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**Abstract**

A numerical model for the equivalent circuit of a Marx type discharge is presented. The treatment includes the effect of finite closure time of the switches which drive the generator. The role of stray capacities in the generation of several oscillations (usually ascribed to "noise") is discussed. The results of numerical calculations for simple configurations are presented.

**1. Introduction**

In pulsed, high voltage discharges it is usually observed in the initial part of the measured magnitudes (like  $di_L/dt$ , the temporal derivative of the current  $i_L$  circulating on a load with impedance  $Z_L$ ), complex high frequency components superimposed to the expected temporal behavior. This phenomenon is commonly ascribed to noise pick-up (or resonances) in the measuring system. Marx generators behave also in this way and, in an effort to clarify the source of these high frequency components, we have developed a numerical model which yields the evolution of the electrical magnitudes in this kind of generators. In what follows, preliminary results will be presented and discussed.

**2. Model and results**

For simplicity, we will consider a two stage Marx generator, discharging on a load impedance  $Z_L$  through a switch  $S_L$  (see Fig. 1).

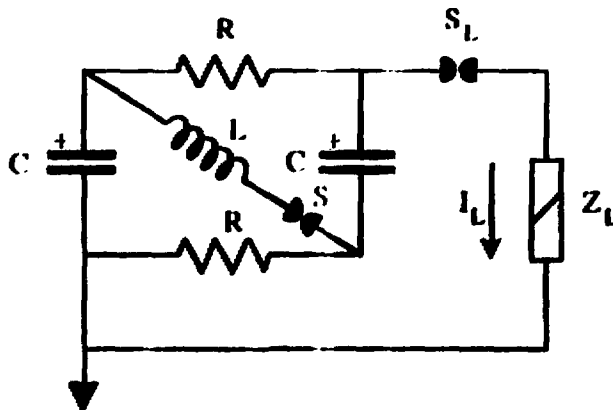


Fig 1: Standard two stage Marx generator.

Both capacitors (capacity  $C$ ) are externally charged up to the voltage  $V_0$  through the resistances  $R$ , and the switches  $S$  and  $S_L$  control the Marx erection and its discharge on the load, respectively. For a proper operation of the circuit,  $R \gg (L/C)^{1/2}$  ( $L$  being the inductance associated with  $S$  and connections), so that when  $S$  begins its closing action the branches containing the  $R$ 's can be considered as open circuits. To simulate the real behavior of the switches, we will treat them as "black box" elements, with voltage drops following a Fermi-Dirac-like function. This representation has been employed in previous works [1,2]. A time delay  $\Delta T$  between the closing of  $S_L$  and  $S$  is allowed for the model. The simplest conceivable equivalent circuit is shown in Fig. 2, where  $V_S$  and  $V_{SL}$  indicate the Fermi-Dirac functions quoted above, and  $L$  is the inductance associated with  $S$ . Note that a stray capacity ( $C_{SL}$ ) associated with  $S_L$  has been included. The presence of this capacity (however small) is essential for a proper description of the voltage erection of the circuit, because it provides a path for the discharge current while  $S_L$  remains open.

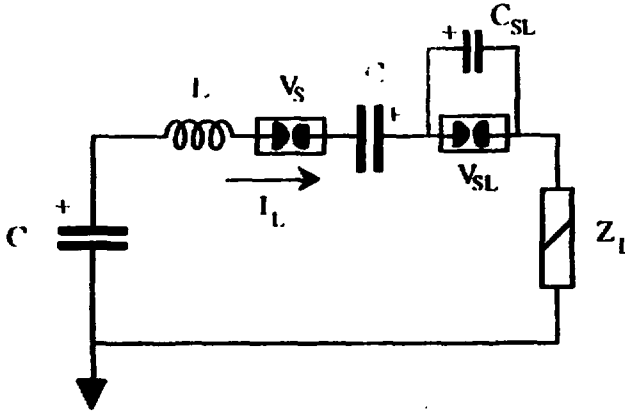


Fig 2: Equivalent circuit to model a two-stage Marx generator.

Kirchhoff's law for the circuit of Fig. 2 writes

$$2V_0 - \frac{2}{C} \int_0^t i_L \, dT = L \frac{di_L}{dT} + V_S + V_{SL} + V_{Z_L}$$

where according to the model chosen for simulating the behavior of the switches, we have employed the following expressions for  $V_S$  and  $V_{SL}$

$$V_S = \frac{V_0 [1 + \exp(-B_S T_S)]}{1 + \exp[B_S (T - T_S)]}$$

and

$$V_{SL} = \begin{cases} V_0 + \frac{1}{C_{SL}} \int_0^T i_L \, dT & T \leq \Delta T \\ \frac{2 V_{SL}(\Delta T)}{1 + \exp[B_{SL} (T - \Delta T)]} & T \geq \Delta T \end{cases}$$

Both  $B_S$  and  $B_{SL}$  are related to the closure speed of the switches:  $4.4 / B_S$  is the time required for  $V_S$  to fall from  $0.9 V_0$  to  $0.1 V_0$  (the equivalent holds for  $V_{SL}$ ). The parameters  $T_S$  and  $\Delta T$  are the "instants" at which the corresponding switches are closing. Solutions of the above equations have been found; as an example, in Fig. 3 we show  $di_L/dT$  (in  $V_0/L$  units) as a function of the time  $T$  (in  $(LC)^{1/2}$  units), for the case  $C_{SL} = 10^{-5} C$  (that is,  $C_{SL}$  values in the pF range),  $L = 1$  and  $Z_L = (LC)^{1/2}$  (purely resistive load). Besides,  $S_L$  is assumed to operate by overvoltage

its voltage drop reaches  $2V_0$ , and with  $\Delta t (LC)^{-1/2} = \Delta T = 0.1$ . The closure speeds of the switches has been chosen as  $B_S = B_{S_L} = 100$ , with  $T_S = 0.01$ .

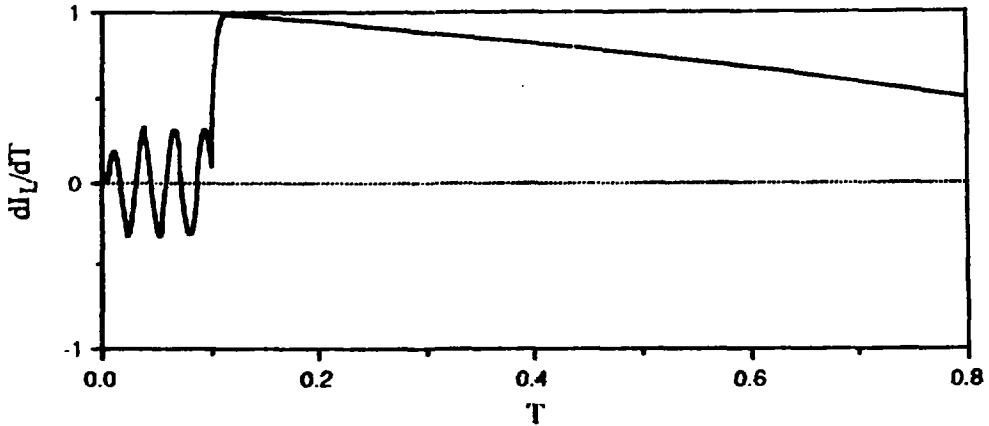


Fig 3: Typical waveform of  $dI_L/dT$  obtained from the circuit of Fig 2.

As can be seen,  $dI_L/dT$  grows up from 0 during the closure time of  $S$  (i.e. while  $C_{S_L}$  charges from  $V_0$  up to  $2V_0$  (in the time interval  $0 \leq T \leq 0.01$  in the example)), then  $dI_L/dT$  has oscillations around 0 while  $S_L$  remains open ( $T < 0.1$ ), and finally the main discharge begins and  $dI_L/dT$  evolves according with what must be expected for a simple RLC series circuit, i.e. a damped sinusoid with a nondimensional period of  $2\pi(7/4)^{1/2}$  for the chosen values of the parameters of this example. Also it can be noted that the high frequency oscillations can be easily predicted as resonances of an  $L C_{S_L}$  circuit in the present example. In more involved cases, the frequency of these oscillations can also be predicted after a small amount of simple algebra.

In a real Marx generator, many other stray capacitances can arise taking into account that during the erection, conductors initially grounded are allowed to charge up to high voltages. The determination of these stray capacitances depends on the geometrical details of the actual device under study, and a full discussion of this subject exceed the scope of the present work. However, due to the fact that is common practice in Marx devices to locate both switches as close as possible (so that the UV emission from the first helps in the triggering of the second), a stray capacitance  $C'$  between  $S$  and  $S_L$  is likely to exist.

In Fig. 4 we show a modified equivalent circuit including  $C'$ , and an inductance  $L'$  which takes into account a partition of the total series inductance.

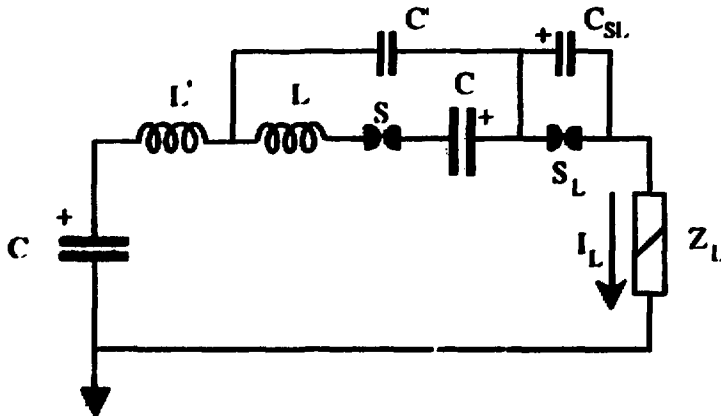


Fig 4: Equivalent circuit to model a two-stage Marx generator with a stray capacitance  $C'$  between switches.

In Fig. 5,  $dI_L/dT$  as a function of the time is given, for the same set of values of the parameters used in Fig. 3 and with  $C' = C_{SL} = 10^{-5}$  C and  $L' = L$ . Now, higher frequency oscillations (both with  $S_L$  open or closed) appear, with periods which can be analytically obtained from a frequency analysis of the circuit and which are in good agreement with those obtained from the numerical calculations. It can be noted that during  $\Delta t$ , the curve of  $dI_L/dT$  has high frequency oscillations superimposed to a fundamental one.

In more elaborated models (with more stages or more stray elements, for instance) one must expect more complex spectral structures. Another point of interest to comment is the amplitude with which the high frequency oscillations can be excited. It is not possible to give an answer to this question in an analytic way, but as a rule, one can say that the higher are the closure speed of the switches, the larger will be the amplitude of the excited harmonics.

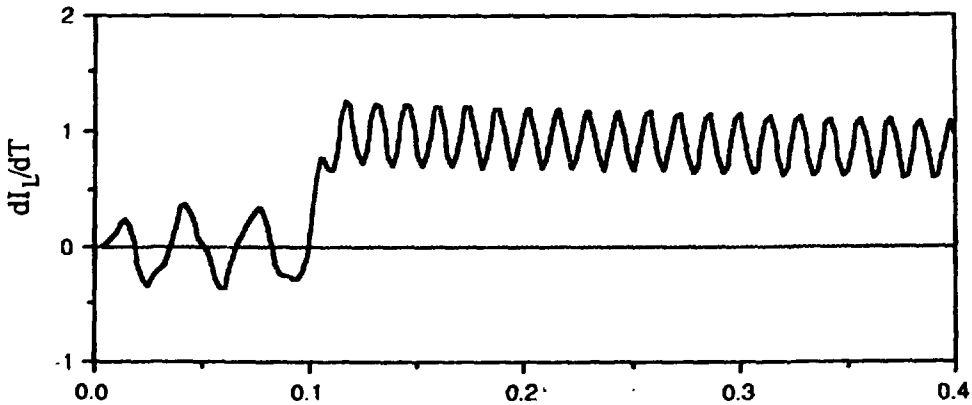


Fig 5: Typical waveform of  $dI_L/dT$  obtained from the circuit of Fig 4.

### 3. Final remarks

It must be noted that the circuit of Fig. 4 does not contain all the stray elements which should be included for reproducing a measured signal. In fact, besides stray capacitances between switches one can add stray capacitances between the capacitors of the Marx, but this procedure should be made for the particular geometrical arrangement of each device. In particular, work in this direction is under way for the Marx generator of the Plasma Physics Laboratory of the Pontificia Universidad Católica de Chile, and the results of such study will be presented elsewhere.

As a final remark, we have shown that a proper electrical model for a Marx discharge requires the inclusion of several stray elements for reproducing high frequency resonances and the erection of the generator. Besides, the inclusion of finite closure times of the switches is also necessary for obtaining the amplitude of these harmonics.

### Acknowledgements

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### References

- [1] Brazzone, H., Kelly, H., Moreno, C., Am. J. Phys. **57**, 63 (1989).
- [2] Brazzone, H., Kelly, H., Moreno, C., IEEE Trans. Plasma Sci. (in press).