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A NEW UNEXPECTED FEATURE OF SUPERDEFORMED NUCLEI: STRANGE DEGENERACIES AND THEIR ORIGIN

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A NEW UNEXPECTED FEATURE OF SUPERDEFORMED NUCLEI: STRANGE DEGENERACIES AND THEIR ORIGIN

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ABSTRACT

Unexpected similarities between the superdeformed bands found recently in many nuclei of the mass $A \sim 150$ and $A \sim 190$ regions and the physical sense of this discovery are discussed. These similarities manifest themselves through the existence of nearly identical sequences of transitions belonging to two neighboring nuclei, and by now numerous nuclear pairs manifesting this feature have been found. The underlying microscopic mechanism is traced back to two independent effects: a readjustement of nuclear deformation diminishing the role of alignment of the least bound nucleous, and, the pseudospin symmetry, responsible for an approximate decoupling of the orbital and the intrinsic (pseudo) spin-degrees of freedom.

I – INTRODUCTION

When this text is being written there are alltogether 19 superdeformed bands discovered in the rare earth ($A \sim 150$) nuclei; the number of superdeformed bands discovered in Mercury- and Cesium-regions of $A \sim 190$ and $A \sim 130$ mass ranges, respectively, is growing with an impressive speed. Four years ago only one such a band has been known [1], yet the experiments aiming at finding the new, extreme deformations are still considered very difficult.

The progress is fast and important, and as it often happened in the past, it originates from the very special, interesting features of the underlying nuclear behavior.

What are these phenomena which appear so intriguiging that so many laboratories have now important programs in the domain of superdeformation?

Here we will give a brief account of several of them, the "strange degeneracy" pattern underlied in the title of this article being one of the most recently discovered intriguing effects.

I.A Unusual population pattern

It has been found that the intensities of transitions corresponding to the superdeformed bands behave in a characteristic, distinct manner. They grow very quickly with decreasing spin and, within a very few transitions, they reach a plateau. For most of the following transitions the intensities stay practically constant; this form of behavior is very different from that observed in normal nuclei in which the discussed intensity grows markedly with the decreasing spin.

I.B Unusual de-population pattern

For reasons which are still debated, the intensity of gamma rays in the superdeformed bands decreases to zero very abruptly within one to two transitions following the sequence of about 15 transitions in which it stayed practically constant. Such an abrupt change remains in a significant contrast to the effect of plateau just mentioned. It appeared fairly unusual to many physicists that a superdeformed configuration may be so stable over so many spin values and loose the stability completely at a lower spin value at such an abrupt manner.

I.C Unusually regular spin-vs.-frequency behavior

The great majority of the known normal (this term is being used in contrast to superdeformed) nuclear configurations produce a characteristic and rather complex I-vs.-ω dependence (I denotes spin, ω - the rotational frequency). This behavior is often so irregular that it leads to a multivalued character of $I = I(\omega)$, giving rise to the so-called back-bending(s), up-bending(s) and other structures. In full contrast to that, the I-vs.-ω for a superdeformed nucleus appears practically structureless, forming nearly a straight line, and only the derivatives of $I(\omega)$ reveal (often extremely interesting) signs of structural rearrangements.

I.D The problem of rigid nuclear rotation

When a sequence of transitions corresponding, at the highest spin limit, to $I \sim (60 \text{ to } 70)\,\hbar$ was observed, one of the first expectations discussed was that the pairing correlations will become neglegible and that the behavior of the effective moment of inertia should resemble a rigid body pattern. Although the spins of the superdeformed states have not been rigorously established in experiment, a "reasonable" evidence suggests that the effective moments of inertia are different from the corresponding rigid body values.

This observation in turn, can be related to one of the fundamental (unsolved) problems of nuclear structure: the relation of the effective mass of a nucleon in the nuclear finite medium to its free mass on one hand and to the rigid nuclear rotation problem, on the other.

I.E Unusual degeneracies

Since the present use of the term "degeneracy" may be misleading let us introduce more precisely the context in which this term is being applied throughout the article. It has been recently established [2] that there exist many pairs of superdeformed nuclei, their mass numbers differing usually by one or two nucleonic masses, in which nearly identical bands have been found. This, in contrast to the usual meaning of the term "degeneracy", does not imply that the corresponding nuclear levels are degenerate but rather that the <u>differences</u> between two consecutive transitions in one nucleus are (nearly) equal to those in a neighbouring nucleus. So understood degeneracies (also referred to as E_γ-degeneracy) which persist over many, many transitions, are in some cases almost an order of magnitude smaller than what one should expect by extrapolating our knowledge about normal nuclear configurations.

The discussion of the problem of "unusual" degeneracies, what appears to be a totally unexpected feature, will be the main subject of this article. We will introduce first some analytical considerations, which reveal a microscopic insight into nuclear readjustements of an even-even superdeformed nucleus being "disturbed" by the presence of an additional nucleon. Then more realistic considerations will be presented, based on the Woods-Saxon average field potential. Finally, the importance of the pseudo-spin symmetry for the problem of "unusual" degeneracies will be discussed.

II – CONDITIONS FOR AND CONSEQUENCES OF E_γ – DEGENERACIES

Let us consider the high-spin limit of nuclear rotation i.e. the case encountered in most of the superdeformed bands where the spins vary typically between 25 $\hbar$ and 65 $\hbar$. In such a case the angular momentum can be considered perpendicular to the main symmetry axis (see below) and in order to fix the notation we will consider the O_x axis to be parallel (or nearly parallel) to spin $\vec{I}$. In such a situation $I_x \simeq |\vec{I}|$ and we will omit the subscript "x" for simplicity.

Now, consider a situation illustrated schematically in Fig. II.1. A nucleus with Z protons and N neutrons ($N + Z \equiv A$) possesses an effective moment of inertia

$$J^{(1)} \equiv I/\omega, \tag{2.1}$$

where I denotes the total spin and ω the frequency of rotation. Since the superdeformed bands obey in most of the cases the rotor formula, we find, for the nucleus A and a neighboring nucleus B:

$$E_A(I) \sim \frac{\hbar^2}{2\mathcal{J}_A}\, I\,(I+1) \quad \text{and} \quad E_B(I') \sim \frac{\hbar^2}{2\mathcal{J}_B}\, I'\,(I'+1)\;. \tag{2.2}$$

Within a good occuracy, the corresponding transition energies are:

$$\underline{\text{Nucleus A}} : E_\gamma^A(I) \equiv E_A(I+2) - E_A(I) = (4I+6)\,\frac{\hbar^2}{2\mathcal{J}_A} \tag{2.3}$$

and

$$\underline{\text{Nucleus B}} : E_\gamma^B(I') \equiv E_B(I'+2) - E_B(I') = (4I'+6)\,\frac{\hbar^2}{2\mathcal{J}_B}\;, \tag{2.4}$$

and, without any loss of generality we will chose Z and N as even (I are integer numbers). Consider now a nucleus B with the mass number close to that of nucleus A. In our discussion it will be useful to think about the nearest neighbor, i.e. a nucleus with the mass number $A+1$. In the corresponding energy spectrum (Eq. (2.4)) I' denote now the (half-integer) spin values.

Let us introduce the difference, ΔJ, by

$$J_B = J_A + \Delta J\;. \tag{2.5}$$

ΔJ may in general be positive, negative or zero, but it is expected to be small (with respect to both J_A or J_B) for it is known that the measured moments of inertia in the superdeformed bands considered are close to each other.

Taking a typical mass number ($A \sim 150$) and the range of spins, ($I \sim 40$), characteristic for superdeformed rare earth nuclei and assuming that the dependence of the rigid body moment of inertia on the mass-number is of the form:

$$J_{RIG} \sim A^{5/3}/70 \; [\hbar^2 \, \mathrm{MeV}^{-1}], \tag{2.6}$$

we find that at $I' \sim I \sim 40\,\hbar$ the shift in the positions of the corresponding nearest peaks in the spectra is

$$E^B_\gamma(I') - E^A_\gamma(I) \equiv \Delta E_\gamma \sim 10\,\mathrm{keV} \; . \tag{2.7}$$

This value is an order of magnitude larger than the observed energy shifts which are often ~ 0 within the experimental error bars.

The use of expression (2.6) corresponds to a classical situation, where the nuclear density is supposed to be uniform and the moment of inertia (of an ideal rigid body) is supposed to grow monotonically with increasing mass number A. Disagreement of the estimate (2.7) with experiment signifies an important influence of quantum mechanical effects which, as we shall see, are sensitive to the quantum numbers of an odd nucleon. It is therefore our purpose to analyse the microscopic mechanisms which determine the nuclear high-spin response to "adding" such a nucleon.

II.A *Polarisation Caused by an Additional Particle (within the Mean Field View Point)*

Within the mean field formalism one finds at least four leading factors expected to manifest the nuclear response to rotation when "adding" a nucleon to a rotating nucleus. These are: the nuclear mass, represented primarily by the particle number (A); the spatial distribution of the nucleons in the nucleus, represented first of all by the nuclear shape (β); the nuclear rotation represented by the quasiclassical frequency ω, and the intensity of the pairing correlations, represented usually by the average pairing-gap Δ. There may also be other structural elements involved, but we limit our considerations to the above mentioned factors.

The dependence of the moment of inertia on the four extensive variables

$$\mathcal{J} = \mathcal{J}\,(A,\beta,\Delta,\omega)\;, \tag{2.8}$$

suggests that the nuclear response to "adding" a nucleon can be represented (in terms of the moments of inertia) in the following way:

$$\delta\mathcal{J} = \left(\frac{\partial\mathcal{J}}{\partial A}\right)\delta A + \left(\frac{\partial\mathcal{J}}{\partial\beta}\right)\delta\beta + \left(\frac{\partial\mathcal{J}}{\partial\Delta}\right)\delta\Delta + \left(\frac{\partial\mathcal{J}}{\partial\omega}\right)\delta\omega\;. \tag{2.9}$$

It has been argued [3], that the influence of the pairing correlations on the moment of inertia in the superdeformed states can be considered very weak at the high spin limit which is considered here. For this reason we will limit our discussion below to the three remaining principal forms of dependence i.e. on A, β and ω.

Relations (2.8-9) stress the mean-field formalism point of view involving such concepts as deformation or the average pairing field.

It will be instructive to express the effect of polarisation of a nucleus in a way which does not rely to any particular model-formalism and refers only to the observables accessible experimentally. In the case considered we have two groups of observables: $(E_\gamma^{(I)}, I)$ – in nucleus A, and, $(E_\gamma^{(I')}, I')$ – in nucleus B. Since

$$E_\gamma(I) \equiv E(I+2) - E(I) \cong 2\left(\frac{E(I+2)-E(I)}{\Delta I}\right)_{\Delta I=2} = 2\omega\,(I) \tag{2.10}$$

we can, equivalently, use the notation in terms of frequency, $\omega\,(I)$, rather than the transition energy, $E_\gamma\,(I)$.

We consider a pair of nuclei in which the "degenerate" bands exist. In such a case, there exists a pair of spin values I and I', refering to the nuclei A and B, respectively, such that

$$\omega_A\,(I) \simeq \omega_B\,(I') \text{ over many transitions}\;. \tag{2.11}$$

We have therefore:

$$\delta\omega \equiv \omega_B\,(I') - \omega_A\,(I) = \frac{I'}{\mathcal{J}_B} - \frac{I}{\mathcal{J}_A}\;. \tag{2.12}$$

Since

$$\frac{I'}{\mathcal{J}_B} = \frac{I+\delta I}{\mathcal{J}_A+\delta\mathcal{J}} \cong (I+\delta I)\frac{1}{\mathcal{J}_A}\left(1-\frac{\delta\mathcal{J}}{\mathcal{J}_A}\right) = \frac{I}{\mathcal{J}_A}+\frac{\delta I}{\mathcal{J}_A}-\frac{I}{\mathcal{J}_A}\left(\frac{\delta\mathcal{J}}{\mathcal{J}_A}\right)$$

$$= \frac{I}{\mathcal{J}_A}+\frac{\delta I}{\mathcal{J}_A}-\omega_A(I)\left(\frac{\delta\mathcal{J}}{\mathcal{J}_A}\right), \qquad (2.13)$$

(where only the terms linear in small quantities $(\delta I, \delta\mathcal{J})$ have been retained), we obtain:

$$\delta\omega = \frac{\delta I}{\mathcal{J}_A} - \omega_A(I)\ \left(\frac{\delta J}{J_A}\right)\ . \qquad (2.14)$$

Dividing the last relation by $\omega_A(I)$ and noticing again that $\mathcal{J}_A\omega_A(I) = I$ we get:

$$\frac{\delta\mathcal{J}}{\mathcal{J}} = \frac{\delta I}{I} - \frac{\delta\omega}{\omega}\ ; \qquad \delta I \equiv I' - I, \qquad (2.15)$$

where we have omitted the obvious index A in $\mathcal{J}_A \to \mathcal{J}$, $I_A \to I$ and $\omega_A(I) \to \omega$. The above relation indicates that in general, the measured polarisation of the moments of inertia, $\delta\mathcal{J}/\mathcal{J}$, has a hyperbolic dependence on spin (note that $I' - I \equiv \delta I$ should be viewed as a constant in function of spin) and that the relative shift in frequencies, $\delta\omega/\omega$, can be the only source of fluctuations of $\delta\mathcal{J}/\mathcal{J}$. The above observation is interesting, given the fact that with the present day experimental techniques only the frequency shifts can be measured, while the spins $I = I_0, I_0+2, I_0+4, \ldots$ and $I' = I'_0, I'_0+2, I'_0+4, \ldots$ remain, strictly speaking unknown. Relation (2.15) will also be convenient when comparing the experimental and theoretical results in the next sections. The difficulty in extracting directly the quantity $\delta\mathcal{J}/\mathcal{J}$ from experiment follows from an uncertainty of the I_0 and I'_0.

II.B Competition between the Shape and the Alignment Effects (Analytical Results)

Having introduced the problem of degeneracies we will turn now to the analysis of the underlying mechanisms. We will start with the harmonic oscillator model of a system of n nucleons what will allow to obtain analytical results. Later on our consideration will be generalised to the case of a realistic mean field.

Consider the harmonic oscillator hamiltonian

$$\widehat{H} = \hat{t} + \frac{1}{2} m \left(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2\right). \tag{2.16}$$

For a given number of particles n, we may specify each nucleonic state $|i> \equiv |n_x(i), n_y(i), n_z(i)>$. It will be convenient to introduce the following auxiliary quantities:

$$\left.\begin{aligned} \Sigma_1 &\equiv \sum_{i=1}^{n} \left(n_x(i) + \tfrac{1}{2}\right); \\ \Sigma_2 &\equiv \sum_{i=1}^{n} \left(n_y(i) + \tfrac{1}{2}\right); \\ \Sigma_3 &\equiv \sum_{i=1}^{n} \left(n_z(i) + \tfrac{1}{2}\right). \end{aligned}\right\} \tag{2.17}$$

The corresponding total energy, defined as the sum of the single-particle contributions is

$$E = \sum_{j=1}^{3} \hbar\omega_j \, \Sigma_j \, . \tag{2.18}$$

In order to follow the shape-adjustements of nucleus A after "adding" a nucleon, we will minimise the total energy assuming the self-consistency condition [4]. For this purpose we observe that the equation of the equipotential surface corresponding to (2.16):

$$\frac{x^2}{\left(\frac{1}{\omega_x}\right)^2} + \frac{y^2}{\left(\frac{1}{\omega_y}\right)^2} + \frac{z^2}{\left(\frac{1}{\omega_z}\right)^2} = \text{const} \, . \tag{2.19}$$

defines the semiaxes of the related ellipsoid as: $a \sim \omega_x^{-1}$; $b \sim \omega_y^{-1}$, $c \sim \omega_z^{-1}$. The self-consistency requirement, more precisely, will be formulated in terms of the geometrical correspondence between the shape of the deforming equipotential surfaces and the corresponding surfaces of the density. Here we will limit ourselves to the condition that the anisotropies of the density and that of the potential are equal. The former will be specified, after Ref. [4], by employing the mean square values

$$< x_i^2 >_{\text{all nucleons}} = \left(\frac{\hbar}{m\omega_i}\right) \Sigma_i \, . \tag{2.20}$$

Thus the self-consistency condition reads:

$$\frac{< x_1^2 >}{\left(\frac{1}{\omega_1}\right)^2} = \frac{< x_2^2 >}{\left(\frac{1}{\omega_2}\right)^2} = \frac{< x_3^2 >}{\left(\frac{1}{\omega_3}\right)^2} = \text{const.} , \tag{2.21}$$

or

$$\left(\frac{\hbar}{m\omega_1}\right)\Sigma_1\omega_1^2 = \left(\frac{\hbar}{m\omega_2}\right)\Sigma_2\omega_2^2 = \left(\frac{\hbar}{m\omega_3}\right)\Sigma_3\omega_3^2 ,$$

which results in the well known relation

$$\Sigma_1\omega_1 = \Sigma_2\omega_2 = \Sigma_3\omega_3 . \tag{2.22}$$

Suppose that a particle has been "added" to an even-even core. Such a particle will be characterised by its quantum numbers (n_x, n_y, n_z), what will lead to modifications of the Σ_1, Σ_2 and Σ_3:

$$\Sigma_1 \rightarrow \Sigma_1 + \delta\Sigma_1,\ \Sigma_2 \rightarrow \Sigma_2 + \delta\Sigma_2,\ \Sigma_3 \rightarrow \Sigma_3 + \delta\Sigma_3 , \tag{2.23}$$

where we introduced the notation

$$\delta\Sigma_1 \equiv n_x + \frac{1}{2}; \quad \delta\Sigma_2 \equiv n_y + \frac{1}{2}; \quad \delta\Sigma_3 \equiv n_z + \frac{1}{2} . \tag{2.24}$$

The selfconsistency condition will imply that in general also the oscillator frequency must be modfied:

$$\omega_i \rightarrow \omega_i + \delta\omega_i; \qquad i = x, y, z . \tag{2.25}$$

The volume conservation condition requires that

$$\omega_1\omega_2\omega_3 = \overset{0}{\omega}{}^3 \equiv \frac{v}{\hbar^3}; \tag{2.26a}$$

and

$$(\omega_1 + \delta\omega_1)(\omega_2 + \delta\omega_2)(\omega_3 + \delta\omega_3) = (\overset{0}{\omega}{}')^3 \equiv \frac{v'}{\hbar^3} . \tag{2.26b}$$

We set usually

$$(\hbar\overset{0}{\omega})^3 = v \quad \left(= \frac{41}{A^{1/3}}\text{MeV}\right)$$

Increase in the volume resulting from introducing the $(A+1)^{st}$ particle gives

$$v' = v + \delta v; \quad \delta v = -\frac{v}{A^2}\ ; \tag{2.27}$$

where (as in the following) only the terms linear in "small" quantities (i.e. $\delta\Sigma_i, \delta\omega_i$) are retained. The left-hand side of Eq. (2.26b) gives, within this order:

$$(\omega_1 + \delta\omega_1)(\omega_2 + \delta\omega_2)(\omega_3 + \delta\omega_3) \approx \omega_1\omega_2\omega_3 \left\{1 + \frac{\delta\omega_1}{\omega_1} + \frac{\delta\omega_2}{\omega_2} + \frac{\delta\omega_3}{\omega_3}\right\}\ . \tag{2.28}$$

Substituting $\omega_1\omega_2\omega_3$ from (2.26a) and comparing with Eq. (2.27) gives

$$\frac{\delta\omega_1}{\omega_1} + \frac{\delta\omega_2}{\omega_2} + \frac{\delta\omega_3}{\omega_3} = \frac{\delta v}{v}. \tag{2.29}$$

Similarly, we will write down the selfconsistency condition for the nucleus B in the form:

$$(\omega_1 + \delta\omega_1)(\Sigma_1 + \delta\Sigma_1) = (\omega_2 + \delta\omega_2)(\Sigma_2 + \delta\Sigma_2) = (\omega_3 + \delta\omega_3)(\Sigma_3 + \delta\Sigma_3)\ . \tag{2.30}$$

Recall, that with a nucleon placed on a given orbital (n_x, n_y, n_z), we may treat $\{\delta\Sigma_i\}$ as known, while the shape modifications $\{\delta\omega_i\}$ are still to be determined. Retaining as before, only the quantities linear in terms of the small increments we get from Eq. (2.30):

$$\omega_1\Sigma_1\left(1 + \frac{\delta\omega_1}{\omega_1} + \frac{\delta\Sigma_1}{\Sigma_1}\right) \simeq \omega_2\Sigma_2\left(1 + \frac{\delta\omega_2}{\omega_2} + \frac{\delta\Sigma_2}{\Sigma_2}\right) \simeq \omega_3\Sigma_3\left(1 + \frac{\delta\omega_3}{\omega_3} + \frac{\delta\Sigma_3}{\Sigma_3}\right) \tag{2.31}$$

what, in view of (2.22) gives

$$\frac{\delta\omega_1}{\omega_1} + \frac{\delta\Sigma_1}{\Sigma_1} = \frac{\delta\omega_2}{\omega_2} + \frac{\delta\Sigma_2}{\Sigma_2} = \frac{\delta\omega_3}{\omega_3} + \frac{\delta\Sigma_3}{\Sigma_3} \equiv \Delta. \tag{2.32}$$

Equations (2.32) (three relations) together with Eq. (2.29) form a system of four algebraic equations for the unknowns Δ, $\delta\omega_1$, $\delta\omega_2$ and $\delta\omega_3$. After eliminating the auxiliary term Δ we find the solution

$$\frac{\delta\omega_i}{\omega_i} = \frac{\delta v}{3v} + \frac{1}{3}\left[\sum_{j=i}^{3} \frac{\delta\Sigma_j}{\Sigma_j}(1 - 3\delta_{ji})\right]. \tag{2.33}$$

In what follows, it will be convenient to introduce the nuclear elongation parameter

$$\alpha \equiv \omega_2/\omega_3 \simeq \omega_1/\omega_3 \ . \tag{2.34}$$

Then, the self-consistency condition implies in particular

$$\omega_2\Sigma_2 = \omega_3\Sigma_3 \Leftrightarrow \alpha\omega_3\Sigma_2 = \omega_3\Sigma_3 \Rightarrow \Sigma_2 = \frac{1}{\alpha}\Sigma_3 \ . \tag{2.35}$$

Using the fact that for an axially symetric (e.g. superdeformed) nucleus we have

$$\omega_1 = \omega_2 \equiv \Omega\,; \qquad \Sigma_1 = \Sigma_2 \equiv \Sigma\ , \tag{2.36}$$

and neglecting again all the terms quadratic in our "small quantities" we find that the effect of the core polarisation induced by the additional particle can be expressed as follows:

$$\delta\omega_1 = \frac{\Omega}{3v}\delta v + \frac{\Omega}{3\Sigma}\left[\delta\Sigma_2 - 2\delta\Sigma_1 + \frac{1}{\alpha}\delta\Sigma_3\right] \tag{2.37a}$$

$$\delta\omega_2 = \frac{\Omega}{3v}\delta v + \frac{\Omega}{3\Sigma}\left[\delta\Sigma_1 - 2\delta\Sigma_2 + \frac{1}{\alpha}\delta\Sigma_3\right] \tag{2.37b}$$

$$\delta\omega_3 = \frac{\Omega}{3\alpha v}\delta v + \frac{\Omega}{3\alpha\Sigma}\left[\delta\Sigma_1 - \delta\Sigma_2 + \frac{2}{\alpha}\delta\Sigma_3\right] \ . \tag{2.37c}$$

So far we have considered the effect of the shape polarisation expressed in terms of modifications of the semi-axes which follow from the "additional" particle. We will now turn to the problem of angular momentum aligment. In order not to complicate this discussion we will limit ourselves here to the small rotational frequencies

$$\omega_{rot} \equiv \omega \to 0 \ . \tag{2.38}$$

Under this simplifying condition we may use directly the Inglis form for the moment of inertia which, according to Eq. (4.116) in Ref. [4], reads:

$$\mathcal{J} = \frac{1}{2\omega_2\omega_3}\left\{(\Sigma_3 - \Sigma_2)\frac{(\omega_2+\omega_3)^2}{\omega_2-\omega_3} + (\Sigma_3+\Sigma_2)\frac{(\omega_2-\omega_3)^2}{\omega_2+\omega_3}\right\} \ , \tag{2.39}$$

where we assume a one-dimensional rotation pattern (with the O_x axis chosen as the rotation axis). The presence of an additional particle will "polarise" the nuclear inertia tensor and in particular

$$\mathcal{J} \to \mathcal{J} + \delta\mathcal{J} \ . \tag{2.40}$$

Now we would like to express $\delta\mathcal{J}$ as a function of $\delta\omega_i$ and $\delta\Sigma_j$. For that purpose we will substitute

$$\omega_i \to \omega_i + \delta\omega_i \quad \text{and} \quad \Sigma_j \to \Sigma_j + \delta\Sigma_j \ , \tag{2.41}$$

into Eq. (2.39), and retain, as before, only the terms linear in small quantities. For an axially symmetric nucleus we may use the relations

$$\omega_2 \pm \omega_3 = \Omega\left(1 \pm \frac{1}{\alpha}\right) \tag{2.42a}$$

$$\Sigma_3 \pm \Sigma_2 = \Sigma\,(\alpha \pm 1) \ , \tag{2.42b}$$

which gives also:

$$(\Sigma_3 - \Sigma_2)\frac{(\omega_2 + \omega_3)^2}{\omega_2 - \omega_3} = \Sigma\Omega\frac{(\alpha+1)^2}{\alpha} \tag{2.43a}$$

$$(\Sigma_3 + \Sigma_2)\frac{(\omega_2 - \omega_3)^2}{\omega_2 + \omega_3} = \Sigma\Omega\frac{(\alpha-1)^2}{\alpha} \ . \tag{2.43b}$$

The above relations give, after elementary substitutions

$$\mathcal{J} = \frac{\Sigma}{\Omega}(\alpha^2 + 1) \tag{2.44}$$

$$\frac{\delta\mathcal{J}}{\mathcal{J}} = \frac{1}{3\Sigma}\left\{-\Sigma\frac{\delta v}{v} - \Sigma(\delta\Sigma_1) + \frac{5-\alpha^2}{1+\alpha^2}(\delta\Sigma_2) + \frac{5\alpha^2 - 1}{\alpha(\alpha^2+1)}(\delta\Sigma_3)\right\} \ . \tag{2.45}$$

The above expression shows also the way in which the angular momentum alignment, ΔI, is influenced by a polarising particle, since

$$\Delta I \sim \omega\delta\mathcal{J} \ . \tag{2.46}$$

More precisely, since $\delta\Sigma_1 \equiv \left(n_x + \frac{1}{2}\right)$, $\delta\Sigma_2 \equiv \left(n_y + \frac{1}{2}\right)$ and $\delta\Sigma_3 = \left(n_z + \frac{1}{2}\right)$, one finds directly the sought relation

$$\Delta I = \Delta I(n_x, n_y, n_z)$$

in the small-ω limit.

Relation (2.45) can be given an interesting physical interpretation. First let us observe that the coefficient in front of $\delta\Sigma_1$ is always negative what implies that the stronger the nucleonic oscillation along the axis of rotation, the smaller the moment of inertia contribution induced by that particle. In contrast, the nucleonic oscillator quanta along the O_z axis (elongation axis perpendicular to the axis of rotation) contribute always an increase of the moment of inertia. It is interesting to notice that this latter contribution becomes neglegible (~ 0) only for the strongly oblate nuclear distortions with $\alpha \sim \frac{1}{\sqrt{5}}$.

Given the fact that the nucleonic oscillations along the O_x and O_z axes contribute in Eq. (2.45) with opposite signs, the decive role is going to be played by the third (O_y) contribution, which changes sign at $\alpha \sim \sqrt{5}$. This characteristic change of sign takes place at the elongations corresponding approximately to the superdeformation, the contribution from $\delta\Sigma_2$ becoming more and more negative when elongation increases. For the hyperdeformed configuration, for instance ($\alpha = 3$), the discussed term $\sim 2/5(\delta\Sigma_2)$.

It becomes therefore clear, that Eq. (2.45) provides a criterion for the E_γ-degeneracy driving odd-n orbitals: they should have the maximum number of the x-oscillation quanta and the minimum number in the z-oscillation quanta.

The corresponding competition between the three oscillation modes of a nucleon in an elongated nucleus, leading to possible cancelations and thus implying that $\delta\mathcal{J}/\mathcal{J}$ gets close to zero, results in the presented derivation of Eq. (2.45) from the self-consistency condition. In other words, the deformation adjustements of the bulk of nucleonic matter follow the character of the quantum oscillations of an "additional" nucleon. If it oscillates mainly in the direction of the axis of rotation, its effect of attracting the other nucleons decreases the probability of finding other nucleons for away from the axis of rotation - thus implying always a negative contribution to $\Delta\mathcal{J}$.

III – THE $\Delta\mathcal{J}/\mathcal{J}$ POLARISATION IN THE WOODS-SAXON POTENTIAL

In what follows we would like to confront our theoretical considerations with experiment. This will require a more realistic treatment (as compared to the simple harmonic oscillator model) of the single particle spectrum.

We are going to use the Woods-Saxon Hamiltonian

$$\hat{H}_{WS} = \hat{t} + V_{WS} + V_{WS}^{SO} + \frac{1}{2}(1+\tau_3)V_{COUL.} . \quad (3.1)$$

where

$$V_{WS} = \frac{V_0\left[1 \pm \kappa\frac{N-Z}{N+Z}\right]}{1+\exp\{dist_\Sigma(\vec{r}, def)/a\}} \quad (3.2)$$

and

$$V_{WS}^{SO} = -\lambda(\nabla V_{WS} \wedge \vec{p}).\vec{s} \quad (3.3)$$

according to standard definitions and notation (cf. Ref. [5] for details). In Eq. (3.2) V_0, κ and $\underline{a}$ are the potential parameters, "def" represents all the deformation parameters, and the symbol $dist_\Sigma$ represents the distance of a point specified by $\vec{r}$ and the nuclear surface Σ.

Typical spectra of the eigenvalues of the Schrödinger equation

$$\hat{H}\psi_\nu = e_\nu\psi_\nu \quad (3.4)$$

are illustrated in figures (III.1a-b) and (III.2a-b) for neutrons and protons, respectively. The single particle energies are plotted here in function of the nuclear elongation encompassing a transition from spherical to superdeformed nuclear shapes.

In contrast to a customany representation of such Woods-Saxon diagrams, the interactions between single-particle states corresponding to the level crossings presented in Figs. III.1b and III.2b have been artificially removed; this way of representing the spectra is a useful simplification of the illustrations.

The labelling of the single particle states corresponds to the $[N, n_z, \Lambda]\Omega$ asymptotic. The labels have been attributed to the corresponding states by taking the corresponding leading term in the expansion in terms of the axially symetric harmonic oscillator basis

$$\psi_\nu^{WS} = \sum_{Nn_z\Lambda\Sigma} C_{\nu;Nn_z\Lambda\Sigma}\varphi^{HO}_{Nn_z\Lambda\Sigma} ; \tag{3.5}$$

for each ν, the label corresponds to this harmonic oscillator state which produces the maximum $|C_{\nu;Nn_z\Lambda\Sigma}|^2$.

In order to benefit from our consideration in the preceding section, a relation between the axially symetric basis used here and the more convenient (n_x, n_y, n_z, Σ) basis used in Sect. II will be needed.

For the purpose of this discussion we will chose, as a measure of the $\Delta\mathcal{J}/\mathcal{J}$ polarisation in the Woods-Saxon potential the quantity

$$(\Delta\mathcal{J}/\mathcal{J})_\nu^{WS} = \sum_\mu C^2_{\nu\mu}(\Delta\mathcal{J}/\mathcal{J})_\mu^{HO} \tag{3.6}$$

where $\mu \equiv (n_x, n_y, n_z, \Sigma)$, ν labels the corresponding Woods-Saxon states and $(\Delta\mathcal{J}/\mathcal{J})_\mu^{HO}$ are given by Eq. (2.45).

The results of the corresponding calculations are illustrated in Figs. (III.3) and (III.4) for neutrons and protons, respectively. To avoid the complications, which follow from the level crossings (and which are of less importance here) the crossings were artificially removed, similarly to what has been mentioned when commenting Figs. (III.1) and (III.2).

Two important features deserve noticing. First, most of the $(\Delta\mathcal{J}/\mathcal{J})_\nu^{WS}$ curves decrease, with increasing deformation, just rendering the "strange degeneracy" phenomena more and more likely, when deformation increases. Secondly, for deformation close to the superdeformed equilibrium values, there are some of the $(\Delta\mathcal{J}/\mathcal{J})_\nu^{WS}$ contributions which change sign (passing through zero). This property illustrates the special role played by the superdeformed configurations. It is only at those large elongations where rather numerous single particle orbitals bring vanishing $(\Delta\mathcal{J}/\mathcal{J})$ contributions and consequently produce the "strange degeneracy" pattern observed with such an abundance in e.g. rare earth nuclei.

The theoretical predictions of Figs. (III.3-4) have been confronted with experiment using the results for several pairs of nuclei. Here we will limit ourselves to a pair represented by:

Nucleus A : $^{150}Gd_{86}$ ("first excited" band)

and

Nucleus B : $^{151}Tb_{86}$ ("first" band).

The important structural difference between the two bands is, according to our interpretation, that one of the two signature-partner orbitals of the [301]1/2 level is inoccupied in ^{150}Gd (cf. Fig. (III.4), while both signature partners are occupied in ^{151}Tb.).

The calculations corresponding to these configurations are presented in Fig. (III.5). The top part illustrates the comparison between theory and experiment for the $\mathcal{J}^{(2)}$ moments of inertia defined as usually by

$$\mathcal{J}^{(2)} = \left[\frac{d^2E}{dI^2}\right]^{-1} . \tag{3.7}$$

A direct comparison of the $\Delta\mathcal{J}/\mathcal{J}$ quantities extracted from experiment and the Woods-Saxon value corresponding to the [301]1/2 orbital are given in the bottom part of the figure and indicate a very good agreement with the theoretical predictions. Similar agreement has been achieved also for nuclei where the orbitals with higher n_z-values (cf. Eq. (2.45)) are expected to play a role. In those cases the extracted $(\Delta\mathcal{J}/\mathcal{J})$ values and those calculated with the Woods-Saxon code are larger (as componed to the results in Fig. III.5) but confirm the proposed interpretation.

IV – COMMENT ON THE PSEUDO-SPIN SYMMETRY AND E_γ–DEGENERACIES

The mechanism of the pseudo-spin symmetry in heavy nuclei is illustrated schematically in Fig. IV.1.

One of the first discussions of the pseudo-spin symmetry effects in the super-deformation context, Ref. [6], has been extended recently, Ref. [7] to the analysis of the experimental data on the E_γ- degenarate bands in a few Rare Earth ($A \sim 150$) nuclei. The main argument used was based on the extension of the decoupled rotor formula

$$E(I) = \frac{\hbar^2}{2J}\left[I(I+1) + a(-1)^{I+\frac{1}{2}}(I+\frac{1}{2})\right] \tag{4.1}$$

to a superdeformed configuration. Here

$$a = - < |j_+| > \tag{4.2}$$

denotes the decoupling parameter.

Although espression (4.1) is in general <u>not</u> applicable at rotational frequencies as large as those found in the superdeformed configurations one can find arguments that in the particular case analysed in Ref. [7] it can still be applied.

Expression (4.1) leads to particular symmetries between the spectrum of an even-even and the corresponding odd-even system (rotor-plus-particle) if $|a| = 1$. This symmetry manifests itself by either equality of the type

$$E_\gamma^A(I) = E_\gamma^B(I'); \tag{4.3}$$

or

$$E_\gamma^A(I) = \frac{1}{2}\left(E_\gamma^B(I'+2) + E_\gamma^B(I')\right) \ , \tag{4.4}$$

both types of symmetries actually seen in experiments. It is important to remark that changing the sign of a, for instance from $(+1)$ to (-1) implies exchanging the roles of the symmetries of the type (4.3) by (4.4) and vice versa. It was thus interesting to check the microscopic significance of the condition $|a| = 1$. In the well known "Nilsson" asymptotic limit

$$a = (-1)^N \delta_{\Lambda O} \tag{4.5}$$

according to standard notation.

However, a comparison with experimental data based on the typical, realistic single particle scheme led to the observation precisely opposite to the result of Eq. (4.5).

According to the rules of the pseudo-spin approach, the quantum characteristics of a state with the Nilsson asymptotic $[N n_z \Lambda]\Omega$ transform into $[\widetilde{N}, \widetilde{n_z}, \widetilde{\Lambda}]\widetilde{\Omega}$, where the corresponding numbers in the pseudo-spin representation are

$$\widetilde{N} \equiv N-1\ , \qquad \widetilde{n_z} = n_z\ , \qquad \widetilde{\Lambda} = \Lambda \pm 1 \quad \text{and} \quad \widetilde{\Omega} \equiv \Omega\ . \tag{4.6}$$

It can further be shown that within the above representation

$$a \rightarrow \tilde{a} = (-1)^{\widetilde{N}} \delta_{\widetilde{\Lambda} O} \tag{4.7}$$

what may change the predictions drastically. Since such a change corresponds well with the present day evidence about some of the E_γ-degenerate nuclei, the suggestion of the pseudo-spin asymptotic being realised in superdeformed nuclei seemed motivated.

The main problem with the above interpretation is that it is applicable only if the moments of inertia (cf. Eq. (4.1)) in nucleus A and B, according to notation in Sect. II, are within a good accuracy identical ($\mathcal{J}_A = \mathcal{J}_B$). The above relation is likely to be satisfied only if the n_z-quantum number of the odd nucleon is 0, as it has been argued in the preceding sections. Again as a matter of "coincidence", the case discussed involved [301]1/2 orbital which actually satisfies all the necessery criteria i.e. of both the condition $\mathcal{J}_A = \mathcal{J}_B$ ($n_z = 0$) and $a = +1$ ($= (-1)^2$).

The above observations close the logi -l unit: we have proposed the physical mechanism in both the criteria for the polarisation in the moments of inertia and the relation of the latter with the discussion of the decoupled band properties in the presence of the pseudo-spin symmetry.

Another, more general consequence of the pseudo-spin symmetry has been discussed recently by Stephens and collaborators, Ref. [8]. There, the main idea presented was that of a (nearly) complete decoupling of the pseudo-spin degrees of freedom from the orbital motion for many (non-intruder) orbitals. Such a mechanism may result in the pseudo-spin alignments with the axis of rotation thus producing the alignments in multiples of 1/2. There exists a relation between such a mechanism and the $\delta\mathcal{J}/\mathcal{J}$ polarisation discussed below: the orbital contributions of the $\delta\mathcal{J}/\mathcal{J}$ polarisations would be neglegible for $n_z \sim 0$ orbitals with the corresponding (small) contributions originating from pseudo-spins (1/2) which should still be measurable.

V – SUMARY AND CONCLUSIONS

We have studied the microscopic conditions for the angular momentum alignments and the nuclear deformation changes in the even-even ("core") nucleus caused by "adding" to it an odd nucleon.

We found out that the $\delta\mathcal{J}/\mathcal{J}$ polarisation depends in a sentive way on the n_z-quantum number. This observation helps significantly in identifying the single-particle orbitals in superdeformed nuclei without resorting to the g-factor measurements which are very difficult at present.

A systematic comparison with experiment shows a good agreement with the predictions of the microscopic calculations based on the Woods-Saxon potential.

VI – ACKNOWLEDGMENT

This article contains the results obtained in collaboration with Z. Szymanski and T. Werner, Ref. [9].

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Figure Captions

Fig. II.1 A schematic illustration of an "unusual" character of E_γ-degeneracies discussed in the text. The $(E_\gamma^A - E_\gamma^B)$ quantity discussed troughout the article would be $\sim 10\,$keV at spins $\sim 40\,\hbar$ and even larger at higher spins if not of specific quantum-mechanical aspects of the motion of the least bound particles.

Fig. III.1a Single-particle Woods-Saxon spectrum of the neutrons in function of nuclear elongation represented by both the quadrupole deformation (β_2) explicitly displayed in the x-axis and by the hexadecapole deformation which changes along the β_2 axis in such a way that the whole scheme is applicable, on the average, for many superdeformed nuclei in the $A \sim 150$ region.

Fig. III.1b Similar to Fig. III.1a, except that now the single-particle level repulsions have been artificially removed in order to simplify the illustration of the so-called pseudo-SU_3 multiplets (marked with the dotted areas). The influence of those multiplets and the problem of abundance of the superdeformed configurations throughout the periodic table is discussed in Ref. [5] where some details and references to the original lifetime may be found.

Fig. III.2a Similar to III.1a but for the protons.

Fig. III.2b Similar to III.1b but for the protons.

Fig. III.3a The quantities $(\Delta\mathcal{J}/\mathcal{J})_\nu^{WS}$, cf. Eq. (3.6) and surrounding text, for the neutron single-particle states in the Woods-Saxon potential. The quantum number ν is replaced in the figure by the asymptotic quantum number labelling in terms of $[Nn_z\Lambda]\Omega$. The labelling corresponds exactly with the labelling of the curves in preceding figures. In this representation the line crossings have been artificially removed to simplify the display (the crossing-removal procedure becomes clear after inspecting the differences between figures III.1a and III.1b). It is important to notice, that the labels play only an auxiliary role in these figures; they change in general when the deformation passes through the crossing-points.

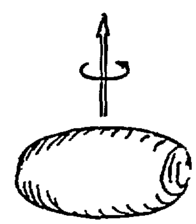
Fig. III.3b Similar to Fig. III.3a but for the protons.

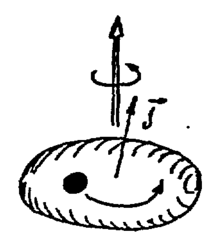
Fig. III.4 The single-particle routhian diagram illustrating the configurations involved when comparing the superdeformed band structures in ^{151}Tb and ^{150}Gd nuclei.

Fig. III.5 A comparison between the calculated and observed dynamical moments of inertia (top) and between the caculated and extracted $(\Delta\mathcal{J}/\mathcal{J})$ quantities (bottom) for the Woods-Saxon results. Several other tests performed on other nuclear couples show a similar type of agreement.

Fig. IV.1 A schematic representation of the role of the spin-orbit interaction leading to the pseudo-spin symmetry. Plotted on the left is a "model spectrum" of an average field potential (characteristic e.g. for Woods-Saxon spherical shape) without spin-orbit term taken into account (only the levels corresponding a large main-shell N one shown). Introducing the spin-orbit interaction leads to the spin-orbit splitting represented schematically by the dashed lines. By what appears as a miracle, the empirical (realistic) strength of the spin-orbit interaction is such, that the double degeneracy pattern illustrated, corresponds to all the quantum numbers being common except for the spin (more precisely pseudo-spin) degrees of freedom, see text.

Why are the SD–Degeneracies Strange?





Rotor:

$$E_\gamma(I) = (4I+6)\,\frac{\hbar^2}{2J}$$

Rotor + Particle:

$$E_\gamma(I) = (4I+6)\,\frac{\hbar^2}{2(J+\Delta J)} \approx$$

$$= E_\gamma(I)\,[\,1-(\Delta J/J)\,]$$

$$= E_\gamma(I) + \Delta E_\gamma(I)$$

Estimate

$J(A) \longrightarrow J_{RIG} \sim A^{5/3}/70$ [$\hbar^2$ MeV^{-1}]

$\Delta J/J \sim 5/(3A) \sim 0.01$ (for $A \sim 150$)

$\Big\}\ \Delta E_\gamma(I) \sim 10$ keV

Figure II.1

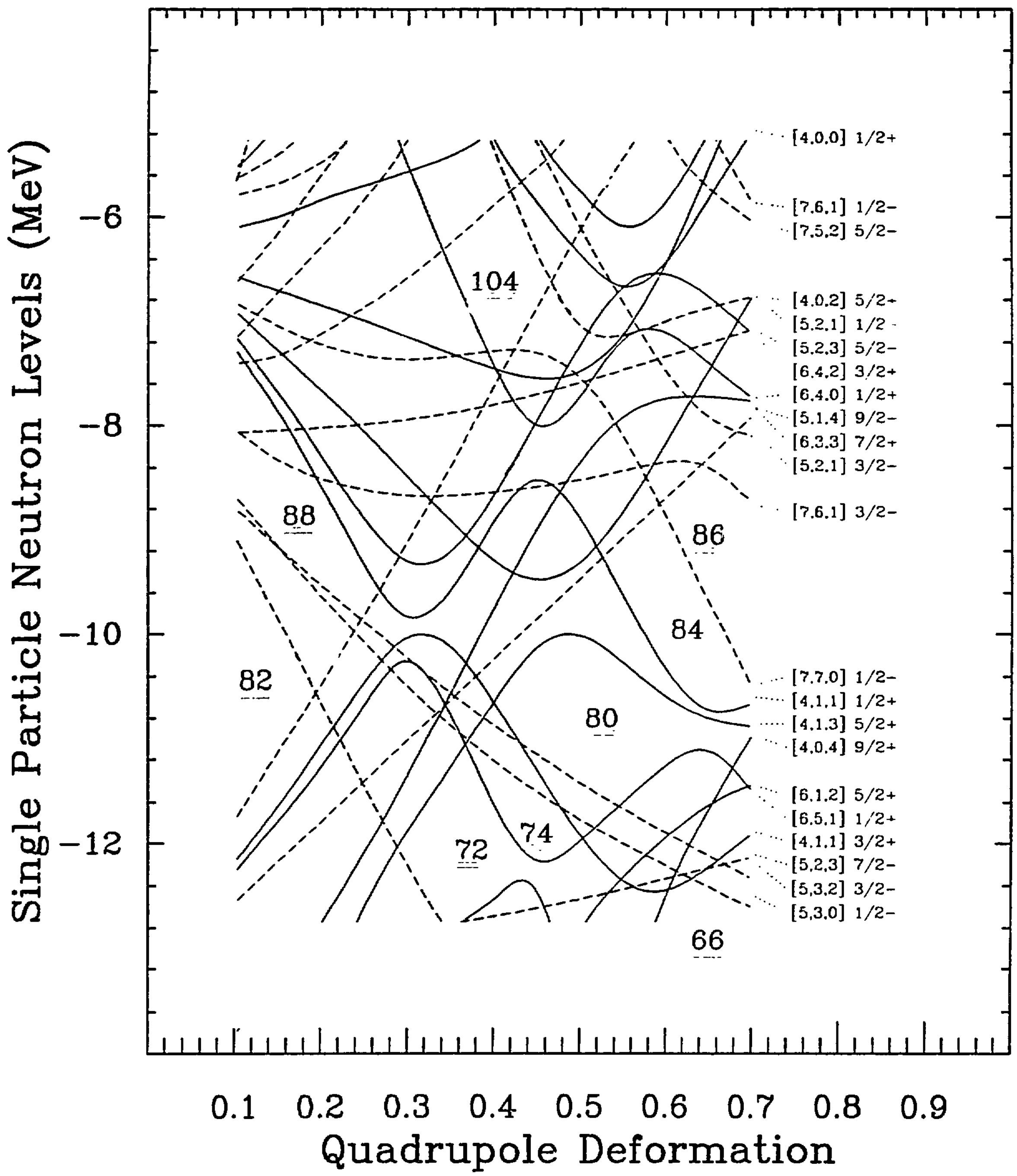


Figure III.1a

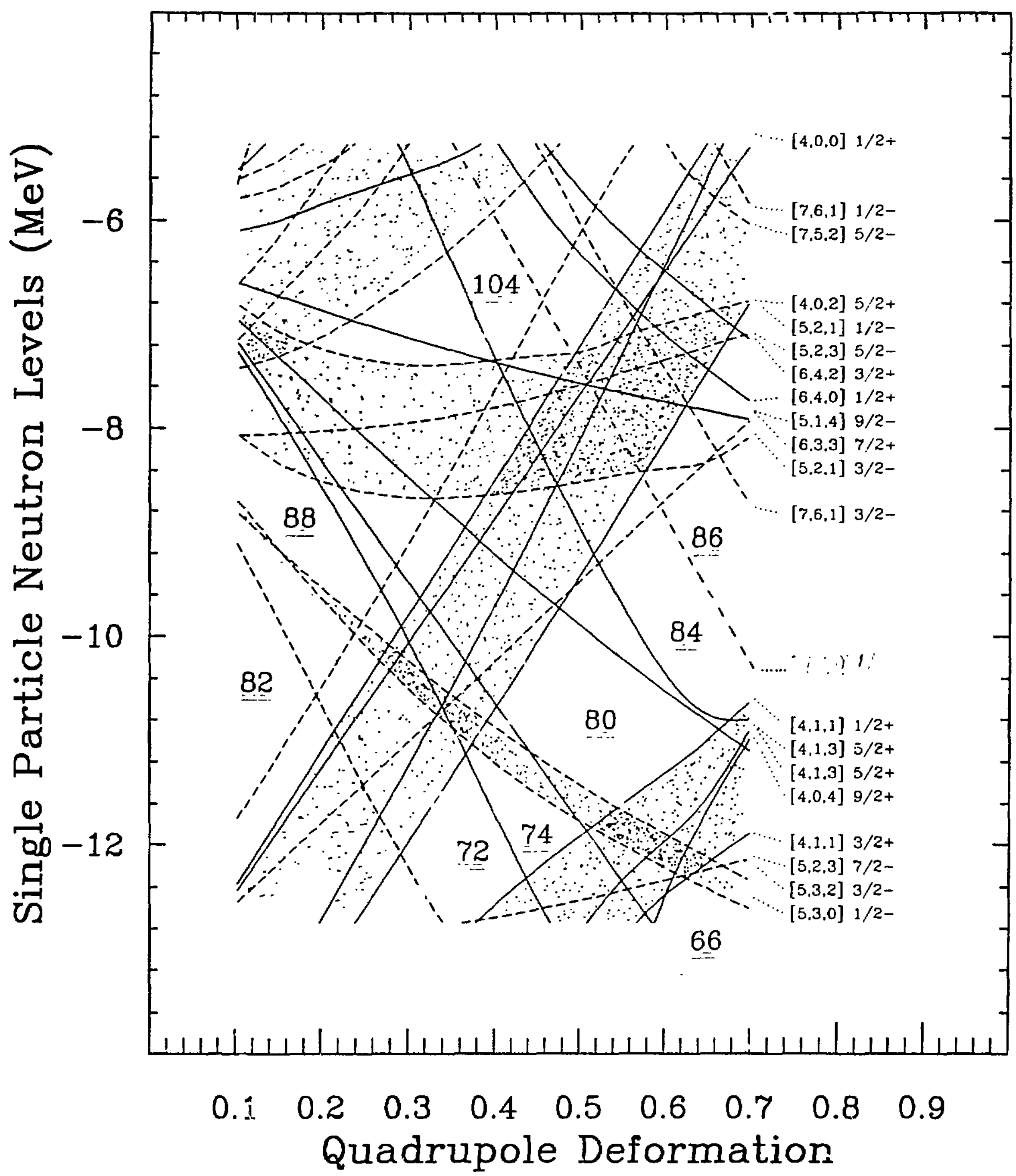


Figure III.1b

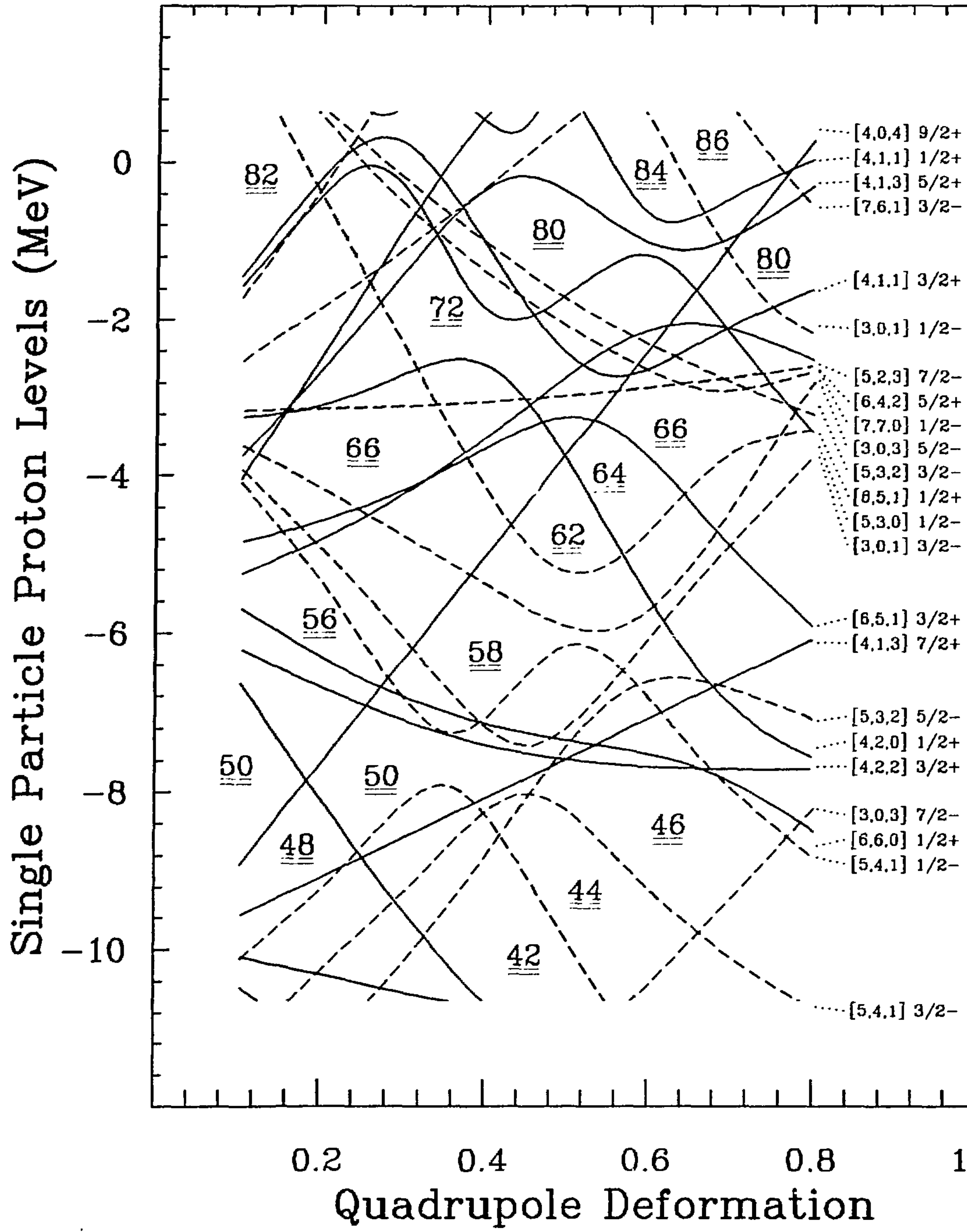


Figure III.2a

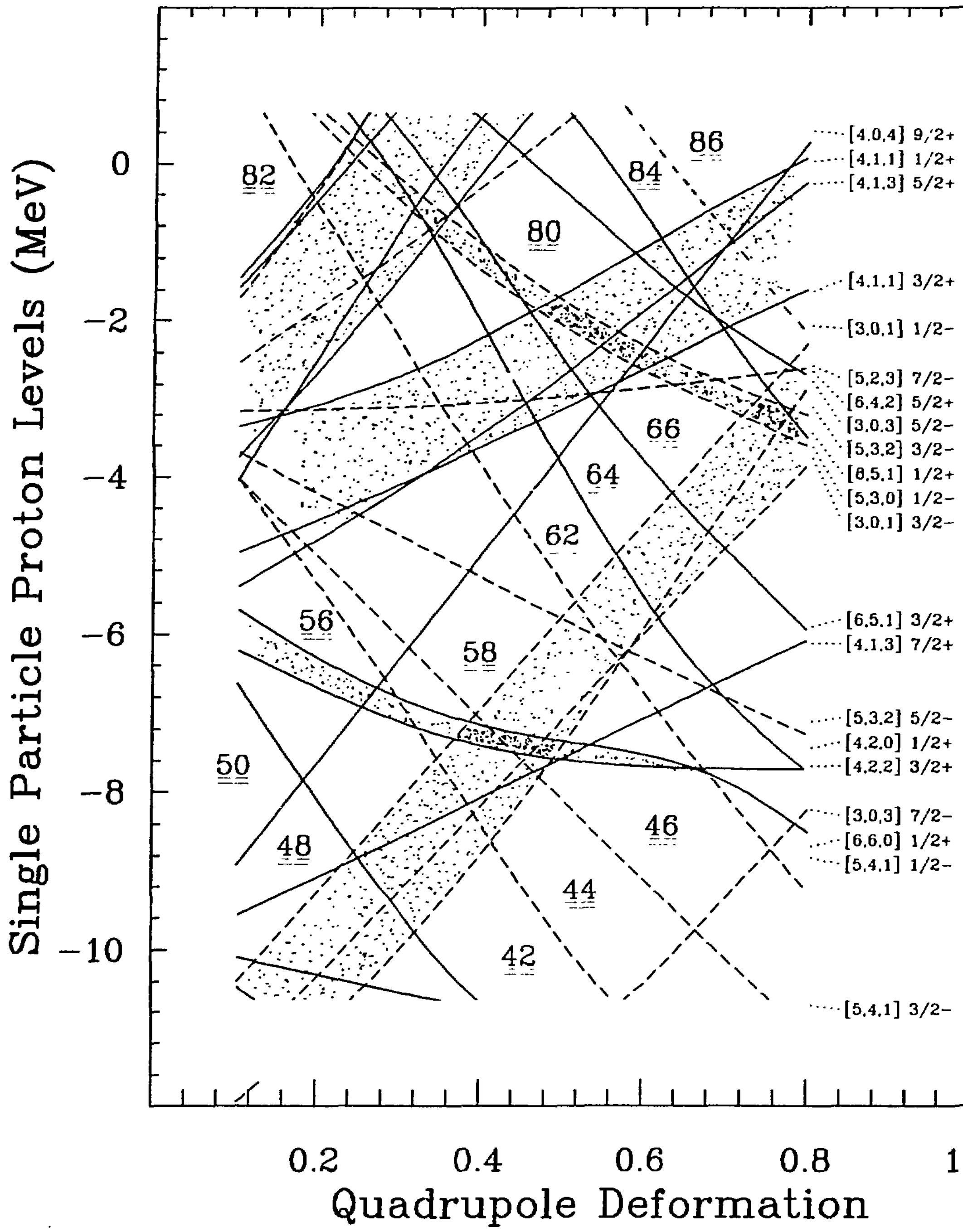


Figure III.2b

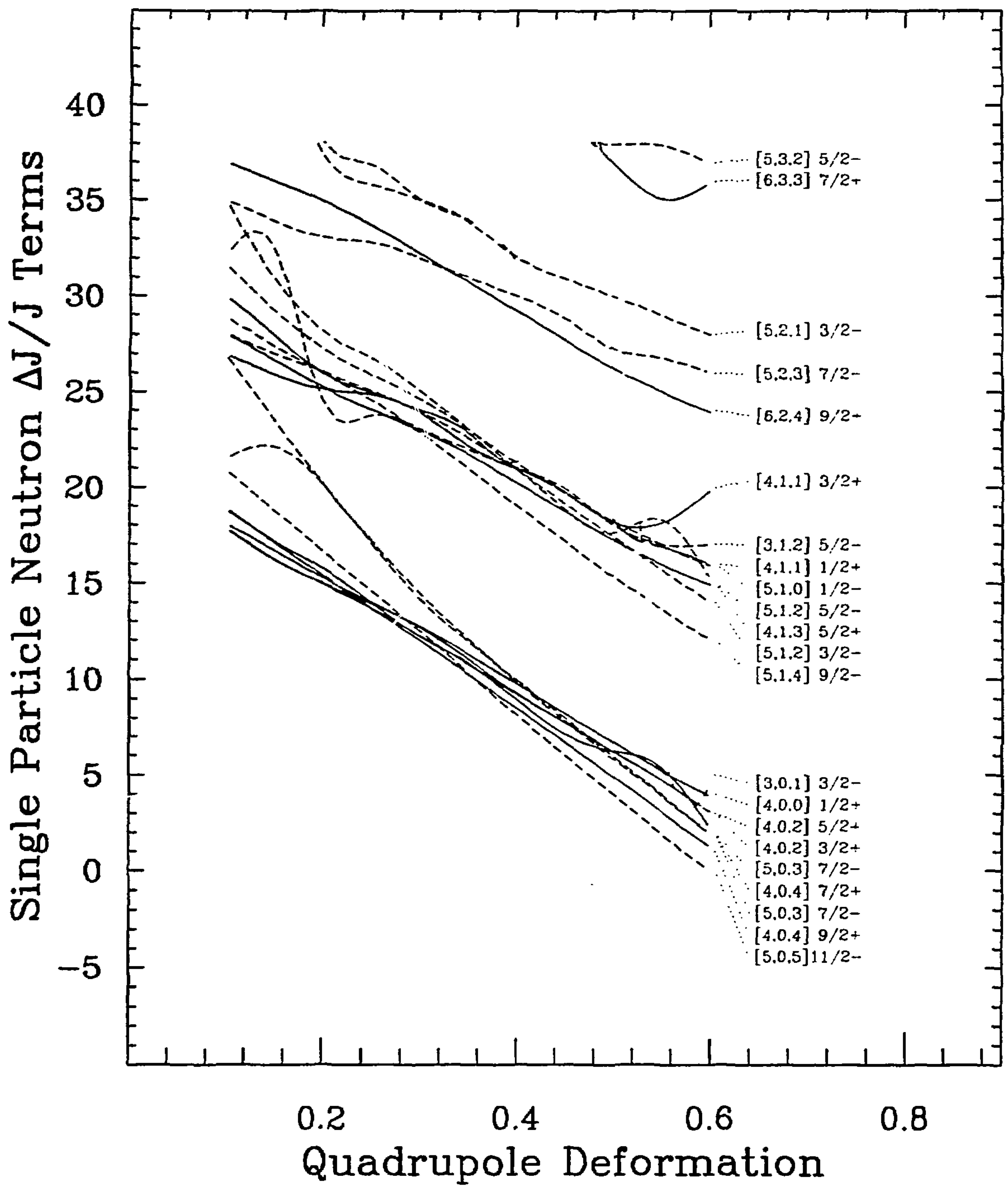


Figure III.3a

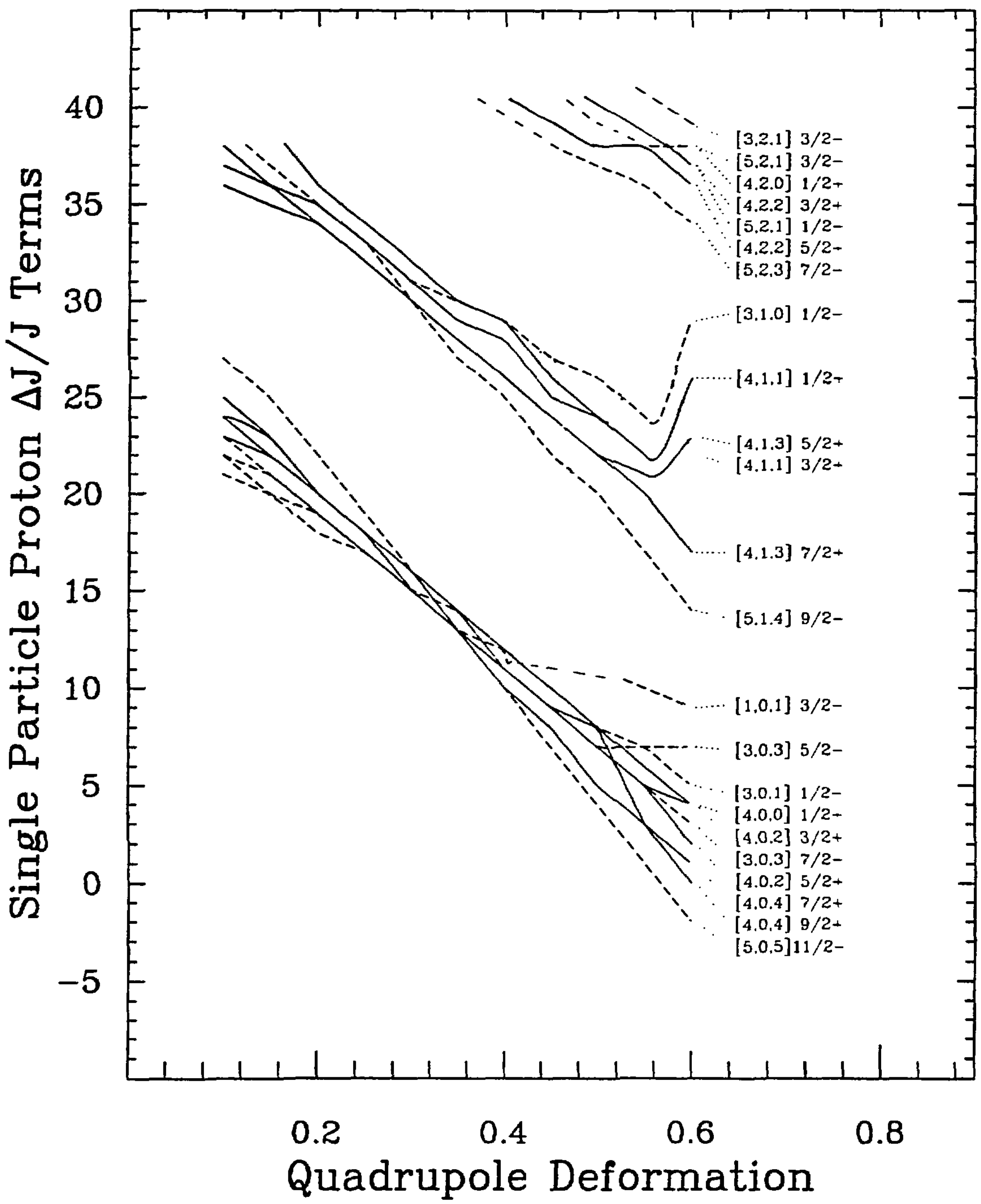
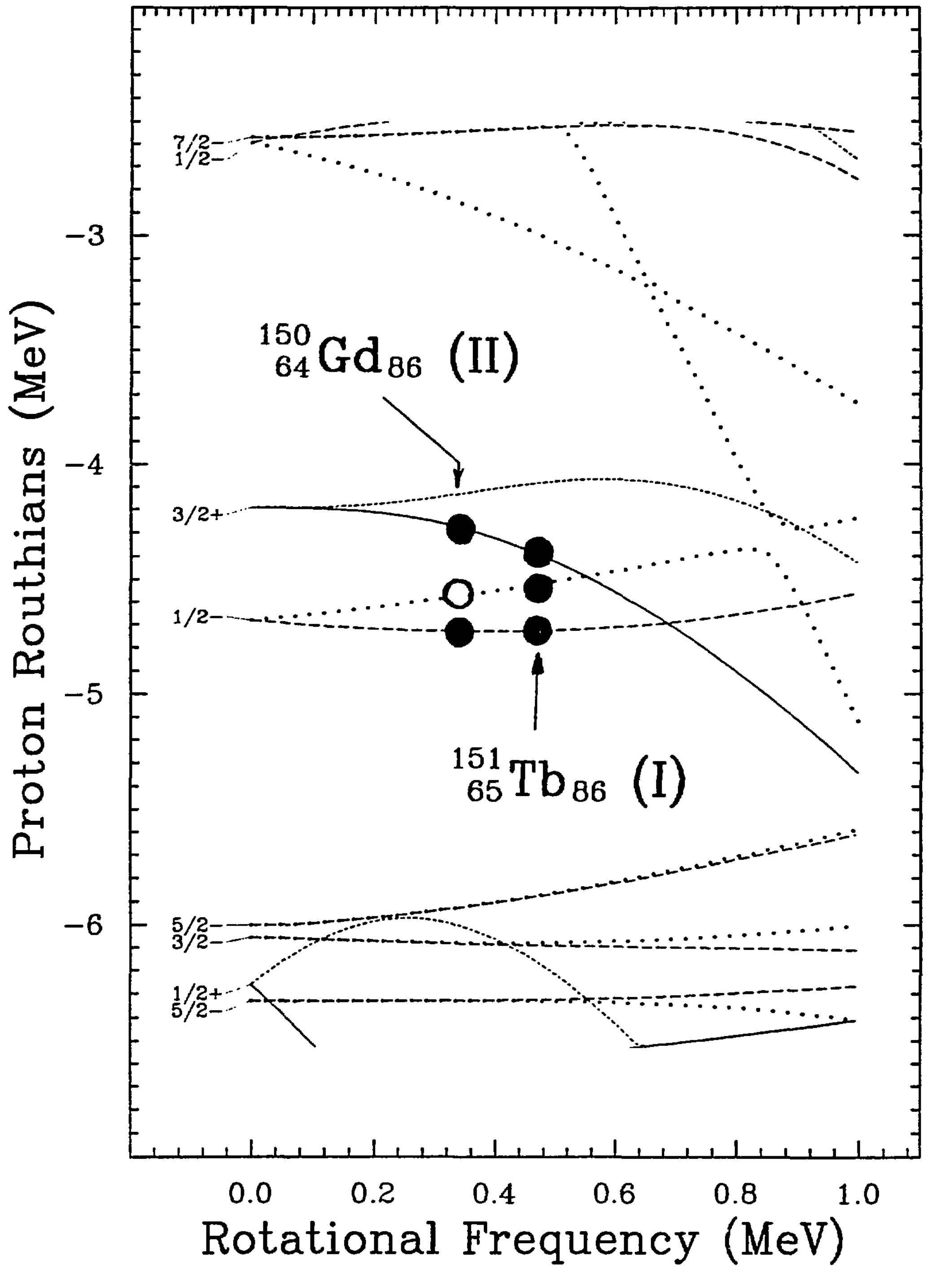
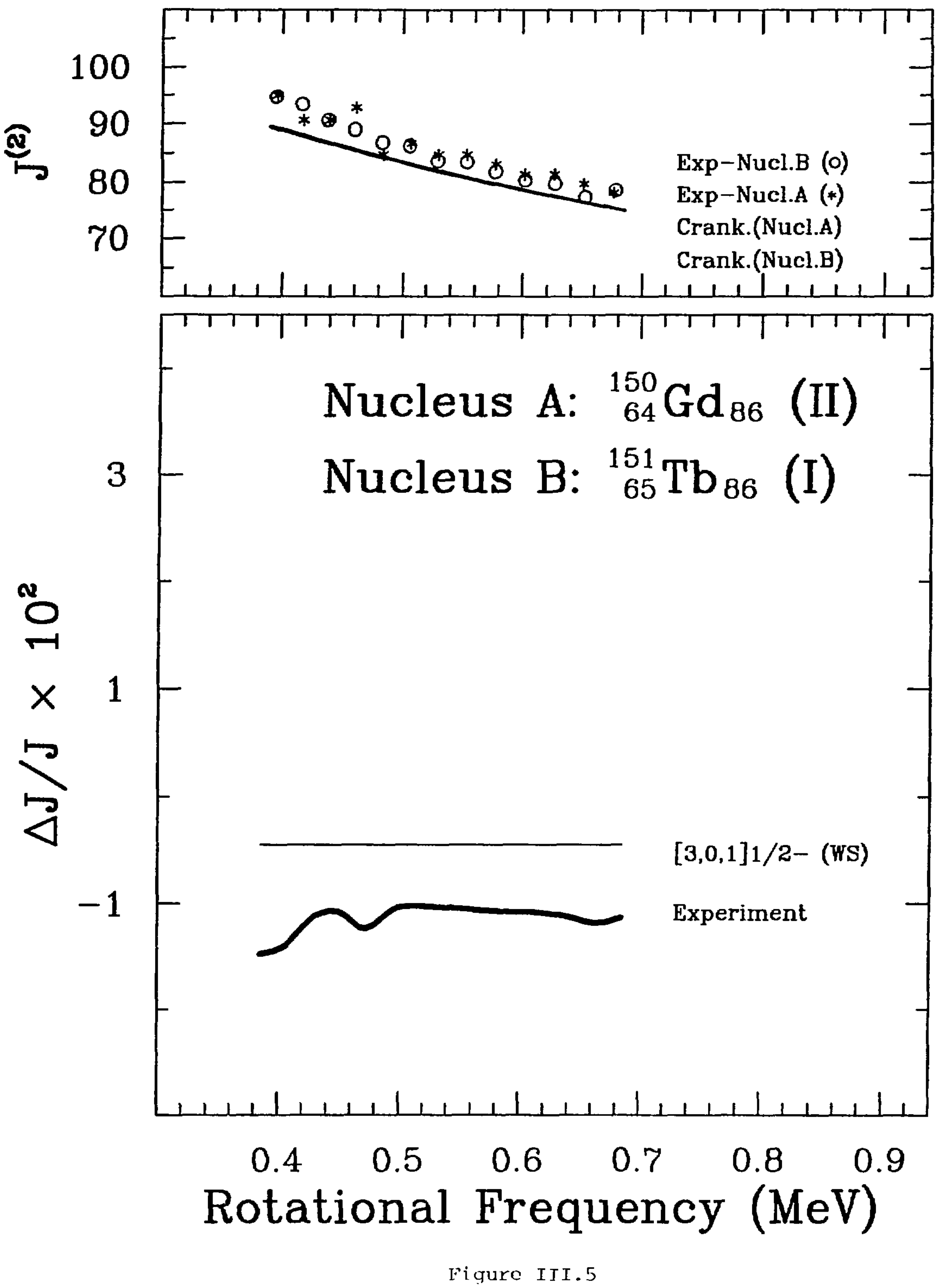


Figure III.3b



Proton Routhians (MeV)
-3
-4
-5
-6
7/2-
1/2-
3/2+
1/2-
5/2-
3/2-
1/2+
5/2-
$^{150}_{64}Gd_{86}$ (II)
$^{151}_{65}Tb_{86}$ (I)
0.0
0.2
0.4
0.6
0.8
1.0
Rotational Frequency (MeV)

Figure III.4



100
90
80
70
$J^{(2)}$
Exp−Nucl.B (o)
Exp−Nucl.A (*)
Crank.(Nucl.A)
Crank.(Nucl.B)
Nucleus A: $^{150}_{64}$Gd$_{86}$ (II)
Nucleus B: $^{151}_{65}$Tb$_{86}$ (I)
3
1
−1
$\Delta J/J \times 10^2$
[3,0,1]1/2− (WS)
Experiment
0.4
0.5
0.6
0.7
0.8
0.9
Rotational Frequency (MeV)

Figure III.5

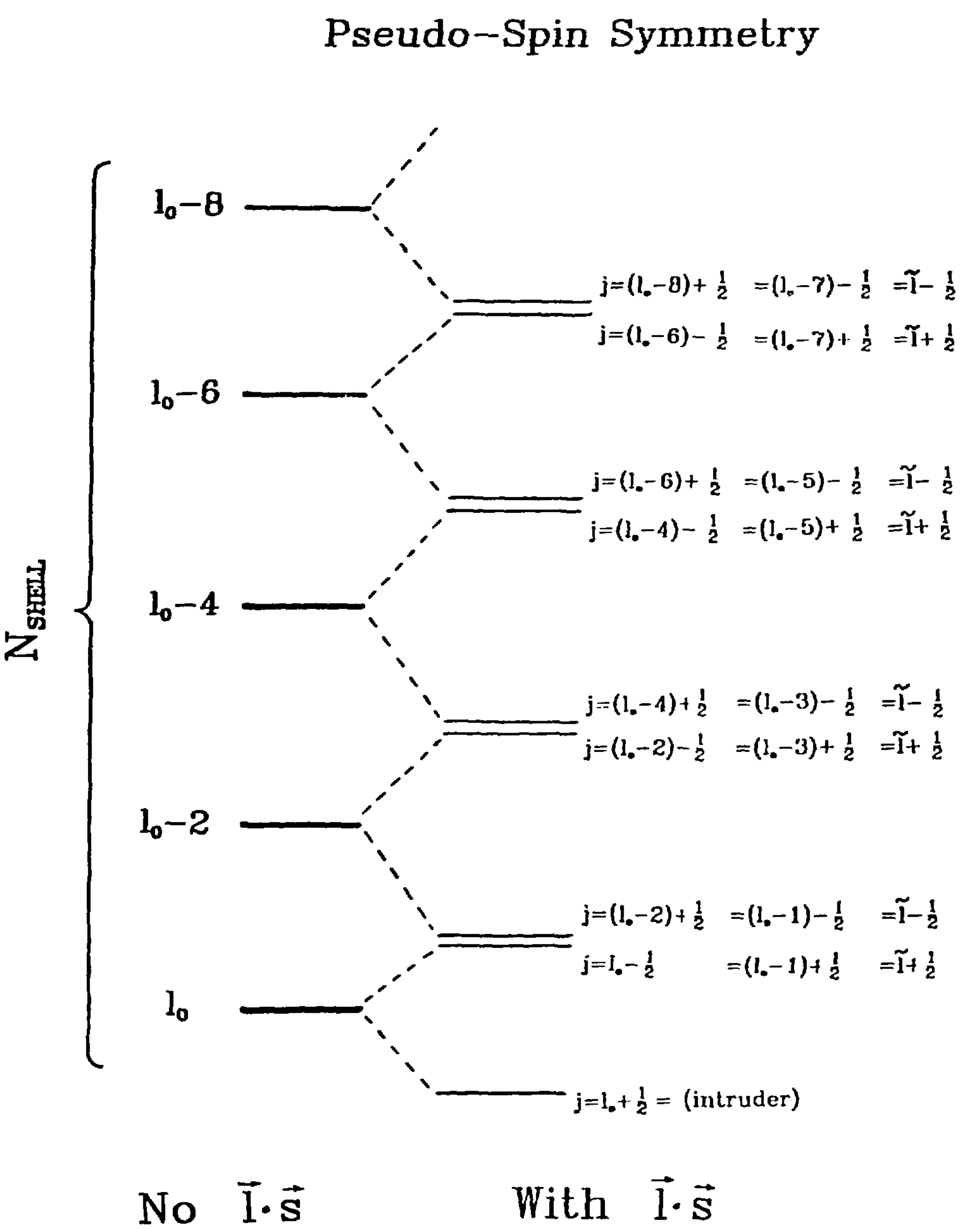

Pseudo–Spin Symmetry
N_{SHELL}
l_0-8
l_0-6
l_0-4
l_0-2
l_0
$j=(l_0-8)+\frac{1}{2} = (l_0-7)-\frac{1}{2} = \tilde{l}-\frac{1}{2}$
$j=(l_0-6)-\frac{1}{2} = (l_0-7)+\frac{1}{2} = \tilde{l}+\frac{1}{2}$
$j=(l_0-6)+\frac{1}{2} = (l_0-5)-\frac{1}{2} = \tilde{l}-\frac{1}{2}$
$j=(l_0-4)-\frac{1}{2} = (l_0-5)+\frac{1}{2} = \tilde{l}+\frac{1}{2}$
$j=(l_0-4)+\frac{1}{2} = (l_0-3)-\frac{1}{2} = \tilde{l}-\frac{1}{2}$
$j=(l_0-2)-\frac{1}{2} = (l_0-3)+\frac{1}{2} = \tilde{l}+\frac{1}{2}$
$j=(l_0-2)+\frac{1}{2} = (l_0-1)-\frac{1}{2} = \tilde{l}-\frac{1}{2}$
$j=l_0-\frac{1}{2} = (l_0-1)+\frac{1}{2} = \tilde{l}+\frac{1}{2}$
$j=l_0+\frac{1}{2}$ = (intruder)
No $\vec{l}\cdot\vec{s}$
With $\vec{l}\cdot\vec{s}$

Figure IV.1

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