R_{pk} fulfill the same symmetry rules as ρ_{pk} , i.e. $R_{pk} = R_{kp}$ and $R_{pk} = R_{(N-p+1)(N-k+1)}$.

The coefficients R_{pk} can be determined from a numerical solution of the Brambilla's equations or, if we neglect the evanescent modes in the waveguide mouths, we can obtain the analytic expressions for them. E.g. the set of equations for two-grill (N=2) has the form (BARANOV and SHCHERBININ, 1977, HURŤÁK and PREINHAELTER, 1989)

$$\begin{aligned} B_1(K_1 + b_1') + e^{i\Delta\phi} B_2 K_2 &= -A_1(K_1 - b_1') - A_2 e^{i\Delta\phi} K_2 \ , \\ B_1 K_2 + e^{i\Delta\phi} B_2(K_2 + b_2') &= -A_1 K_2 - A_2 e^{i\Delta\phi} (K_1 - b_2') \ , \end{aligned} \quad (10)$$

where $b'_l = k_v b_l$. The coupling elements $K_1 = K_{00}(1,1)$ and $K_2 = K_{00}(1,2)$, correspond to the coupling of the fundamental mode in the first waveguide with itself and with the fundamental mode in the second waveguide, respectively. They must be calculated by the numerical integration. E.g. $K_{00}(1,1)$ is given by

$$K_{00}(1,1) = \frac{1}{\pi}\int_0^\infty \frac{dN_z(1-\cos(N_z b'_1))(1-r)}{N_z^2 \; N_\perp(1+r)} , \qquad (11)$$

where $r(N_z)$ is amplitude reflection coefficient of waves ($\sim\exp(ik_z z-i\omega t)$, $N_z=k_z/k_v$, $N_\perp^2=1-N_z^2$) from a plasma and it depends on the plasma surface impedance $\rho(N_z)$. If the plasma density profile is linear $\rho(N_z)$ can be expressed with the help of the Airy functions (GOLANT,1971, HURTÁK and PREINHAELTER, 1989). If we compare the solution of (10) with (9) we acquire

$$R_{11} = R_{22} = \frac{K_2^2 + b_1'^2 - K_1^2}{D} , \quad R_{12} = R_{21} = -\frac{2b'_1 K_2}{D} , \qquad (12)$$

where $D=(K_1+b'_1)^2-K_2^2$ and we suppose $b_1 = b_2$. For N=3 the expressions for R_{pk} are rather complicated and can be found in Appendix.

Now we must match the solution at the junction with the solution at the grill mouth i.e. find out the relation between the primed and unprimed amplitudes. The phase of wave increases by ϕ_p in the p-th waveguide when the wave travels a distance L_g between the junction and the grill mouth. If the first subsidiary waveguide has the same height throughout then $\phi_0 = k_x L_g$.

If we omit an unimportant constant factor $(k_x/(2k_v))^{\frac{1}{2}}$

ensuring the conservation of the wave energy flow at the transition from the rectangular waveguides (at junction) into the infinite parallel-plate waveguides (supposed in the Brambilla's theory) we obtain

$$A_p = A'_p \ , \qquad B_p = \exp(-2i\phi_p) B'_p \ . \tag{13}$$

Making use of these relations we get, from (5) and (9), the final set of the equations for the amplitudes of waves incident at the mouth of the multijunction grill

$$A_p = A' + e^{2i(\phi_0 - \Delta\phi)} \sum_{k,s=1}^{N} \rho_{ps} R_{sk} e^{i(s+k)\Delta\phi} A_k \ . \tag{14}$$

Usually it holds $|\rho R| < 1$ and we can solve the set (14) by the successive approximations. In the zero step we put $A_p = A'$ and obtain the solution of the corresponding conventional grill. In the next step we consider only waves reflected once from the plasma and from the junction. Then we have

$$A_p = A' \left\{ 1 + e^{2i(\phi_0 - \Delta\phi)} \sum_{k,s=1}^{N} \rho_{ps} R_{sk} e^{i(s+k)\Delta\phi} \right\}. \tag{15}$$

In this approximation the incident wave in each subsidiary waveguide is given by the superposition of the transmitted primary wave with the N^2 secondary, double-reflected waves. In Fig. 1 these waves are schematically depicted for the two-waveguide multijunction grill.

In the same approximation the amplitudes of the reflected waves are given

$$B_p = A' \sum_{k=1}^{N} R_{pk} e^{i(k+p)\Delta\phi} \Big\{ 1 + e^{2i(\phi_0 - \Delta\phi)} \sum_{q,s=1}^{N} \rho_{ks} R_{sq} e^{i(s+q)\Delta\phi} \Big\}. \tag{16}$$

Neglecting terms proportional to ρR in the bracket on the right hand side of (16) we obtain the amplitudes of the reflected waves in the conventional grill.

If we use (13) and (16) we can derive, from (4), the formula for the amplitude of the reflected wave in the main waveguide

$$B' = \frac{A'}{N} e^{2i(\phi_0 - \Delta\phi)} \sum_{p,k=1}^{N} R_{pk} e^{i(k+p)\Delta\phi} \times \times \Big\{ 1 + e^{2i(\phi_0 - \Delta\phi)} \sum_{q,s=1}^{N} \rho_{ks} R_{sq} e^{i(s+q)\Delta\phi} \Big\}. \tag{17}$$

The contribution from the multiple reflections in (17) can be omitted as it is usually comparable with the neglected reflections from the faces of the waveguide dividers.

To estimate the effect of the multiple reflections of waves we bring up the exact solution of the set (14) for the two-grill (N=2). The expressions are relatively simple and for A_p we obtain

$$A_{1,2} = A' \Big\{ 1 \mp \frac{2ie^{i(2\phi_0 + \Delta\phi)} R_{11} \rho_{11} \sin\Delta\phi}{1-q} \Big\}, \tag{18}$$

where $q = 2\rho_{11}(R_{11}\cos\Delta\phi - R_{12})e^{i(2\phi_0 + \Delta\phi)}$. If we set q=0 in (18) we consider only the double reflection. It is seen that the contributions from multiple reflections forms the geometrical series with the quotient q.

The amplitudes of the reflected waves for N=2 are

$$B_1 = \frac{A'}{1-q}\left[R_{11} + R_{12}e^{i\Delta\phi} + 2\rho_{11}(R_{12}^2 - R_{11}^2)e^{2i(\phi_0+\Delta\phi)}\right],$$
$$B_2 = \frac{A'}{1-q}\left[R_{11} + R_{12}e^{-i\Delta\phi} + 2\rho_{11}(R_{12}^2 - R_{11}^2)e^{2i\phi_0}\right]. \qquad (19)$$

Finally, for the amplitude of the reflected wave in the main waveguide we have

$$B' = \frac{A'}{1-q}\, e^{i(2\phi_0+\Delta\phi)} \times$$
$$\times\left[R_{11}\cos\Delta\phi + R_{12} + 2\rho_{11}(R_{12}^2 - R_{11}^2)e^{i(2\phi_0+\Delta\phi)}\right]. \qquad (20)$$

3. Comparison of analytic and numerical solutions

As an example we present here the results of the optimization of the three-waveguide multijunction grill installed at small tokamak CASTOR[2]. This grill works at a frequency of 1.25 GHz and it is used in the experiments investigating the lower hybrid current drive (BADALEC et al. ,1988).

The geometrical dimensions are: the main waveguide height a=16cm, its width b=4.6cm, the width of the subsidiary waveguides b_p=1.4cm (p=1,2,3), the thickness of the dividers d_p=0.2cm and the length of the structure L_g=95cm (it corresponds to ϕ_0=45°). The measured plasma density in front of grill and its gradient are n_0=30n_{crit} (n_{crit}=2×10^{10}cm^{3}) and dn_0/dx=8×10^{11}cm^{-4} ,respectively.

---

[2] Three multijunction grills were used at this tokamak: the four-waveguide grill which was optimized by PREINHAELTER (1989), the three-waveguide grill described here and seven-waveguide grill. All are described by ŽÁČEK et al. (1990).

The numerical results are obtained by the solution of the full set of equations as it is described by PREINHAELTER (1989). We choose 7 modes in each subsidiary waveguide and, at the junction, we add one mode on each divider face. Thus in the main waveguide we take 23 modes. These results are then compared with those following from the approximate formulae in Appendix.

In Fig. 2 we compare the powers travelling to the grill mouth in the successive subsidiary waveguides P_p^+ $(\sim|A_p|^2 b_p)$ computed numerically with the same powers determined on the bases of the analytic formulae. The dependence of P_p^+ on the phase shift $\Delta\phi$ is demonstrated here. If we use the approximate values of ρ_{pk} and R_{pk} the full agreement of the numerical and analytical solutions cannot be reached even if we take into account the multiple wave reflections between the junction and the grill mouth. If we take, for ρ_{pk} and R_{pk}, the precise values given by the full numerical treatment of the problem and if we consider the multiple wave reflections the agreement is good (see Fig. 3).

In Fig. 4-6 we depicted the dependence of the powers travelling back to the junction and the phases of the waves incident on and reflected from the plasma on the phase shift $\Delta\phi$. The large differences between the numerical and analytical solutions in Fig. 6 arise for those $\Delta\phi$ for which the powers P_p^- are practically zero and the phases of the reflected waves are undetermined.

Roughly speaking, the approximate expressions for A_p and B_p give a good qualitative picture of the dependence of these quantities on $\Delta\phi$ and ϕ_0 (the dependence on ϕ_0 is less pronounced and the corresponding figures are not given here). A better agreement is obtained by the expressions in which the multiple reflections are included, namely if the exact values of ρ_{pk} and R_{pk} are taken. In the last case the results are practically undistinguishable.

The reflection coefficient of this grill is extremely small for a broad region of the phase shifts round $\Delta\phi=90^{\circ}$. It is confirmed by Fig. 7. It is also seen here that the expression (A4) gives good estimate of R_t. The inclusion of the triple reflected waves in R_t gives the same results as the inclusion of the multiple reflections and the difference between the numerical and analytical solutions is given mainly by the unprecise values of R_{pk}.

The effect of the evanescent modes on A_p, B_p and R_t manifest itself only through the values of ρ_{pk} and R_{pk}. The shape of the spectrum, however, depends strongly on the number of the evanescent modes which we consider at the grill mouth. We demonstrate this in Fig. 8 where the spectra of our structure are given for different numbers of these modes. If we consider only the fundamental mode (the curve with the longest dashes in Fig. 8) we obtain only a crude estimate of the spectrum. The full line correspond to the 'exact' solution based on the fundamental and 6 evanescent modes. The parameters of the structure ($\Delta\phi=126^{\circ}$ and $\phi_0=45^{\circ}$) were chosen so that the spectrum contains a prominent peak for $N_z>1$. At the junction we take always proper number of the evanescent modes to obtain 'exact' solution. It is important that the position of the peaks in the spectrum is approximately given by the fundamental mode alone. To determine the heights of peaks precisely we need the solution with at least two evanescent modes.

The grill which was used in the experiments had $\Delta\phi=90^{\circ}$ and $\phi_0=45^{\circ}$. Its spectrum is shown in Fig. 9 together withthe spectra for $\phi_0=90^{\circ}$ and $\phi_0=162^{\circ}$.

As a second example we show that, in some cases, the analytical results for the two-waveguide multijunction grill can be applied to the JET lower hybrid antenna (LITAUDON and MOREAU, 1990).

This grill is composed of eight multijunction sections in one row and each section consists of four waveguides. The main waveguide of the section is narrowed at the junction and the central dividing plate between the second and third waveguides is extended so that the excitation of the higher modes is suppressed. Thus the structure consists of 16 two-waveguide multijunction grills which can have, for $\Delta\phi=90^{\circ}$, very small coupling at the corresponding central dividers.

This was verified for $\delta\phi_{4v}=0^{\circ}$ (the phase shift between the four-waveguide sections). In this case the reflection coefficient of the structure is very small for all ϕ_0 (it corresponds to l_1 in (LITAUDON and MOREAU, 1990)) and the spectra computed for the structure consisting of 16 two-waveguide multijunction grills ($\delta\phi_{2v}=180^{\circ}$, $n_0=5n_{crit}$, $d/dx(\ln n_0)=1\ \mathrm{cm}^{-1}$, $f=3.7$ GHz) coincide exactly with the spectra of the real JET structure. The dependence of the spectra on ϕ_0 is very weak in this case.

Similar results can be obtained also for $\delta\phi_{4v}=-90^{\circ}$ and $\phi_0>90^{\circ}$, where the reflection coefficient is also small.

An interesting situation arises when we investigate the radiation of the JET structure into the dense plasma. In this case the reflections of waves back into the same waveguide plays the most important role, at least, in the central waveguides. E.g. for n_0=20n_{crit} and the other parameters being the same as in the preceding case we have $R_{16,16}$=-0.4-0.02i, $R_{17,16}$=$R_{15,16}$=-0.23-0.13i and $R_{18,16}$= $R_{14,16}$=-0.04-0.087i. Therefore, we can neglect coupling between remote waveguides ($|R_{n+2,n}|\ll|R_{n,n}|$)[3]. If $\delta\phi_{4v}=0^\circ$ the waves reflected from a plasma into the adjacent waveguides cancel mutually (we suppose that $\Delta\phi=90^\circ$ all the time) and the structure behaves as 16 independent two-waveguide multijunction grills with $R_{12}\equiv 0$.

To determine approximately the amplitudes of the incident and reflected waves we can use the formulae (18) and (19) where we set R_{12}=0:

$$A_{1,2}=A'\left[\,1\ \pm\ 2e^{2i\phi_0}\ R_{11}\rho_{11}\right],\qquad B_{1,2}=\ R_{11}A_{1,2}. \qquad (21)$$

In Fig. 10 we depicted the powers travelling to the grill mouth in the subsidiary waveguides of the fourth section of the JET structure as functions of ϕ_0. We can see that the exact results for the first two-grill of the section (the 13th and 14th waveguides) coincide very well with that of the second two-grill (the 15th and 16th waveguides). It confirms their independence. It is also clear that the approximate values (dashed and dotted lines) based on (21)

3 In the rarefied plasma it is not true: e.g. for n_0=5n_{crit} and the same other parameters $R_{16,16}$=-0.106-0.075i, $R_{17,16}$=-0.28-0.175i and $R_{18,16}$=-0.048-0.147i.

yield a good estimate of the real quantities. A better agreement can be achieved if $R_{16,16}$ is used instead of R_{11} in (21) (in Fig.10 it corresponds to the dotted lines). If we use the approximate value R_{11}= −0.44−0.17i, given by (12)[4], the agreement with the exact solution is worse. In (21) we use the value ρ_{11}=0.32+0.38i, given by (7) (a=7.7cm, b=2cm) which slightly differs from ρ_{11}=0.4+0.33i, ρ_{12}=−0.35−0.33i resulting from the exact theory of the two-waveguide multijunction grill. The exact powers travelling to the junction are equal to a half of the expected value $|R_{11}|^2P_k^+$.

If we neglect the evanescent modes at the grill mouth we obtain the following expression for the spectrum (PREINHAELTER, 1989; HURTÁK, 1990) of 16 independent two-waveguide multijunction grills in array ($\Delta\phi$=90°, $\delta\phi_{2v}$=180°, R_{12}=0)

$$P(N_z)\sim 16|A'|^2\,\frac{\sin^2\left[16(\frac{1}{4}\pi-k_z(b_1+d_1))\right]}{\sin\left[\frac{1}{4}\pi-k_z(b_1+d_1)\right]}\,\frac{\sin^2(\frac{1}{2}k_zb_1)}{N_z^2}\,|1+R_{11}|^2\times$$

$$\left|\cos\left[\tfrac{1}{2}(k_z(b_1+d_1))-\tfrac{1}{4}\pi\right]-2i\rho_{11}R_{11}e^{2i\phi_0}\cos\left[\tfrac{1}{2}(k_z(b_1+d_1))+\tfrac{1}{4}\pi\right]\right|^2\times$$

$$\frac{\mathrm{Re}\,\rho_{plasma}}{|\rho_{plasma}|^2}, \qquad (22)$$

where $\rho_{plasma}(N_z)$ is the plasma surface impedance (BARANOV and SHCHERBININ, 1977). The spectrum practically does not

4 Here The values K_1=1.3+0.9i and K_2=0.1+0.87i are determined from (11) for n_0=20n_{crit}, d(ln n_0)/dx=1 cm^{-1}, b_1=0.9k_v and k_v= 0.775 cm^{-1}.

depend on ϕ_0 because, in the peaks where $\sin\left[\frac{1}{2}\pi-k_z(b_1+d_1)\right]=0$, one of the cos terms in the absolute value on the right hand side of (22) is always equal to zero.

Fig. 11 shows the spectrum (22) and Fig. 12 represents the same results of the exact numerical calculation of the JET structure. As it is to be expected the agreement is not very good. The omission of the short-wavelength evanescent modes leads to an increase of the the long-wavelength part of the spectrum (N_z=±1.8).

4. Conclusions

The approximate analytic theory, based on the omission of multiple mouth-junction reflections, provide tractable formulae for parameters of multijunction grill consisting of two or three waveguides. The comparison of analytic and numerical results show that these formulae express, at least qualitatively, the role of phase parameters $\Delta\phi$ and ϕ_0 and are useful in preliminary grill design. If the evanescent modes at the junction and the thickness of dividers are neglected, simple formulae for reflection coefficient ρ_{pk} at the junction are obtained. However, the neglect of evanescent modes at the grill mouth leads to a relatively worse approximation of reflection coefficients R_{pk}.

In special regimes, when the JET-launcher behaves as 16 independent juxtaposed two-waveguide multijunction sections, the formulae for two-waveguide multijunction grill can be used to estimate the power distribution among subsidiary waveguides. The one-mode approximation of the formula for the power spectrum yields precise positions of all peaks. However, the short-wavelength components of the spectrum are reduced as they are predominantly formed by evanescent modes in subsidiary waveguides.

Acknowledgements

A part of this work was done at the stay of the second author in the Joint Institute of Nuclear Research in Dubna and he wishes to thank that Institution for hospitality.

Appendix

The approximate expressions for the coefficients ρ_{pk} describing the reflection of waves from the junction in the three-waveguide grill can be derived from (1)-(3) and (5) in the following form

$$\rho_{11}=\rho_{33}=\frac{1}{2}\,\frac{\left(1+\dfrac{ik_x}{\gamma_1}\right)}{\left(1-\dfrac{ik_x}{\gamma_1}\right)}+\frac{1}{6}\,\frac{\left(1+\dfrac{ik_x}{\gamma_2}\right)}{\left(1-\dfrac{ik_x}{\gamma_2}\right)},$$

$$\rho_{12}=\rho_{21}=\rho_{32}=\rho_{23}=-\frac{1}{3}\,\frac{\left(1+\dfrac{ik_x}{\gamma_2}\right)}{\left(1-\dfrac{ik_x}{\gamma_2}\right)}, \qquad \text{(A1)}$$

$$\rho_{13}=\rho_{31}=-\rho_{11}-\rho_{12}, \qquad \rho_{22}=-2\rho_{12}.$$

If we evaluate preceding formulae taking into account the geometrical parameters of the three-waveguide multijunction grill described in Sec. 3 we obtain ρ_{11}=0.6+0.29i (0.63+0.24i), ρ_{12}=-0.32-0.08i (-0.31-0.06i), ρ_{22}=0.65+0.17i (0.67+0.13i) and ρ_{13}=-0.27-0.2i (-0.28-0.17i). The values in the parentheses[5] follow from the numerical solution of our problem where 7 modes are used in each subsidiary waveguide and where we consider the finite thickness of the dividers. We see that the expressions (A1) gives the values which agree with the numerical solution well.

The coefficients R_{pk} describing the reflection of waves from a plasma have, for the three-waveguide grill, the form

$$
\begin{aligned}
R_{11} = R_{33} &= \frac{(b'_1+K_1)(K_3^2 + b'^2_1 - K_1^2) - 2K_2^2(K_1 + K_3)}{D}, \\
R_{12} = R_{21} = R_{23} = R_{32} &= -\frac{2b'_1 K_2}{D_1}, \\
R_{13} = R_{31} &= -\frac{2b'_1[K_2^2 - (b'_1+K_1)K_3]}{D}, \\
R_{22} &= \frac{(b'_1-K_1)(K_3 + b'_1 + K_1) + 2K_2^2}{D},
\end{aligned}
\qquad \text{(A2)}
$$

where $D=(K_1 + b'_1 - K_3)D_1$ and $D_1=(K_3 + b'_1 + K_1)(b'_1 + K_1)-2K_2^2$.

To compare (A2) with the numerical solution we need the values of the coupling elements of our structure. The numerical integration yields: K_1=0.63+0.27i, K_2=0.29+0.37i, K_3=0.06+0.35i. Because the assumption of the zero thickness of dividers brings about no simplification we consider here the real grill dimensions. The reflection coefficients have than the following values (in parentheses the values based

5 ____________

These values do not fulfill exactly the simple relations between the components of ρ_{kp} (e.g. $\rho_{22} \neq -2\rho_{12}$) which are true only for the approximate expressions where we assume the thickness of the dividers is equal to zero.

on the numerical solution are given): R_{11}=-0.27-0.03i (-0.22+0.05i), R_{12}=-0.28-0.13i (-0.31-0.18i), R_{13}=-0.05-0.1i (-0.07-0.15i), a R_{22}=-0.19+0.06i (-0.08+0.15i). The evanescent modes at the grill mouth are more prominent than those at the junction and thus the agreement between R_{pk} given by (A2) and the values determined fully numerically is worse than that for ρ_{pk}.

If N=3 the general expressions for A_p and B_p are very complicated[6] and we state here the formulae for A_p which include only the double reflected waves

$$A_1=A'\left\{1-e^{2i(\phi_0+\Delta\phi)}\left[i(2\rho_{11}+\rho_{12})R_{11}\sin 2\Delta\phi+\rho_{12}R_{11}\cos 2\Delta\phi+\right.\right.$$
$$\left.\left.+\rho_{12}(R_{13}-R_{22})+i(2\rho_{11}+\rho_{12})R_{12}\sin\Delta\phi-\rho_{12}R_{12}\cos\Delta\phi\right]\right\},$$
$$A_2=A'\left\{1+2\rho_{12}e^{2i(\phi_0+\Delta\phi)}\left[R_{11}\cos 2\Delta\phi+R_{13}-R_{22}-R_{12}\cos\Delta\phi\right]\right\}, \quad \text{(A3)}$$
$$A_3=A'\left\{1+e^{2i(\phi_0+\Delta\phi)}\left[i(2\rho_{11}+\rho_{12})R_{11}\sin 2\Delta\phi-\rho_{12}R_{11}\cos 2\Delta\phi-\right.\right.$$
$$\left.\left.-\rho_{12}(R_{13}-R_{22})+i(2\rho_{11}+\rho_{12})R_{12}\sin\Delta\phi+\rho_{12}R_{12}\cos\Delta\phi\right]\right\}.$$

Finally we give a useful and simple expression for the amplitude of the reflected wave in the main waveguide (in the zero approximation)

$$B'=A'e^{2i(\phi_0+\Delta\phi)}\left(2R_{11}\cos 2\Delta\phi+4R_{12}\cos\Delta\phi+2R_{13}+R_{22}\right)/3. \quad \text{(A4)}$$

It is seen that for $\Delta\phi=90^0$ the leading term drops out and thus the reflection of the three waveguide multijunction grill is than very small.

6 ______________________________

They can be handled by REDUCE and expressed in a compact form suitable for the numerical calculation.

References

BADALEC J. et al. (1988) Fiz. plazmy 14, 395.

BRAMBILLA M. (1976) Nucl. Fusion 16, 47.

GORMEZANO G. (1985), Application of R.F. Waves to Tokamak Plasma, Varena, 422.

HEARN A. C. (1985), REDUCE User Manual, Rand Corp., Santa Monica, CA.

HURŤÁK and PREINHAELTER (1989), Computation of coupling elements between waveguide modes for large launching structure, Rep. IPPCZ-290, Institute of Plasma Physics, Prague.

HURŤÁK (1990) Plasma Physics and Controlled Fusion 32, 623.

LITAUDON and MOREAU (1990) Nucl Fusion, 471.

MOREAU and NGUYEN (1983-4), Couplage de l'onde lente au voisinage de la fréquence hybride basse dans les grands tokamaks, Rep. EUR-CEA-FC-1246, Centre d'études nucléaires de Fontenay-aux-Roses.

NGUYEN and MOREAU (1982), in Fusion Technology (Proc. 12th Symp. Julich) 2, Pergamon Press, Oxford, 1381.

PREINHAELTER (1989) Nucl. Fusion 29, 1729.

PREINHAELTER (1990) Czech. J. Phys. B 40, 527

ŽÁČEK et al. (1990), LHCD experiments on tokamak CASTOR, Rep. IPPCZ-304, Institute of Plasma Physics, Prague, see also Proc. of 10th European School on Plasma Physics, Tbilisi, 1990.

Figure captions

Fig. 1 Schematic sketch of the two-waveguide multijunction grill. Four partial double reflected waves and the direct one, the superposition of which gives the incident wave in the first waveguide, are shown in the form of rays.

Fig. 2 Powers travelling to the grill mouth in the successive subsidiary waveguides normalized to the power incident in the main waveguide as functions of $\Delta\phi$. The electric length of the structure ϕ_0=45°, the dimensions of grill and the plasma parameters are given at the beginning of Sec. 3 . Full lines correspond to the numerical solution (7 modes). Dashed lines include only contributions from the double reflected waves to P_k^+. The multiple reflections are taken into account in dotted lines. In the last two cases used the approximate values of ρ_{pk} and R_{pk}, given by (A1) and (A2), are used.

Fig. 3 Same as Fig. 2 only for ρ_{pk} and R_{pk} we take the values derived from the numerical solution.

Fig. 4 Powers travelling to the junction in the successive subsidiary waveguides normalized to the power incident in the main waveguide as functions of $\Delta\phi$. For the rest see Fig. 2.

Fig. 5 Phases of the incident waves in the successive subsidiary waveguides as functions of $\Delta\phi$. For the rest see Fig. 2.

Fig. 6 Phases of the reflected waves in the successive subsidiary waveguides as functions of $\Delta\phi$. For the rest see Fig. 2.

Fig. 7 Reflection coefficients of the three-waveguide multijunction grill as a function of $\Delta\phi$. Dashed line corresponds to the single reflection formula (A4). Dotted line is based on the approximate formula where the multiple reflections are taken into account. In these both cases the approximate values for ρ_{pk} and R_{pk} are used. Full line is given by the numerical solution.

Fig. 8 Dependence of the power spectra on the number of the evanescent modes taken into account at the grill mouth. Full line corresponds to the numerical solution with 7 modes (a fundamental and 6 evanescent). The line with the longest dashes is given by the fundamental mode only. We add one or two evanescent modes to obtain the remaining two dashed lines.

Fig. 9 Power spectra of waves radiated from the three-waveguide multijunction grill into the plasma for $\Delta\phi=90^{\circ}$ computed numerically. Full line corresponds to $\phi_0=45^{\circ}$, dotted line to $\phi_0=90^{\circ}$, dashed line to $\phi_0=162^{\circ}$.

Fig. 10 Powers travelling to the grill mouth, normalized to the total power incident in the main waveguides in one row of array, in the 13th up to 16th waveguides of the JET structure as functions of ϕ_0 for $\Delta\phi=90^{\circ}$ and $\delta\phi_{4v}=0^{\circ}$. Plasma parameters are $n_0=20n_{crit}$ and $d/dx(\ln n_0)=1cm^{-1}$. Full line is given by the numerical solution, dashed and dotted lines follow from formula (21) with R_{11} given by (12) or with $R_{11}=R_{16,16}$.

Fig. 11 Power spectrum of the grill composed of sixteen two-waveguide multijunction grills computed according to (22) with ϕ_0=160°. Plasma parameters are the same as in Fig. 10.

Fig 12 Power spectrum of the JET grill for $\Delta\phi$=90°, ϕ_0=160°, $\delta\phi_{4v}$=0°. Plasma parameters are the same as in Fig. 10.

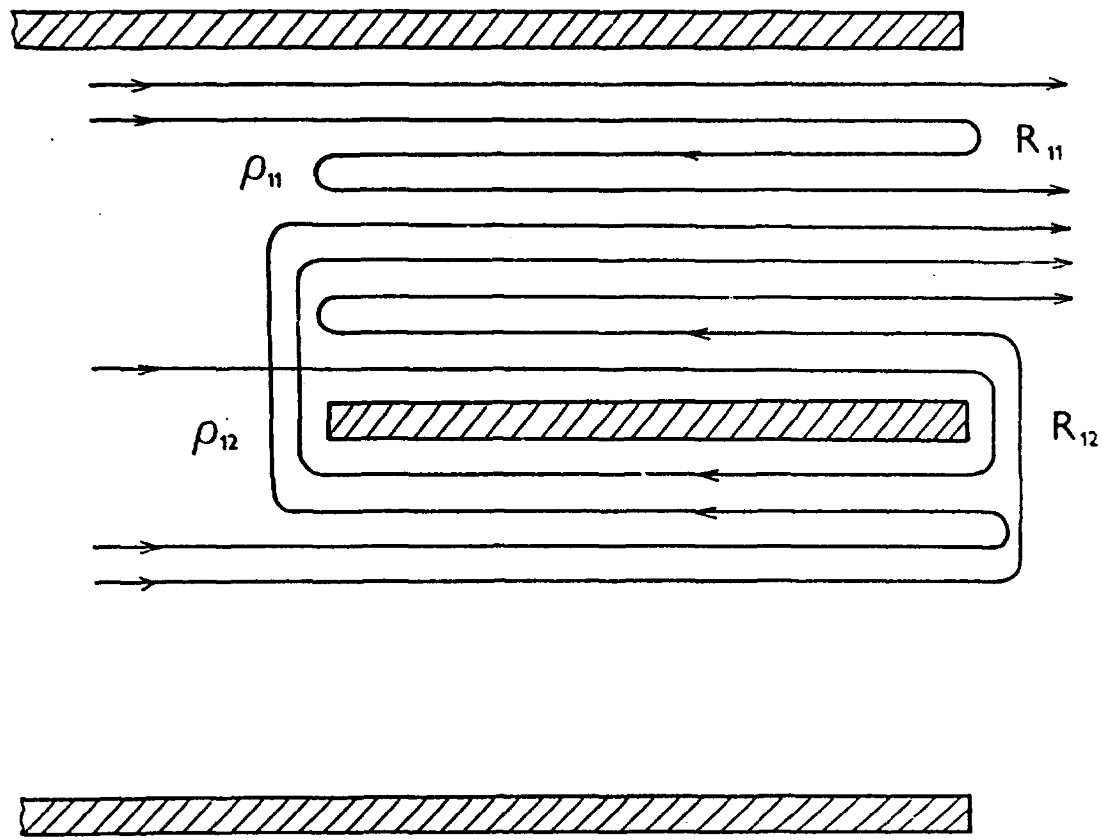


Fig.1

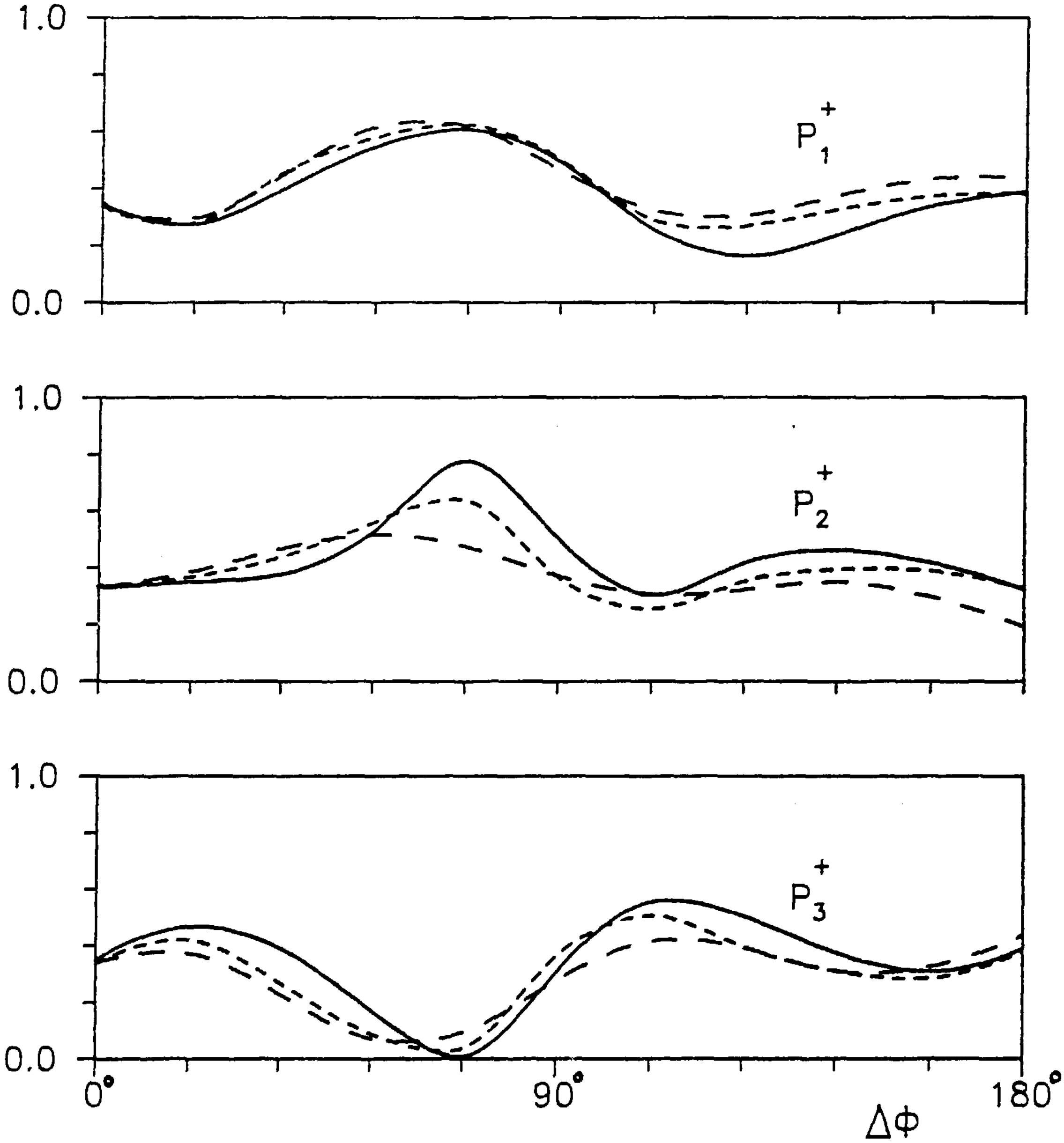


Fig.2

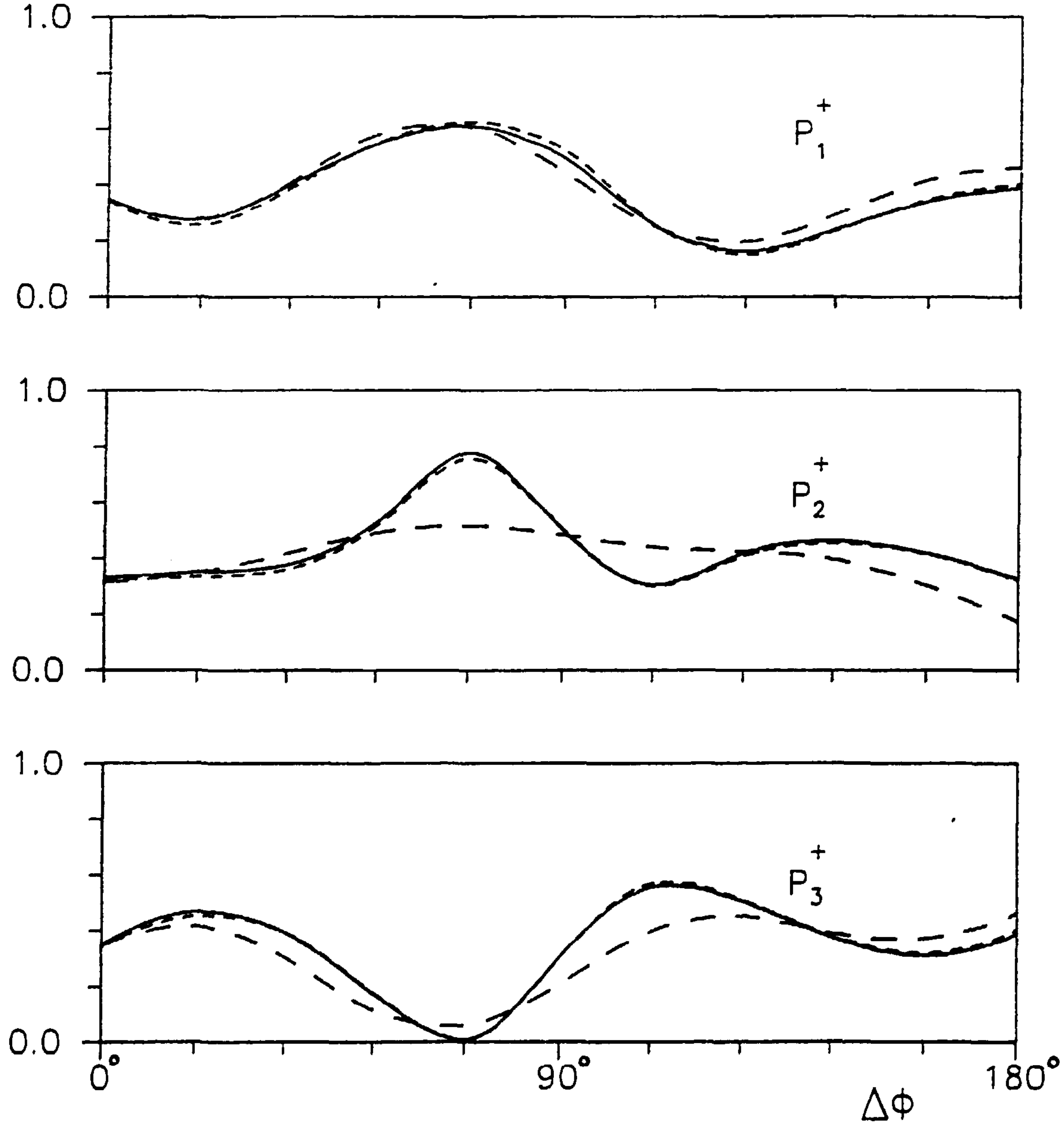


Fig.3

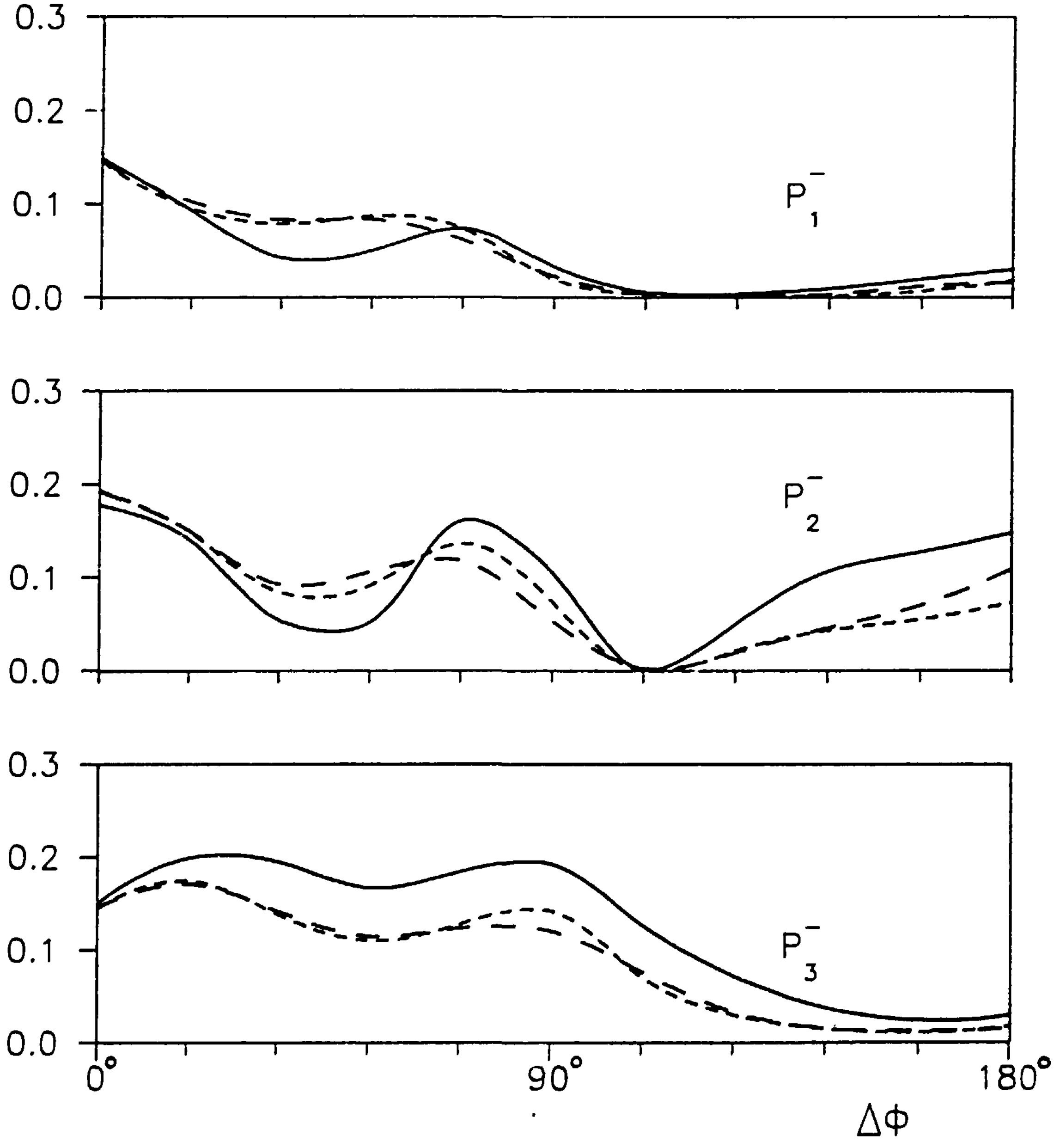


Fig.4

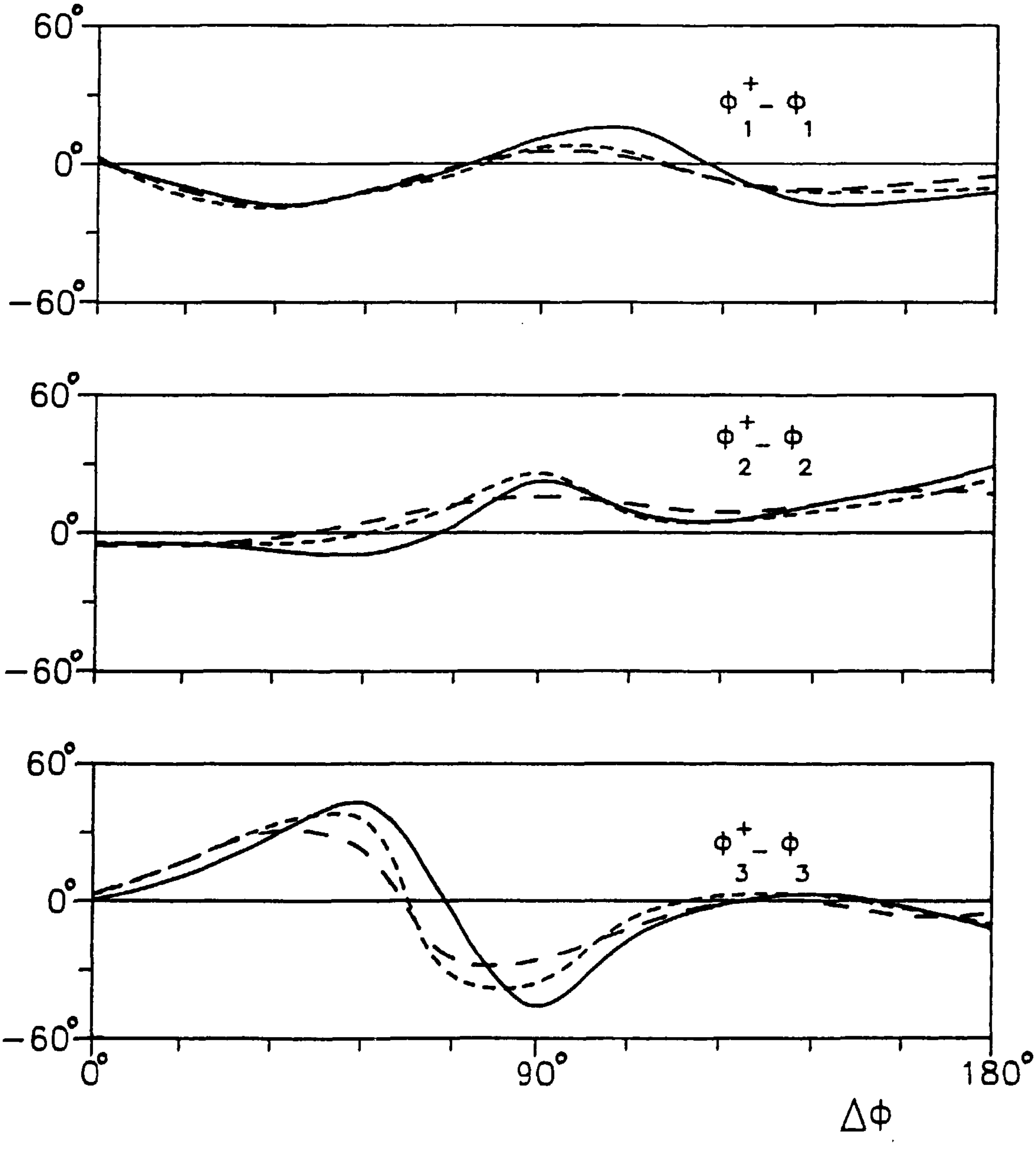
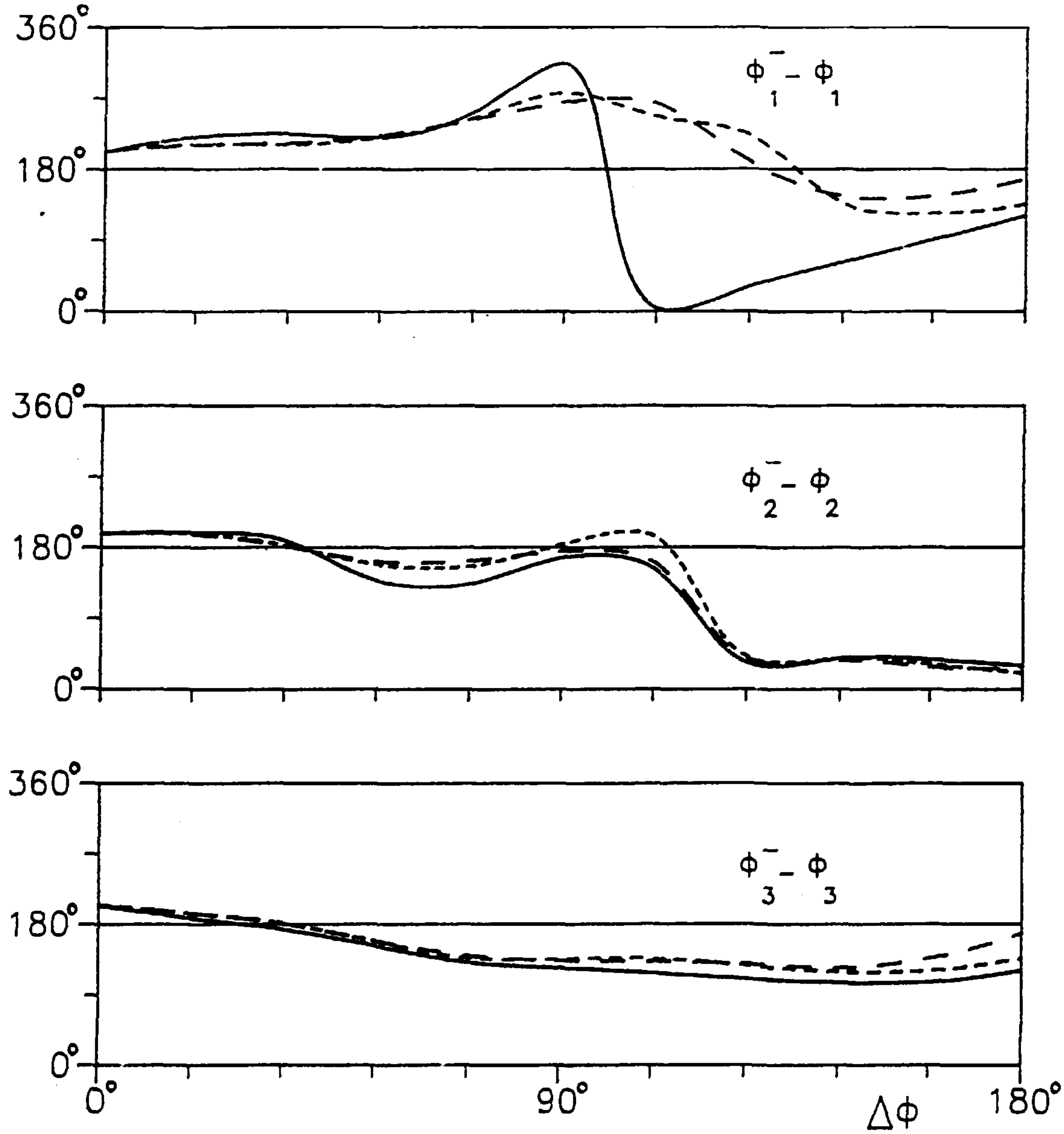


Fig.5



360°
180°
0°
$\phi_1^- - \phi_1$
360°
180°
0°
$\phi_2^- - \phi_2$
360°
180°
0°
$\phi_3^- - \phi_3$
0°
90°
Δφ
180°

Fig.6

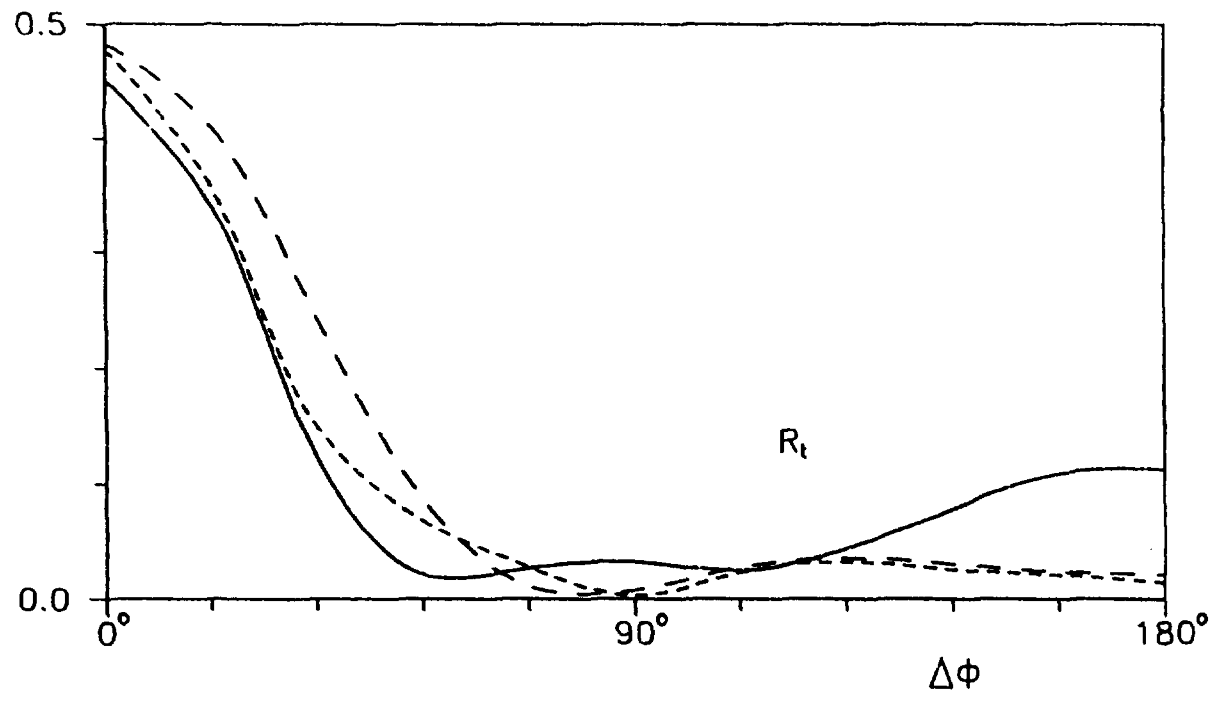


Fig.7

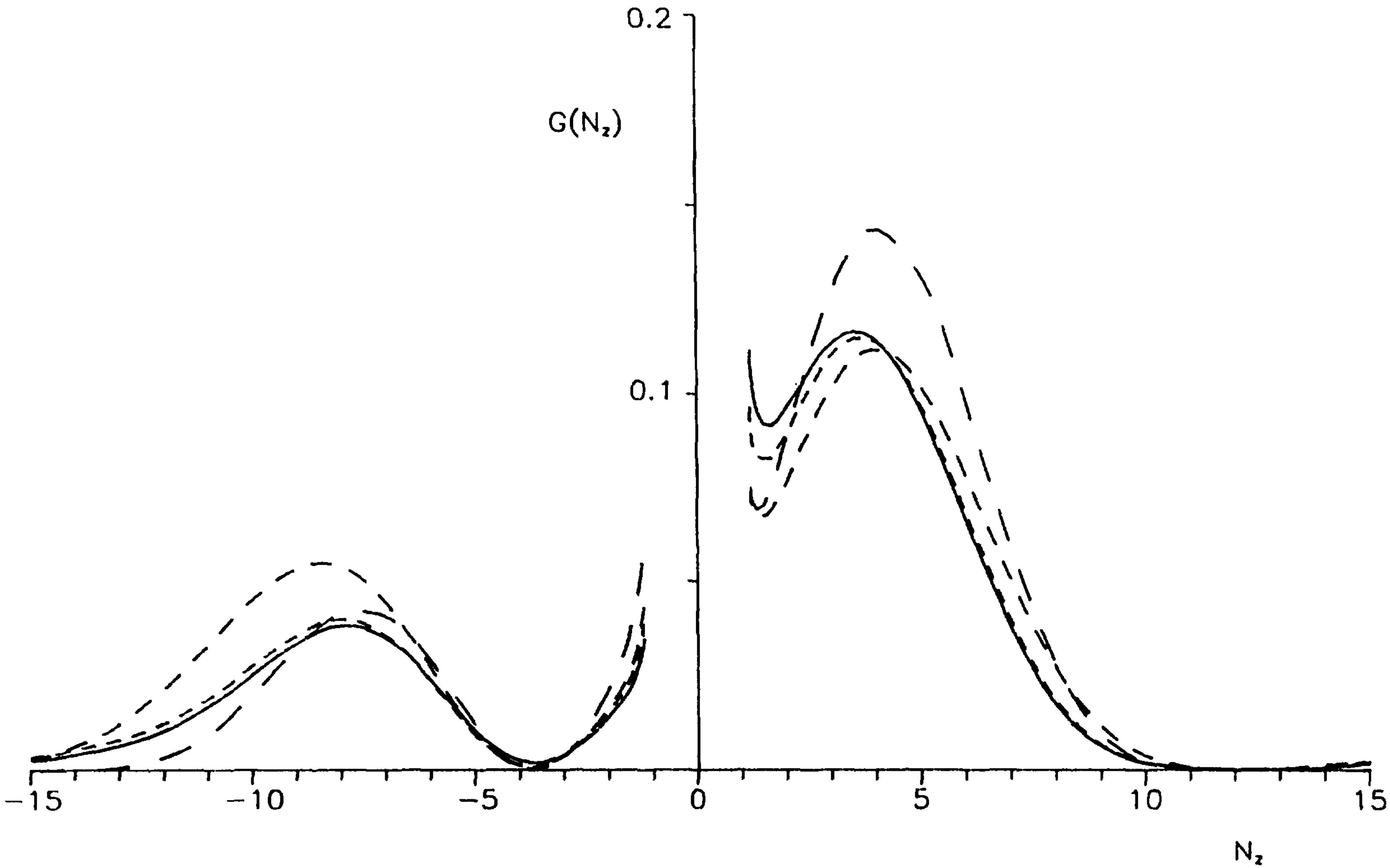


Fig.8

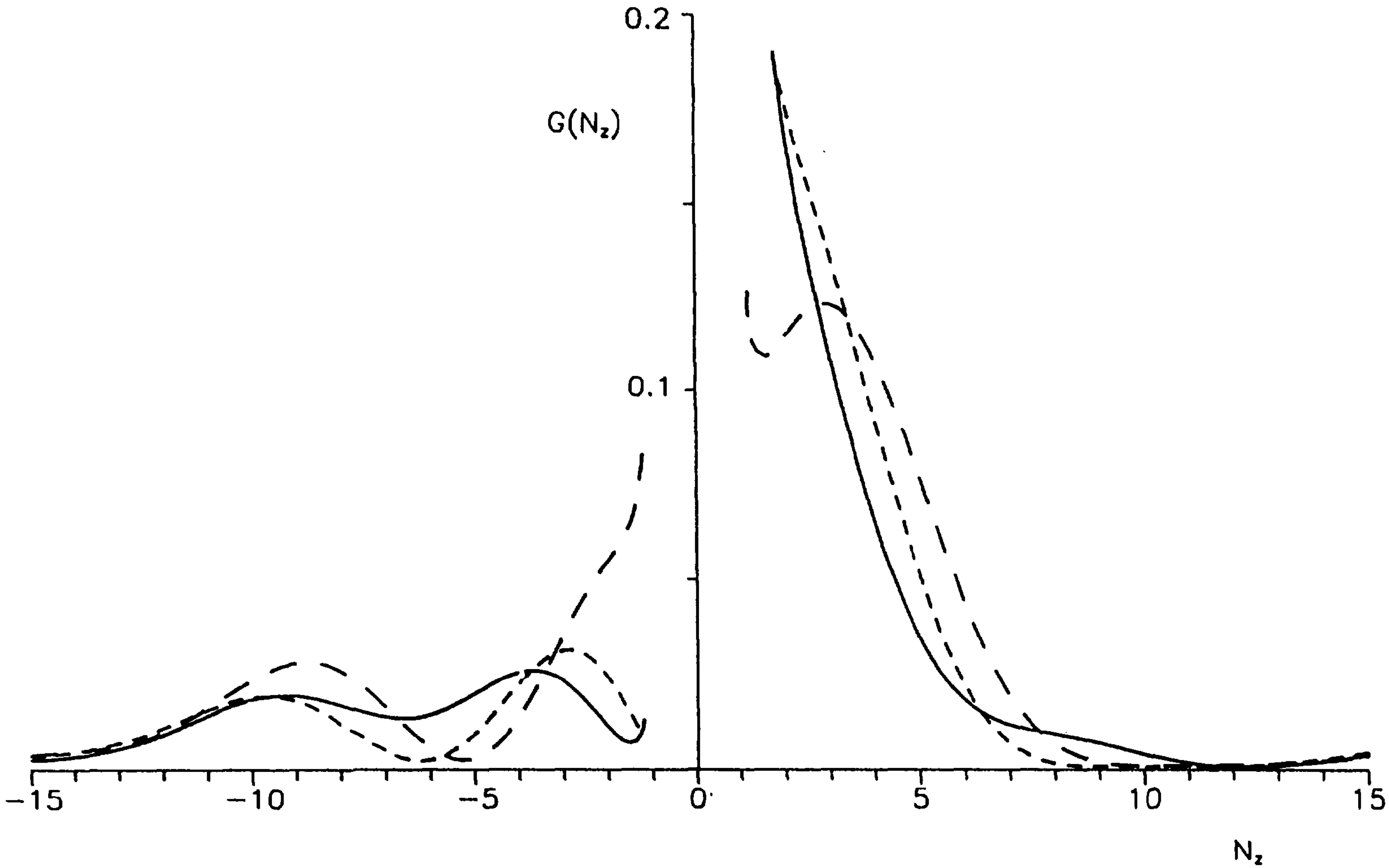


Fig.9

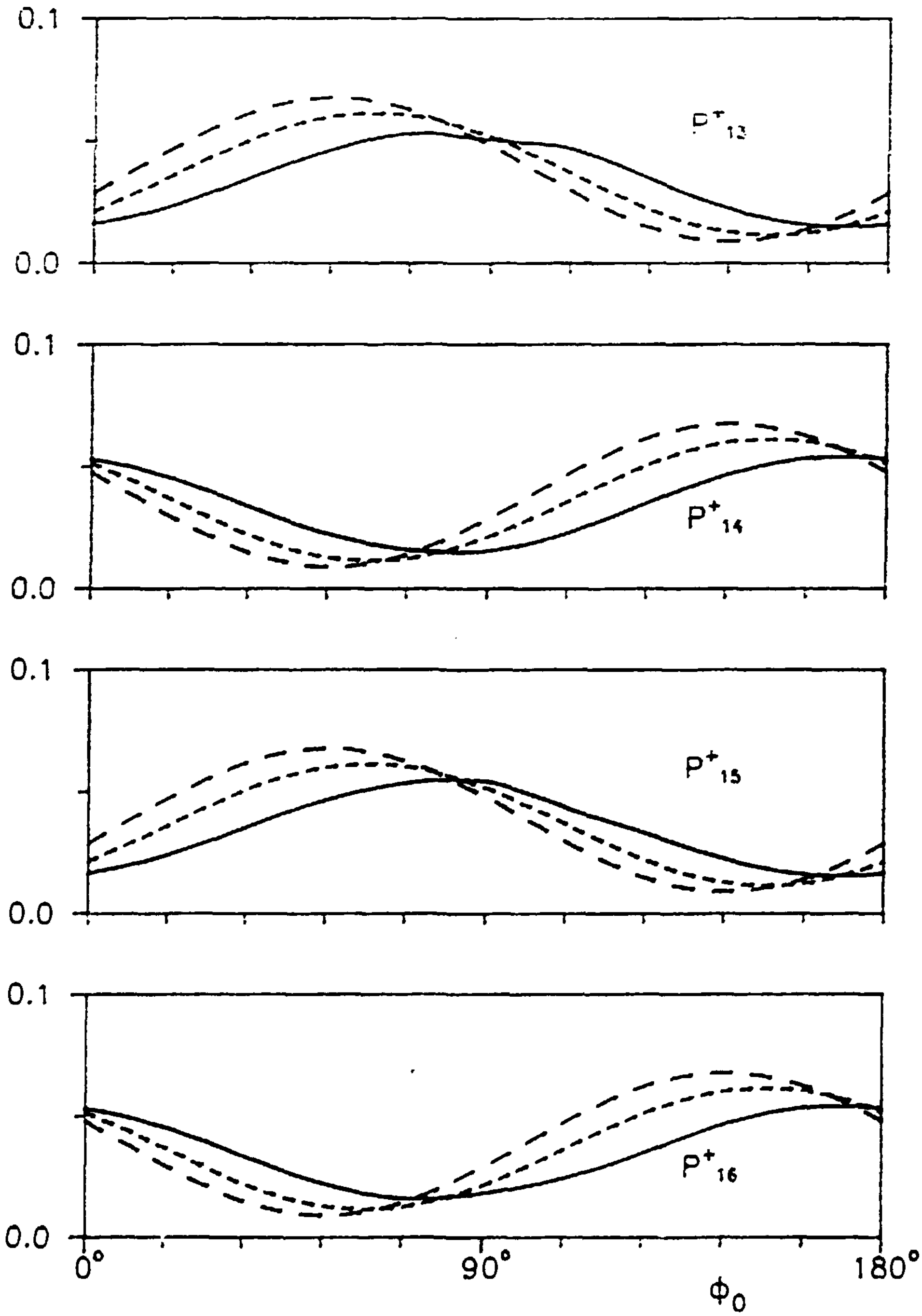


Fig. 10

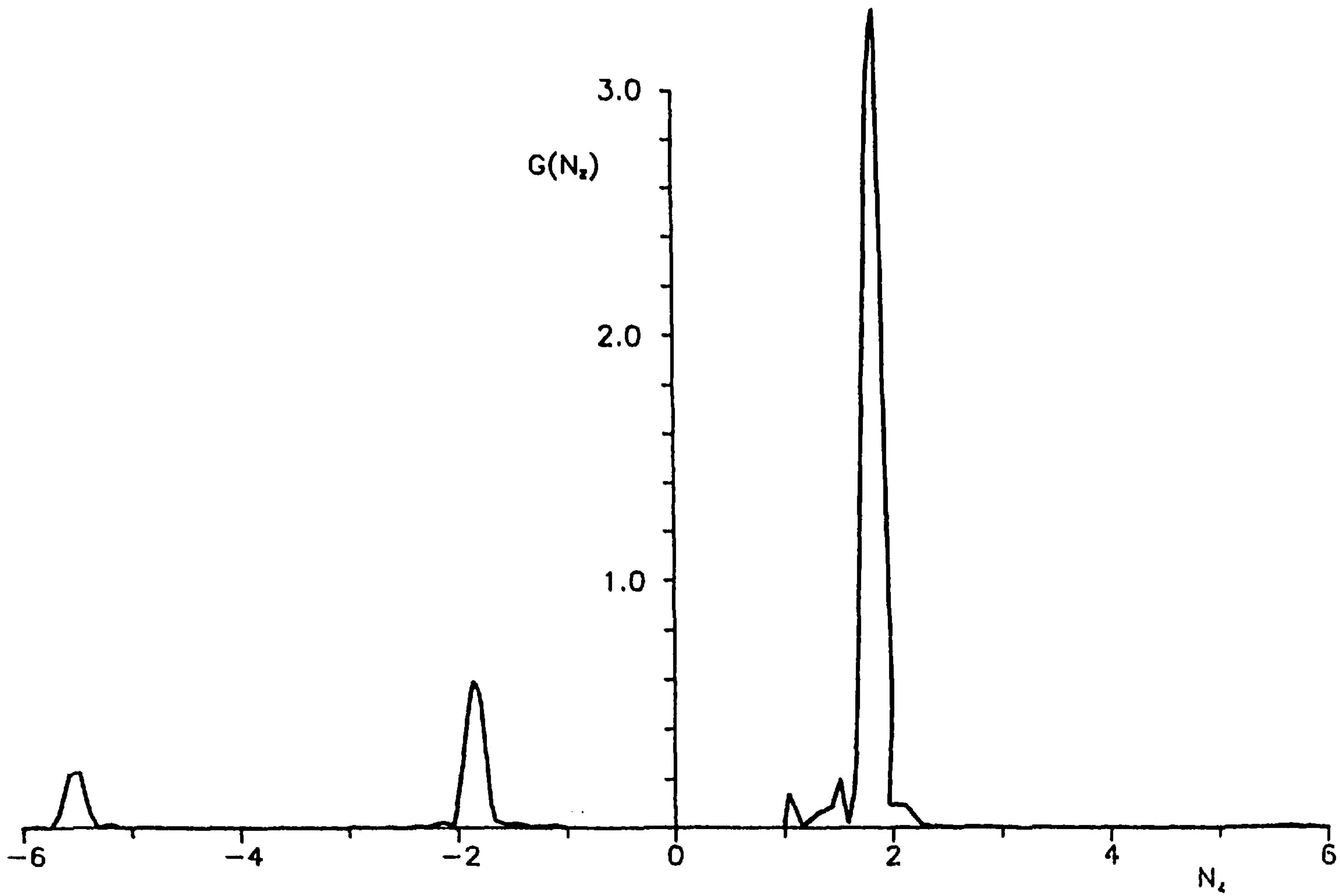


Fig.11

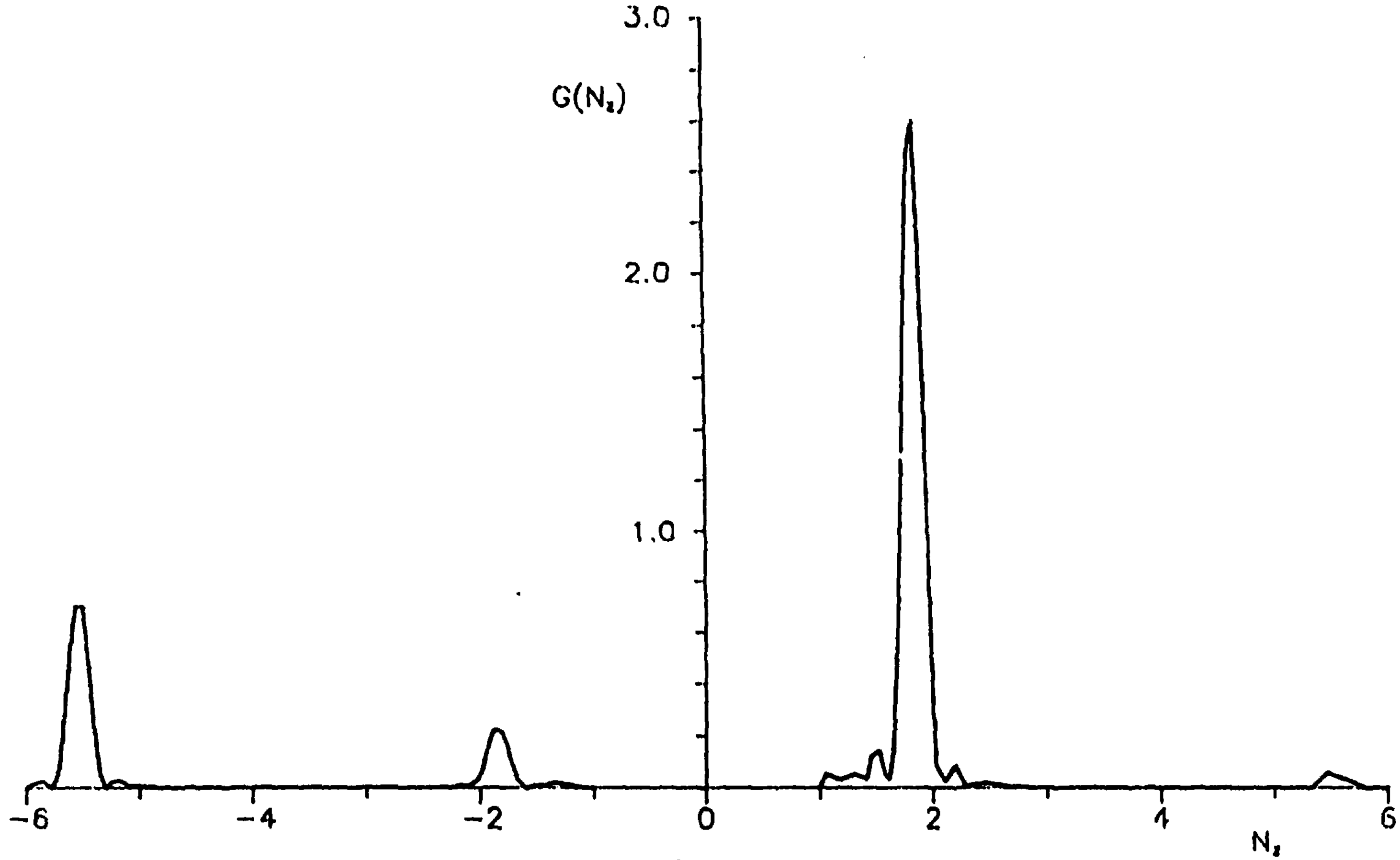


Fig.12