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GLOBAL ALPHA-PARTICLE OPTICAL POTENTIALS

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Abstract:

A search for a global optical potential for alpha-particles is described. It did not prove possible to find such a potential valid for a wide range of energies and nuclei, even treating the absorbing potential as an adjustable parameter for each nucleus. For practical purposes the best that can be done is to define an average potential, and such a potential is compared with a wide range of experimental data. Its energy variation is determined by fitting the total reaction cross-section.

1. Introduction

Global optical potentials are widely used in nuclear reaction calculations and several reliable parameter sets are available for neutrons and protons. There is however no reliable global set for alpha particles. There are several reasons for this including the lack of suitable data on alpha particles, the ambiguities in the potential and the sensitivity of the alpha particle scattering to the nuclear surface.

The aim of this work is to see if a global potential can be obtained for alpha particles. In view of sensitivity of alpha particle scattering to the nuclear surface we expect that it would be necessary to depart from the ideal of a global potential but it was hoped that it would be possible to take this into account by allowing one parameter, probably the strength of the imaginary potential, to vary.

Section 2 describes the choice of potential and some of its characteristics. These are obtained by calculating the differential cross-section for elastic scattering of alpha particles and varying the parameters of the potential one at a time to see the effect of parameters on the scattering. Section 3 describes the search for a global potential starting with that of Delbar *et al*, who obtained excellent fits to very accurate data for the elastic scattering of alpha particles by ^{40}Ca over a wide range of energies from 40 to 166 MeV with a simple energy dependence. It was hoped that a similar potential with different values of the absorbing potential would fit other nuclei and thus form the basis for a global potential. This attempt failed probably because of the rather special structure of ^{40}Ca. The second attempt was based on widely used analysis of McFadden and

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Satchler of the scattering of 24.7 MeV alpha-particles by a range of nuclei. The results of this analysis gave a very high χ^2 to the fits which probably suggests that it is not possible to obtain good fits by varying one parameter W only.

The average potential based on the analysis of McFadden and Satchler is compared with a wide range of data, and the results are given in section 4. Since a reliable potential must be able to fit the differential cross-section as well as the total reaction cross-section we study the energy variation of parameters by fitting the total reaction cross-section of $^{51}V + \alpha$. Section 5 describes this analysis and conclusions are given in section 6.

2. Choice of Potential and Method of Analysis

The optical potential has the form

$$U(r) = V_c(r) + U f_u(r) + iW f_w(r)$$

where the form factors $f_i(r) = \{1 + \exp(r - R_i)/a_i\}^{-N}$ with $R_i = r_i A^{1/3}$, and $N = 1$ or 2.

The potential varies with energy and it is usual to parametrize this variation by keeping the form factor parameters R and a constant and allowing the potential depths U and W to depend on energy.

The differential cross-sections for elastic scattering and the total reaction cross-section can be calculated by standard quantum mechanical and numerical techniques from a potential of the above form. The analysis of a particular set of experimental data is usually carried out by making such a calculation with values of the parameters chosen on general grounds or from previous analyses of similar data. The results of such calculations are compared with the experimental data and a measure of goodness of fit defined and evaluated. Fits to the data are obtained by varying the parameters of potential so as to minimize the mean-square deviation between the experimental and predicted cross-sections

$$\chi^2 = N^{-1} \sum_i \left[\frac{\sigma_{\text{exp}}(\theta_i) - \sigma_{\text{th}}(\theta_i)}{\Delta\sigma \exp(\theta_i)} \right]^2$$

where $\sigma_{\text{exp}}(\theta_i)$ and $\sigma_{\text{th}}(\theta_i)$ are experimental and theoretical cross-sections respectively at angles θ_i, $\Delta\sigma_{\text{exp}}(\theta_i)$ is the error associated with $\sigma_{\text{exp}}(\theta_i)$ and N is the number of data points fitted. The resulting optimum set of parameters define a particular potential that corresponds to a particular data set.

Typical data sets correspond to the scattering of particles of particular energy by a range of target nuclei. Each data set may be analysed individually in the way described above and it is usually found that the parameters are quite similar. In the case of nucleon scattering, close examination of the sensitivity of the calculated observables to the parameters of the potential shows that there are correlated pairs of parameters that can be varied together over certain ranges of values without much affecting the values of the observables. In particular, variations of U and R_u that maintain the constancy UR_u^2 have little effect on the observables and similarly for the product Wa_w. This suggests that it is useful to fix the form factor parameters to average values and then to repeat the whole analysis allowing only the potential depths to vary. Because of the flexibility of the parameters just mentioned the fits to the data obtained in this way are very little inferior to those obtained when all the parameters are allowed to vary.

It is useful to see whether these characteristics are also found for alpha particle scattering. We therefore studied the effect of variation of different parameters of the four parameter average potential on the scattering by varying one parameter at a time. We chose the scattering of 24.7 MeV alpha particles on cobalt and found that the amplitudes of the differential cross-section does not change with changes in U and r_u, increases with increase of diffuseness parameter a and decreases with the increase of absorbing potential W where we set $r_u = r_w$ and $a = a_u = a_w$ here and subsequently. Figs.1a–d show the ratio of differential cross-section to Rutherford for elastic scattering of 24.7 MeV alpha particles on cobalt. (a) shows the effect of varying U, (b) shows the effect of varying W, (c) shows the effect of varying a and (d) shows the effect of varying r_u.

Analysis of the graphs suggests that a general potential that would fit a number of data for different nuclei at different energies could be obtained by a potential having the same U and r_u but different W and a. This also shows that alpha particle scattering is sensitive to the surface and we search for a potential by varying W and a.

3. The Search for a Global Alpha-Particle Potential

(a) The Potential of Delbar *et al* (1978)

The most extensive analysis of alpha-particle elastic scattering over a wide range of energies has been made for ^{40}Ca by Delbar *et al.* They analysed the elastic and inelastic scattering of alpha particles by ^{40}Ca from 40 to 166 MeV using squared Woods-Saxon form factors. They were able to obtain a potential that fitted ^{40}Ca data very well including the effects of anomalous large angle scattering. We therefore began with their best potential and tried to fit the data for other nuclei by allowing the depth of the imaginary potential to vary. The experimental data we use is that for 24.7 MeV alpha-particles scattered by Mn, Co, Cu and Ge analysed by McFadden and Satchler (1966).

Starting with Delbar's potential with values $U = 190.36$ MeV, $W = 10.10$ MeV, $r_u = 1.37$fm, $r_w = 1.75$fm, $a_v = 1.29$fm and $a_w = 1.0$fm and allowing W to vary the best values of W we obtain are as follows for

Mn:	$W = 41.69$ MeV	with	$\chi^2 = 16.3$
Co:	$W = 26.17$ MeV	"	$\chi^2 = 18.2$
Cu:	$W = 26.08$ MeV	"	$\chi^2 = 9.64$
Ge:	$W = 11.14$ MeV	"	$\chi^2 = 19.91$

The results are shown in Figs.2–5.

Since the values of χ^2 are unacceptably high, we allowed the parameter a_w to vary keeping all other parameters fixed to see the effect of the imaginary diffuseness parameter on the scattering. This search also ended with diffuseness

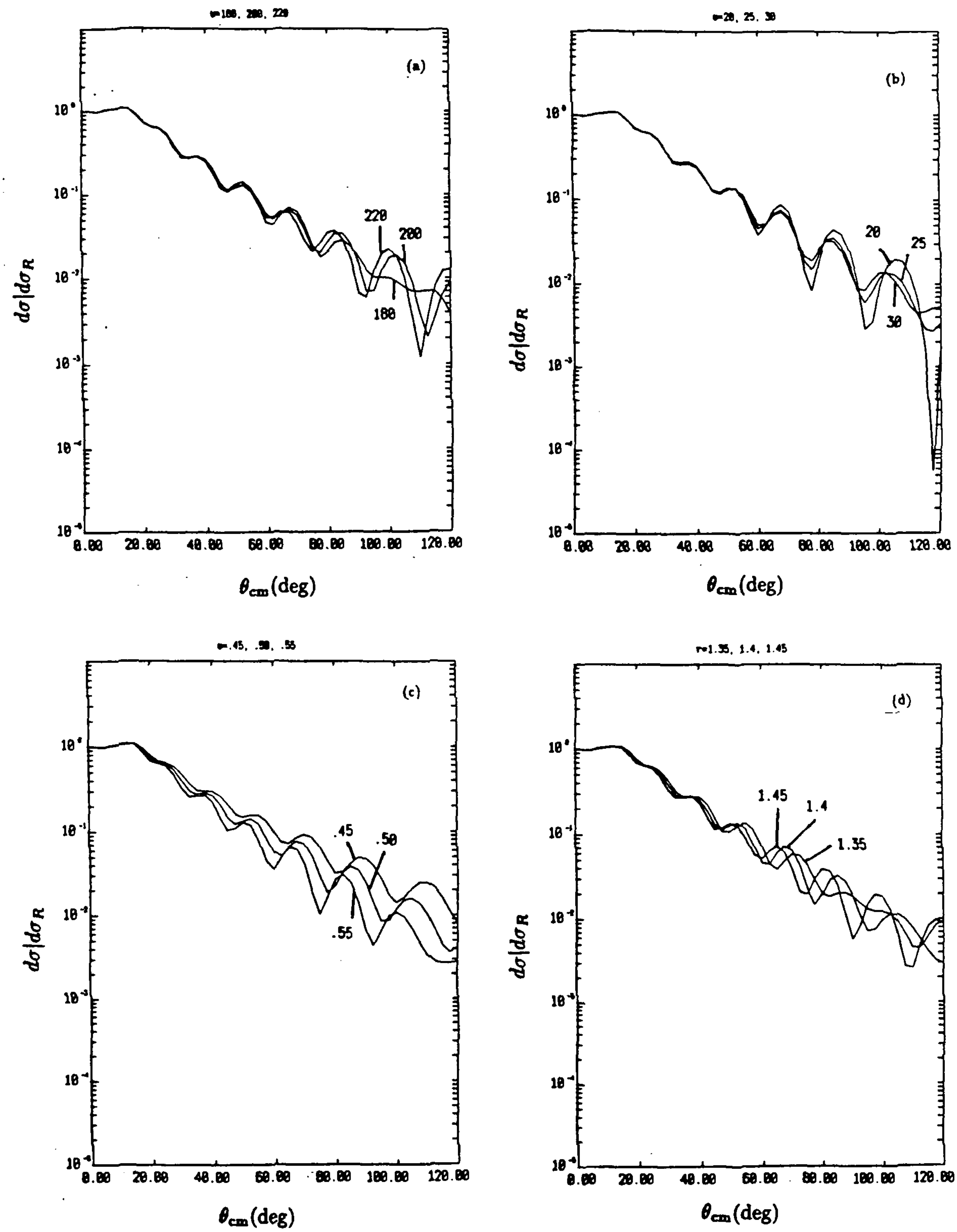


Fig.1. Ratio of differential cross-section to Rutherford cross-section for elastic scattering of 24.7 MeV alpha particles on Cobalt.

(a) Effect of varying the real part of the potential U, U = 180, 200, 220 MeV.

(b) Effect of varying the imaginary part of the potential W, W = 20, 25, 30 MeV.

(c) Effect of varying the diffuseness parameter a, a = 0.45, 0.50, 0.55fm.

(d) Effect of varying the radius of the potential r. r = 1.35, 1.40, 1.45fm.

parameters which are quite different and also very high values of χ^2. The best values of a_w obtained were for

Mn:	$a_w = 0.400$fm	with	$\chi^2 = 343$
Co:	$a_w = 0.324$fm	"	$\chi^2 = 108$
Cu:	$a_w = 0.514$fm	"	$\chi^2 = 118$
Ge:	$a_w = 0.798$fm	"	$\chi^2 = 20.9$

Thus we are unable to find a global alpha particle optical potential based on that of Delbar *et al.* This failure is probably attributable to the rather untypical structure of ^{40}Ca.

(b) The potential of McFadden and Satchler (1966)

We then decided to use an average potential obtained from the set of parameters given by McFadden and Satchler at 24.7 MeV for a range of nuclei. The average potential we use is a four parameter potential having the same geometry for real and imaginary part.

We chose as average potential with $U = 205.88$ MeV, $W = 25.78$ MeV, $r_u = 1.414$fm and $a = 0.529$fm and investigated whether this can serve as global optical potential with one of the parameters absorbing the diffuseness in nuclear structure. The results obtained are as follows:

By allowing W to vary and keeping U, a and r_u fixed we obtain for

Mn:	$W = 34.72$ MeV	with	$\chi^2 = 9.41$	(1.64)
Co:	$W = 25.87$ MeV	"	$\chi^2 = 2.33$	(1.62)
Cu:	$W = 27.74$ MeV	"	$\chi^2 = 7.53$	(3.23)
Ge:	$W = 32.28$ MeV	"	$\chi^2 = 10.6$	(.757)

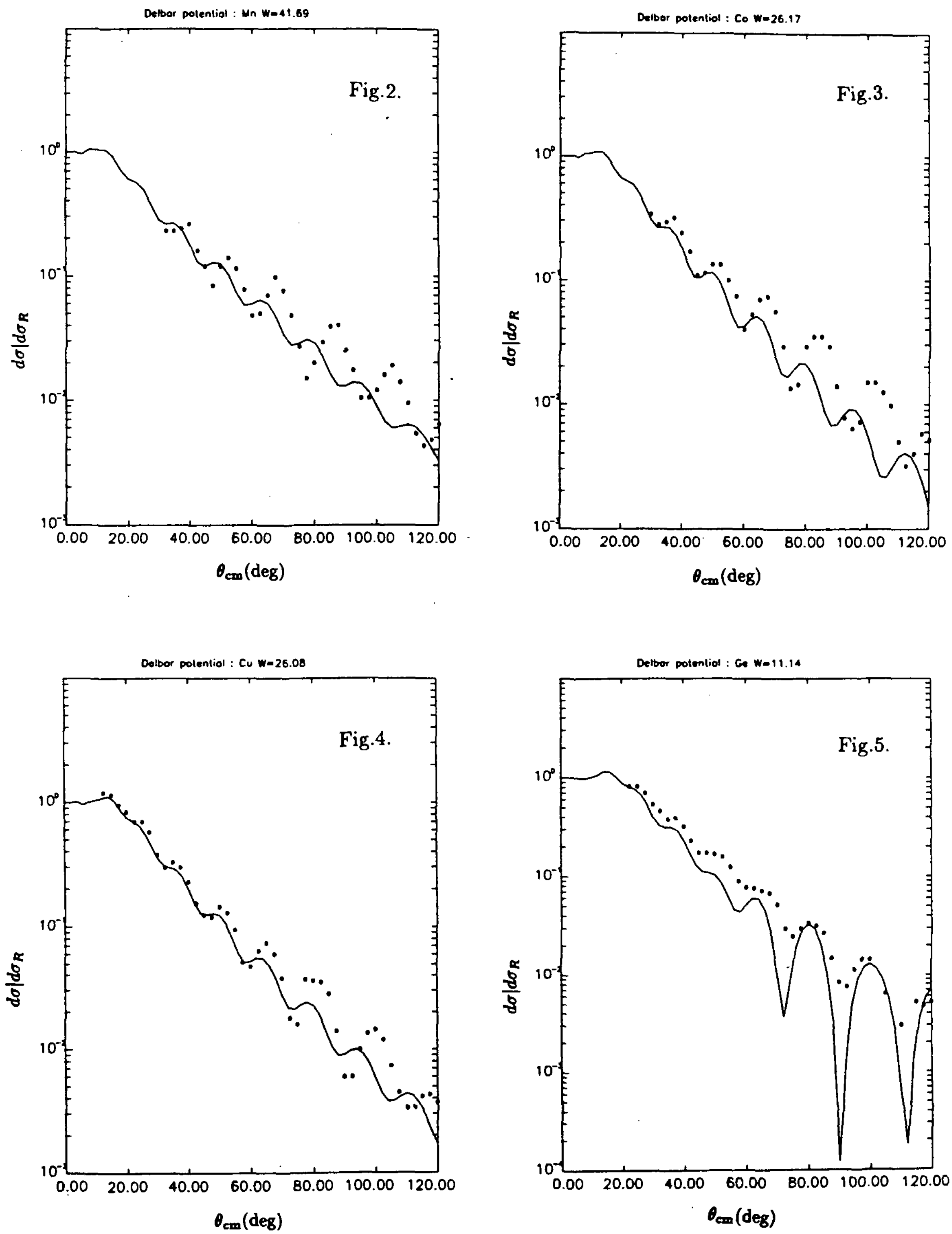
Figs.6–9 show the ratio of differential cross-section to Rutherford cross-section for elastic scattering of 24.7 MeV alpha particles on Mn, Co, Cu and Ge together with the experimental values of McFadden and Satchler. The calculated values are obtained by using the average potential except for W, which are the best values obtained by varying W only.

The values of χ^2 for this analysis are quite high compared with the values obtained by McFadden and Satchler for their best fits to the data (obtained by varying all parameters) which are shown in brackets.

This suggests that the scattering may depend not only on W but also on other parameters such as a as shown by earlier analysis of the variation of different parameters. Allowing a to vary and keeping U, W and r_u fixed we obtain:

Mn:	$a_u = a_w = 0.515$fm	with	$\chi^2 = 9.66$
Co:	$a_u = a_w = 0.525$fm	"	$\chi^2 = 2.10$
Cu:	$a_u = a_w = 0.547$fm	"	$\chi^2 = 4.99$
Ge:	$a_v = a_w = 0.540$fm	"	$\chi^2 = 16.71$

These values of χ^2 are not very much different from those obtained for the variation of W. Analysis by varying r_u and keeping U, W and a fixed gives



Figs.2–5. Comparison of experimental and calculated values of ratios of differential cross-section to Rutherford cross-section for scattering of 24.7 MeV alpha particles. The calculated values are obtained by using Delbar potential except for the values of W shown, which are the best values obtained by varying W. The experimental values are those analysed by McFadden and Satchler.

similar results without any improvement on the values of χ^2. Thus by allowing r_u to vary and keeping U, W and a fixed we obtain:

Mn:	$r_u = r_w = 1.423$fm	with	$\chi^2 = 10.43$
Co:	$r_u = r_w = 1.414$fm	"	$\chi^2 = 2.32$
Cu:	$r_u = r_w = 1.409$fm	"	$\chi^2 = 7.87$
Ge:	$r_u = r_w = 1.378$fm	"	$\chi^2 = 8.27$

(c) Comparison with data of Fulmer *et al* (1968)

It is useful to see how well this average potential fits data for other nuclei and other energies. For this we choose Fulmer *et al* data of 21 MeV alpha-particle scattering on $A = 58 - 64$ targets. Using the average potential and keeping U, r_u and a fixed but varying W we obtain for

^{58}Fe:	$W = 22.51$ MeV	with	$\chi^2 = 22.08$
^{58}Ni:	$W = 22.38$ MeV	"	$\chi^2 = 11.13$
^{62}Ni:	$W = 23.94$ MeV	"	$\chi^2 = 45.59$
^{64}Ni:	$W = 29.55$ MeV	"	$\chi^2 = 45.70$
^{62}Zn:	$W = 19.82$ MeV	"	$\chi^2 = 59.64$

Figs.10–13 show the ratio of differential cross-section to Rutherford cross-section for elastic scattering of 21 MeV alpha particles on ^{58}Fe, ^{58}Ni, ^{62}Ni, ^{64}Ni and ^{64}Zn, together with the experimental values of Fulmer *et al.* By allowing a to vary and keeping U, W and r_u fixed as we did before. We obtain for

^{58}Fe:	$a_u = a_w = 0.559$fm	with	$\chi^2 = 5.46$
^{58}Ni:	$a_u = a_w = 0.543$fm	"	$\chi^2 = 9.81$
^{62}Ni:	$a_u = a_w = 0.545$fm	"	$\chi^2 = 40.04$
^{64}Ni:	$a_u = a_w = 0.537$fm	"	$\chi^2 = 44.22$
^{64}Zn:	$a_u = a_w = 0.519$fm	"	$\chi^2 = 127.4$

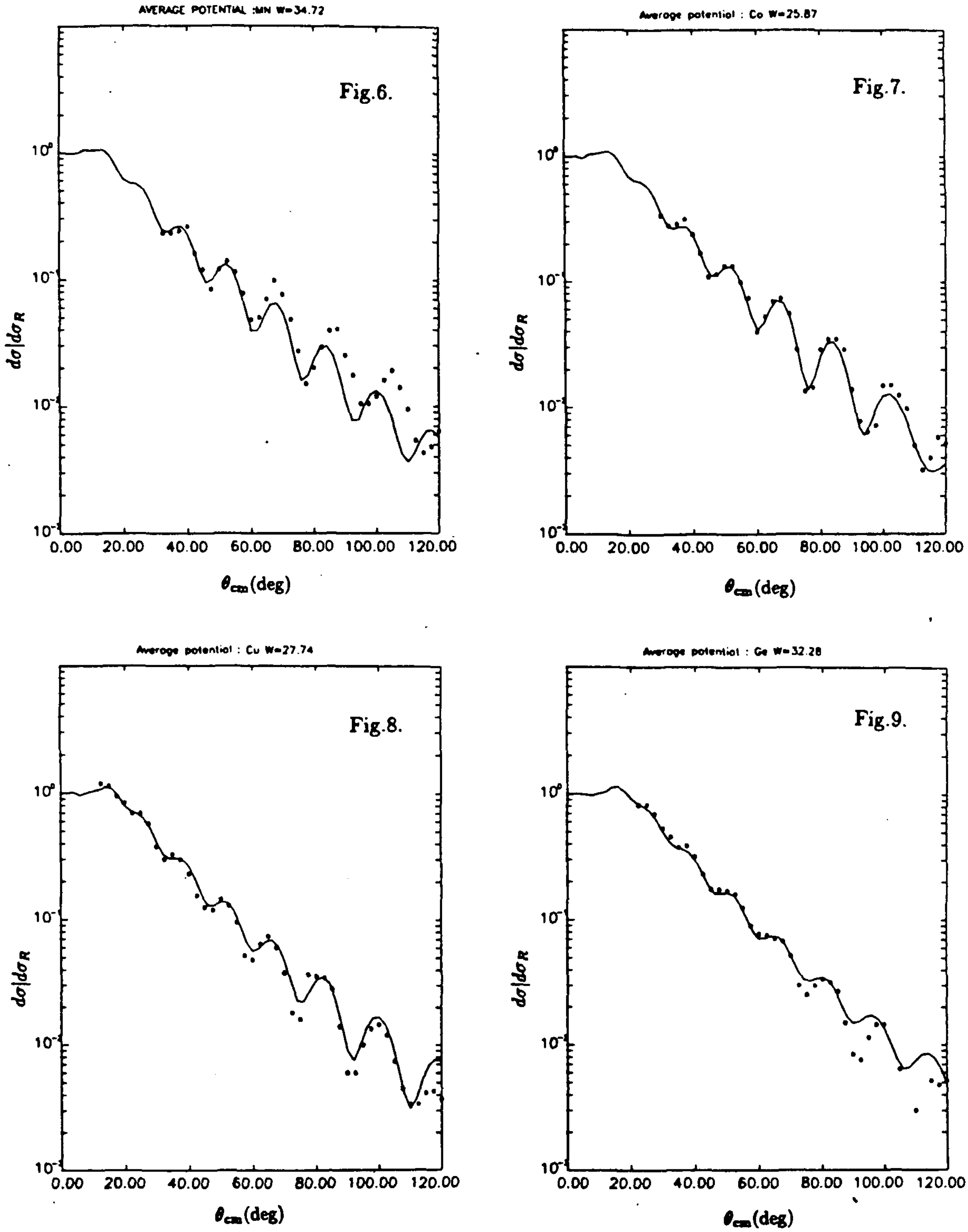
And finally allowing r_u to vary keeping U, w and a fixed we obtain for

^{58}Fe:	$r_u = r_w = 1.426$fm	with	$\chi^2 = 18.93$
^{58}Ni:	$r_u = r_w = 1.423$fm	"	$\chi^2 = 11.58$
^{65}Ni:	$r_u = r_w = 1.392$fm	"	$\chi^2 = 32.14$
^{64}Ni:	$r_u = r_w = 1.403$fm	"	$\chi^2 = 38.43$
^{64}Zn:	$r_u = r_w = 1.397$fm	"	$\chi^2 = 91.14$

As we can see that the values of χ^2 are again unacceptably high like those obtained from earlier analyses.

(d) Comparison with data of Nolte *et al* (1987)

We then studied the potential of Nolte *et al* (1987) who obtained a global potential for alpha energies higher than 80 MeV. They chose a fixed geometry for Woods-Saxon form factor as suggested by Put and Paans (1977) and allowed the depths of the real potential U and imaginary potential W to vary with energy and target number. We used this potential to calculate the differential



Figs.6–9. Comparison of experimental and calculated values of ratios of differential cross-section to Rutherford cross-section for scattering of 24.7 MeV alpha particles. The calculated values are obtained by using Average potential based on those of McFadden and Satchler except for the values of W shown, which are the best values obtained by varying W.

cross-section at 24.7 MeV for different nuclei. Though Nolte *et al* were able to produce good fits to the corresponding data with values of χ^2 less than 20.0 for a range of nuclei and for E_α above 80 MeV, their potential when used at 24.7 MeV gave $\chi^2 = 35.21$ and 45.66 for Cu and Co respectively compared to $\chi^2 = 8.33$ and 2.33 for average potential and $\chi^2 = 4.90$ and 2.99 for the McFadden and Satchler potential for Cu and Co respectively. Thus the potential suitable for higher energies from 80 to 160 MeV cannot be used at a lower energy of 24.7 MeV.

Probably the reason why Nolte *et al* obtained a global potential for energies above 80 MeV is that the medium energy data are well reproduced by a single Woods-Saxon form factor as well as by a squared Woods-Saxon form factor. And since the low energy data suffer from discrete and continuous ambiguities in the optical model parameters (Nolte *et al*) it is difficult to find a more general potential for a range of nuclei at lower energy.

The above analysis shows that it is not possible to obtain good fits on the basis of average potential simply by varying one parameter. Fits of the quality of those of McFadden and Satchler can only be obtained by varying all the parameters simultaneously. We thus confirm the conclusion of McFadden and Satchler that it is not possible to obtain a global alpha particle potential of the same generality and quality as those found for nucleons.

4. The Average Potential

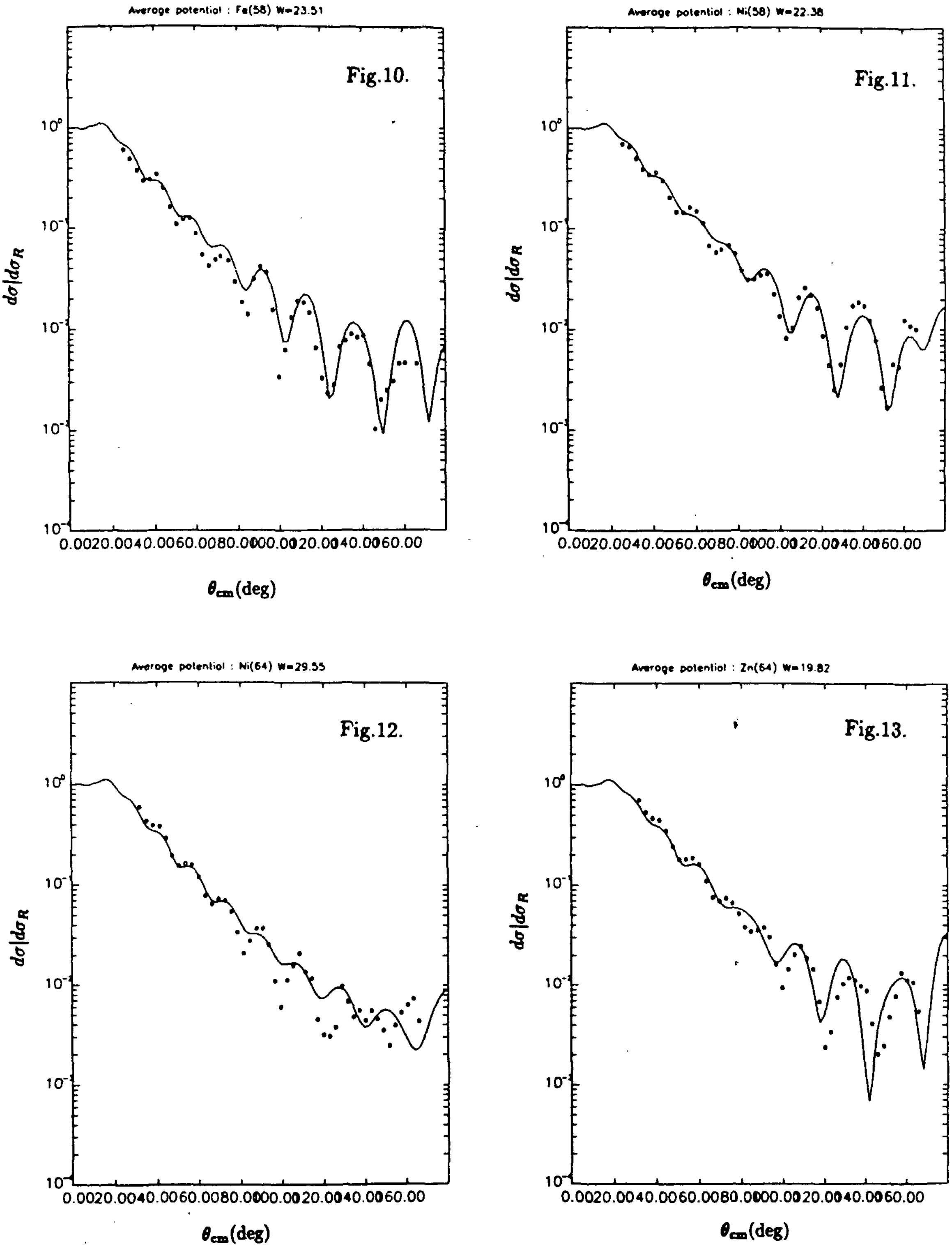
The analyses described in the previous section did not lead to a successful global alpha-particle potential. In many practical applications it is nevertheless desirable to have a potential for general use, and to know the degree of accuracy that may be expected of it. We therefore in this section compare in Fig.14 the cross-sections given by the average potential specified in section 3 with a range of experimental data. On the whole it is seen to give fairly well the overall behaviour of the cross-sections, but not the detailed structure.

The average potential has $U = 205.9$ MeV, $W = 25.8$ MeV, $r_u = 1.414$fm and $a = 0.529$fm.

5. Determination of Energy Variation

The energy variation of the alpha particle potential is required in order to fit the cross-sections of reactions with the alpha particle as projectile or ejectile. Calculations by Avrigeanu *et al* (1989) showed that it is not possible to fit $^{51}V(n,\alpha)$ cross-sections as a function of energy by using the alpha particle potential of McFadden and Satchler, but that a fit can be obtained by increasing the diffuseness parameter to $a = 0.64$fm. Unfortunately this gives total reaction and (n,α) cross-sections that are much too high.

Since the total reaction cross-section is obtained directly from the alpha particle potential and must be fitted by any acceptable potential we therefore analysed the total reaction cross-section using the average potential, and determined the energy variation of r_u, a and U by fitting the experimental values of Vonach *et al.* Analysis showed that using the average potential i.e. $U = 205.88$ MeV, $W = 25.78$ MeV, $r_u = 1.414$fm and $a = 0.529$fm the experimental values



Figs.10–13. Comparison of experimental and calculated values of ratios of differential cross-section to Rutherford cross-section for scattering of 21 MeV alpha particles. The calculated values are obtained by using the average potential based on those of McFadden and Satchler except for values of W shown, which are the best values obtained by varying W.

Table 1a: Fitting σ_R by varying r

E_α (MeV)	r (fm)	σ_R (mb Calculated)	σ_R^* (mb experiment)
6.295	1.380	8.6	8.4 ± 0.3
6.795	1.390	24.0	23.9 ± 0.7
7.826	1.393	104.5	104 ± 3
8.830	1.405	254.8	255 ± 8
9.841	1.412	422.0	421 ± 13
10.860	1.412	568.2	566 ± 18
11.864	1.412	690.6	690 ± 22

Table 1b: Fitting σ_R by varying a

E_α (MeV)	a (fm)	σ_R (mb Calculated)	σ_R^* (mb experiment)
6.295	0.500	8.2	8.4 ± 0.3
6.795	0.510	23.5	23.9 ± 0.7
7.826	0.515	104.4	104 ± 3
8.830	0.524	255.8	255 ± 8
9.841	0.527	420.7	421 ± 13
10.860	0.527	566.8	566 ± 18
11.864	0.528	691.2	690 ± 22

Table 1c: Fitting σ_R by varying U

E_α (MeV)	U (MeV)	σ_R (mb Calculated)	σ_R^* (mb experiment)
6.295	153.0	8.4	8.4 ± 0.3
6.795	170.0	23.9	23.9 ± 0.7
7.826	175.0	104.0	104 ± 3
8.830	194.5	255.8	255 ± 8
9.841	202.0	421.3	421 ± 13
10.860	202.0	567.4	566 ± 18
11.864	203.0	690.6	690 ± 22

* Experimental values are from Vonach *et al.*

of σ_R for $\alpha + {}^{51}V$ reaction from 6.2 MeV to 11.8 MeV can be fitted well either by varying r_u, a or U but not by varying W.

Tables 1a–c and Fig.15a–c show the variation of optical model parameters obtained by using the average potential. (a) shows fitting σ_R by varying r_u, (b) shows fitting σ_R by varying a and (c) shows fitting σ_R by varying U.

These potential will be tested by using them to calculate the cross-sections of various reactions.

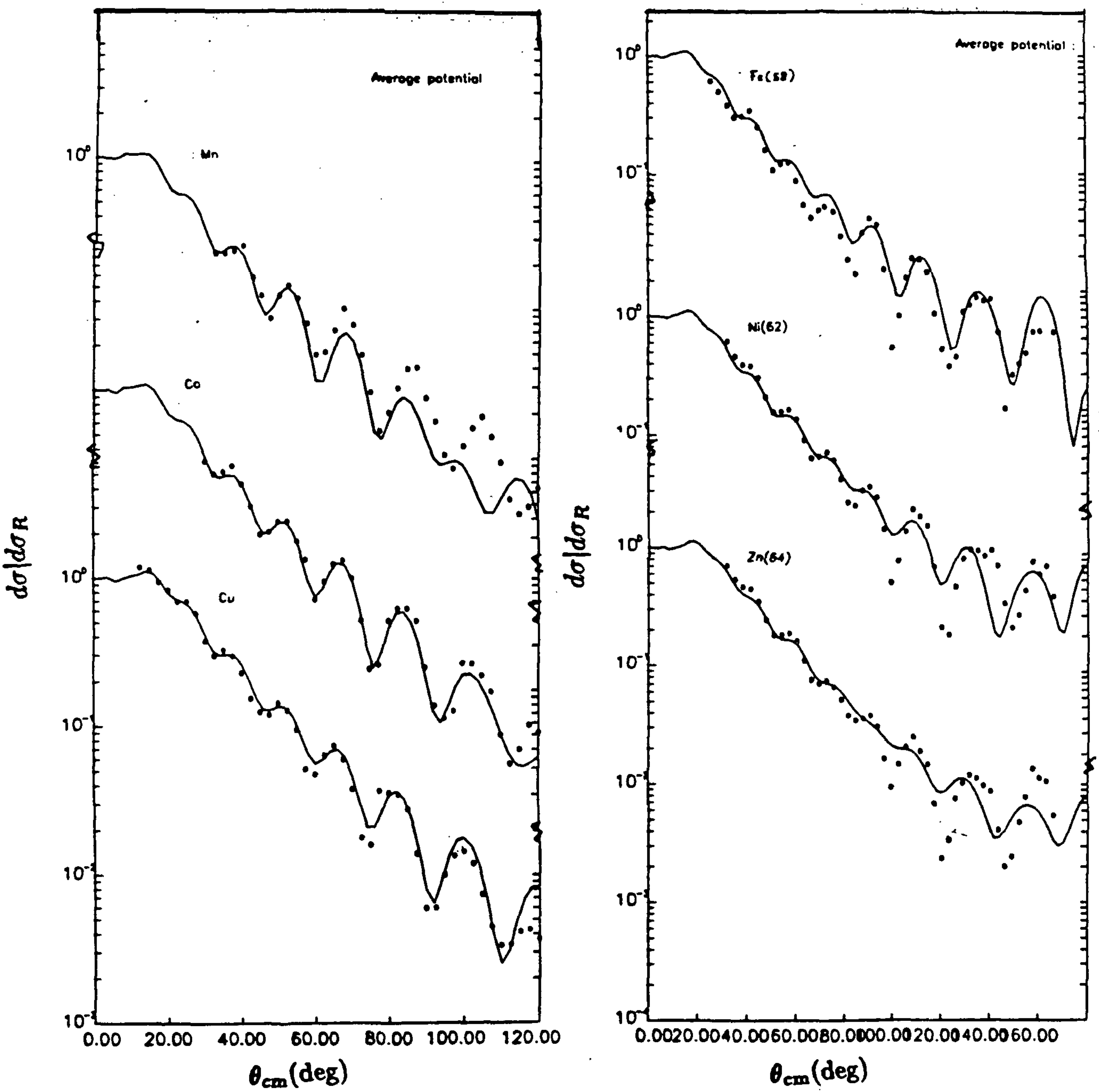
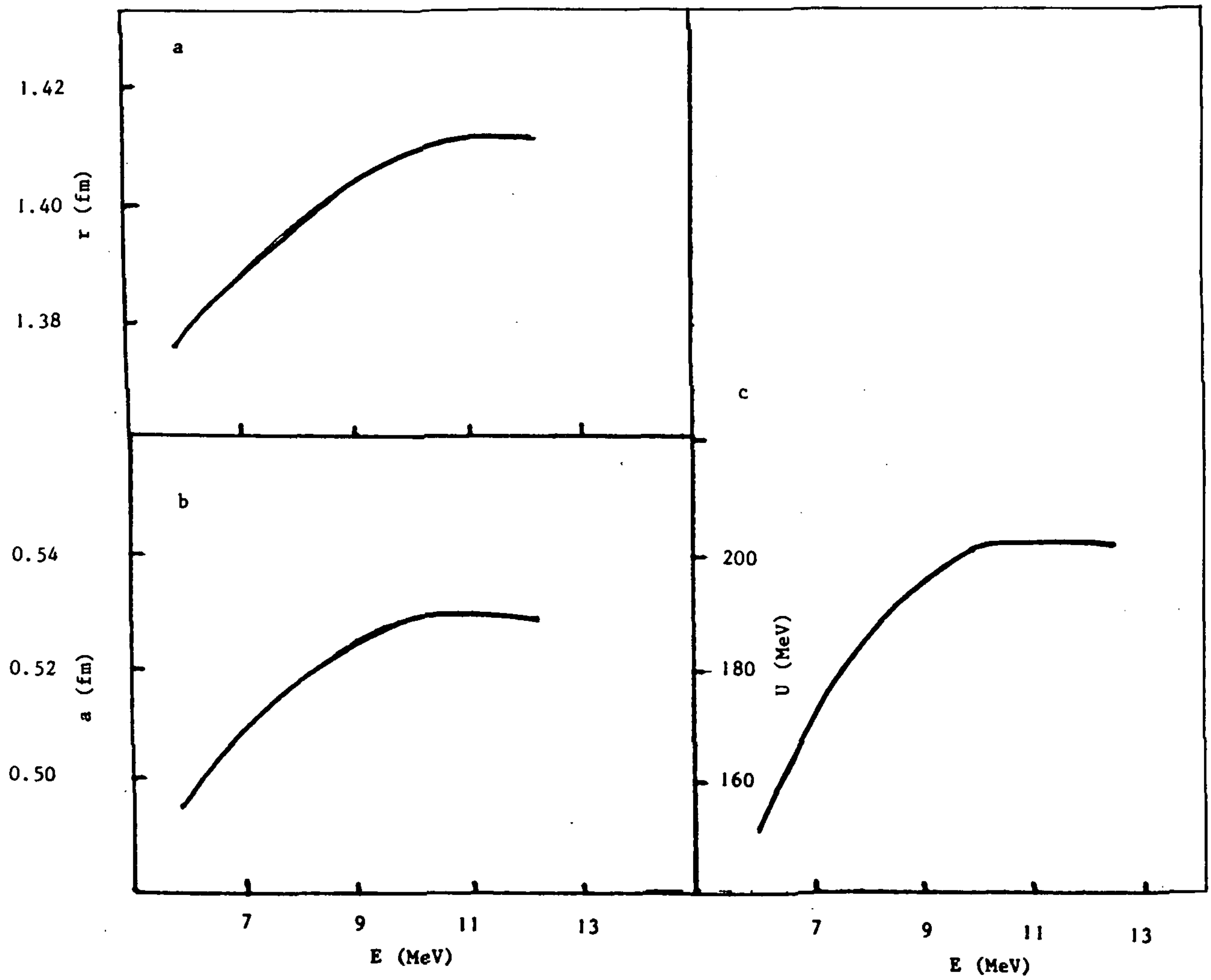


Fig .14. Comparisons between the cross-sections calculated from the average potential U = 205.9 MeV, W = 25.8 MeV, r = 1.414fm, a = 0.529fm and the experimental data.

6. Conclusion

A search for a global potential for alpha particles is described. This has been made by choosing some potential and fixing U, r_u and a of the potential but allowing W to vary to fit the differential cross-section of elastic scattering of alpha-particles on various nuclei at different energies. This has been done because alpha particle scattering is sensitive to the nuclear surface and this suggests that it is unlikely that a simple global potential exists. It was hoped that different values of W for different nuclei would take care of this sensitivity,



Figs.15a–c. Energy variation of optical model parameters required to fit the experimental total reaction cross-section: (a) the radius parameter (b) the diffuseness parameter and (c) the real potential depth.

but the values of χ^2 for the fits to the data are high which suggests that probably it is not possible to find a reliable global potential for alpha-particles by varying W only. Energy variation of total reaction cross-sections have also been studied for some of the reactions, using the potential which could serve as some practical purposes like calculating various reaction cross-sections.

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