Reconnection of Magnetic Field Lines*

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Abstract

Magnetic field line diffusion in a plasma is studied on the basis of the non-linear boundary layer equations of dissipative, incompressible magnetohydrodynamics. Non-linear steady state solutions for a class of plasma parameters have been obtained which are consistent with the boundary conditions appropriate for reconnection. The solutions are self-consistent in connecting a stagnation point flow of a plasma with reconnecting magnetic field lines.

The range of the validity of the solutions, their relation to other fluid models of reconnection, and their possible applications to space plasma configurations are pointed out.

1. Introduction

Reconnection of magnetic field lines is a phenomenon of particular interest in space plasmas in the vicinity of the Sun and planetary magnetospheres and in the context of laboratory plasma confinement configurations. E. g., the sudden release of energy in solar flares and geomagnetic substorms as well as the transfer of solar wind plasma into the magnetosphere is often explained in terms of magnetic reconnection.

Plasma acceleration in current layers has been first studied by Sweet (1958) and Parker (1963). In these models the resistive dissipation of magnetic flux heats the plasma which, in turn, is sqeezed out in narrow-angle regions by the enhanced pressure.

Due to the usual extremely high magnetic Reynold's number in cosmical systems, this process is very inefficient when considered on a global scale. Petschek (1964) proposed a model where the resistvity is important only in a small 'diffusion region' which allows for the reconnection of magnetic field lines. Excited large amplitude magnetohydrodynamic (mhd) waves then lead to effective global energisation and acceleration of the plasma.

In the present paper we give a solution of the non-linear mhd boundary layer equations which looks promissing to fill the gap between the ideal mhd wave solution and the dissipative central region which provides the necessary reconnection of magnetic field lines.

2. The Model

We look for a planar boundary layer solution of the viscous-resistive, incompressible mhd equations. If we assume the y-axis normal to the boundary, the governing equations are

$$\nu \psi_{yyy} - \psi_y \psi_{xy} + \psi_x \psi_{yy} + \frac{1}{4\pi\rho}(A_y A_{xy} - A_x A_{yy}) = \frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}x} \tag{1}$$

$$\nu_m A_{yy} + \psi_x A_y - \psi_y A_x = -cE_0. \tag{2}$$

Here we introduced the Stoke's functions $\psi(x,y)$ and $A(x,y)$ for plasma flow and magnetic field, $\mathbf{v} = \nabla \times \psi \mathbf{e}_z$, $\mathbf{B} = \nabla \times A\mathbf{e}_z$, ν and $\nu_m = c^2/4\pi\sigma$ are the kinamatic viscosity and the magnetic

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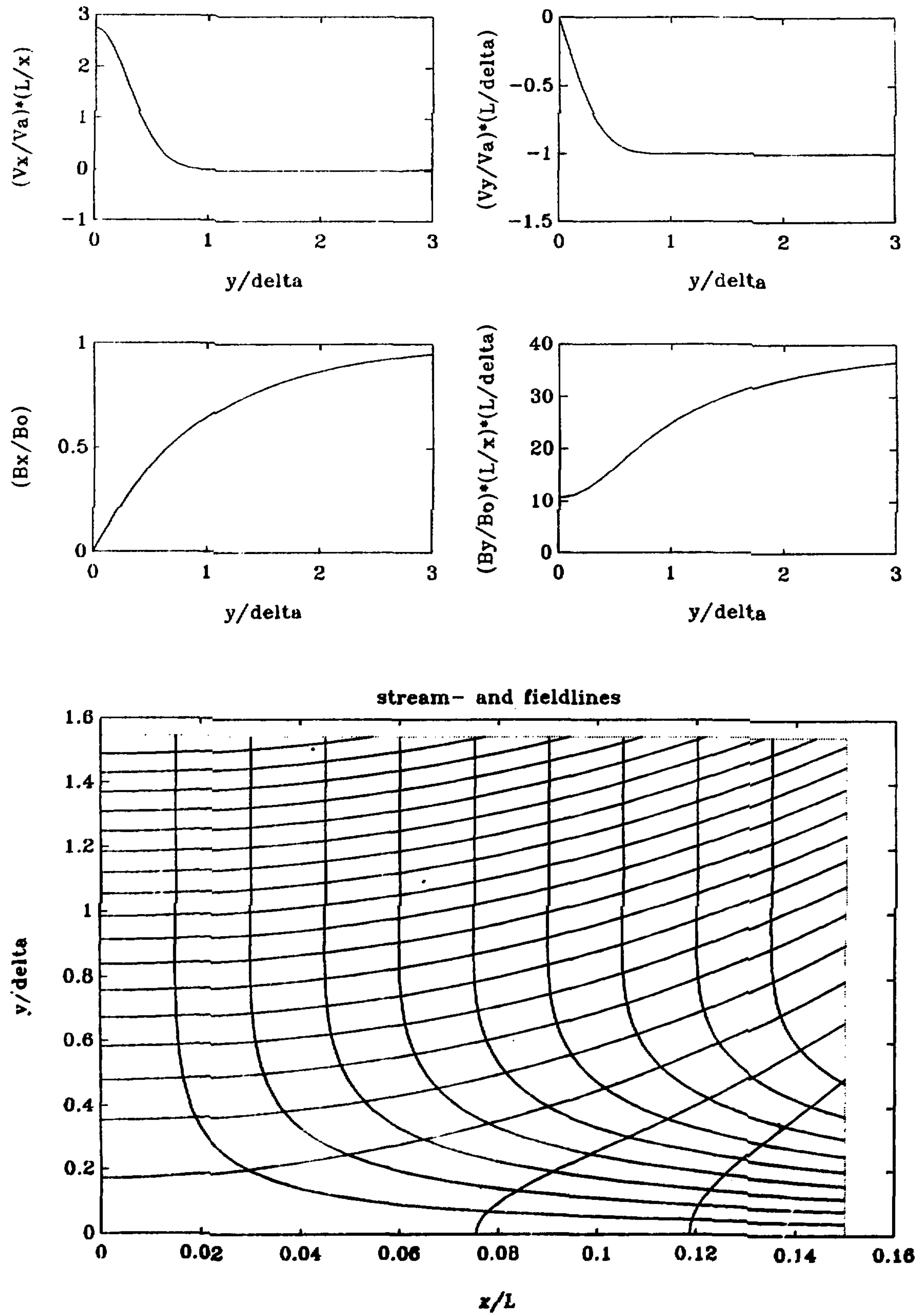

Figure 1: Velocity and magnetic field for $\nu/\nu_m = 0.1$.

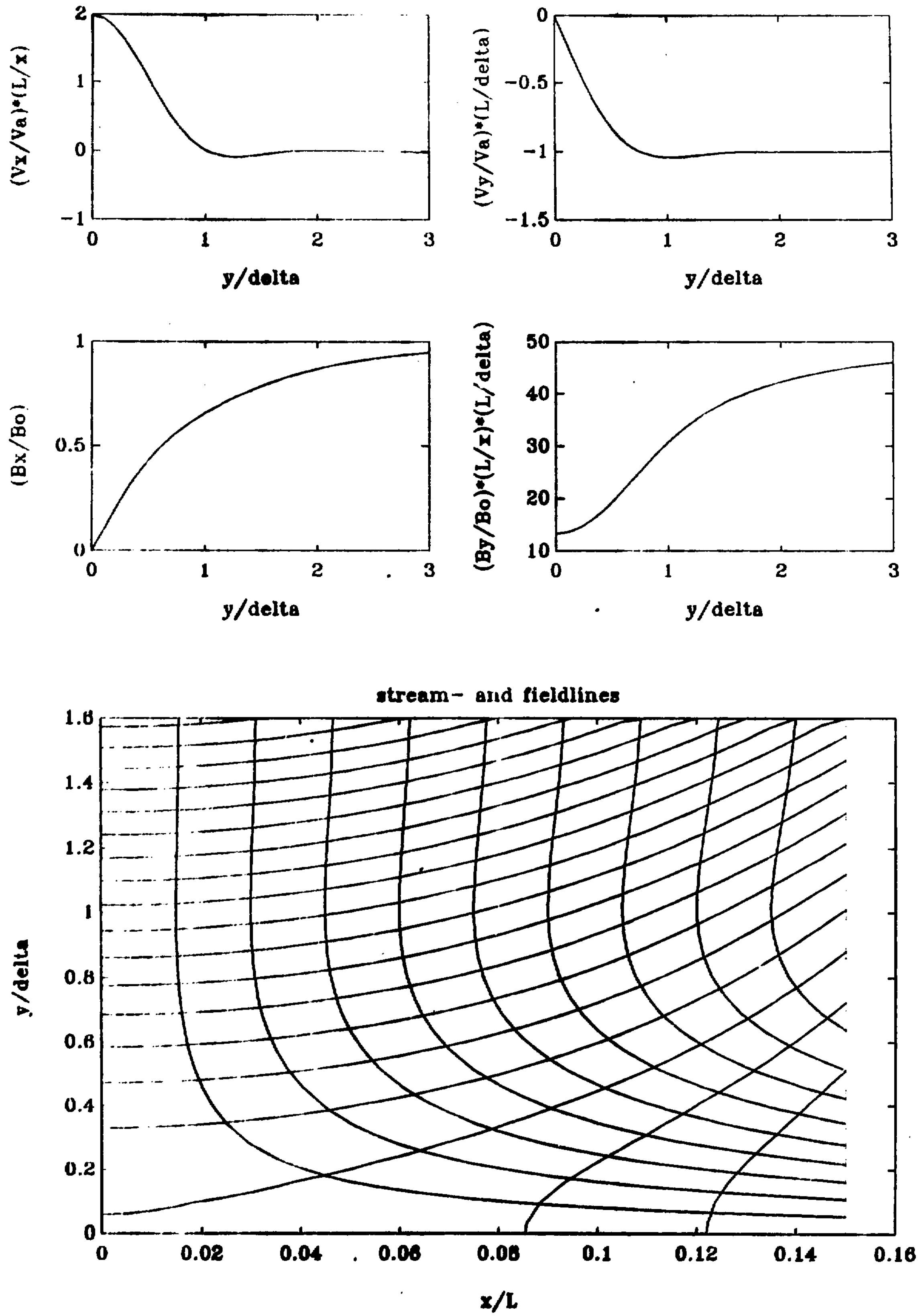

Figure 2: Velocity and magnetic field for $\nu/\nu_m = 1$.

diffusivity, E_0 is a constant electric field in the z-direction and $P(x) = p + B_x^2/8\pi$ is the total pressure which is assumed to be known in boundary layer theory (Schlichting, 1951).

We look for a self-similar solution of the type

$$\psi = \frac{x}{L}\,\delta\,V_a\,f(\frac{y}{\delta}) \tag{3}$$

$$A = \delta\,B_0\left[\phi\left(\frac{y}{\delta}\right) + \frac{1}{2}\left(\frac{x}{L}\right)^2 \varphi\left(\frac{y}{\delta}\right)\right], \tag{4}$$

where B_0 is some constant magnetic field at infinity and $V_a = B_0/\sqrt{4\pi\rho}$ the corresponding Alfvén velocity. Equations (1) and (2) may then be written with $\alpha = \nu/\nu_m$ as

$$\alpha f''' - f'^2 + f f'' + \phi'\varphi' - \phi''\varphi = -\frac{1}{\rho V_a^2}\frac{\mathrm{d}P}{\mathrm{d}x} \tag{5}$$

$$\phi'' + f\phi' = 1 \tag{6}$$

$$\varphi'' + f\varphi' - 2f'\varphi = 0, \tag{7}$$

where we have put $\delta^2/L = \nu_m/V_a$ and $cE_0 = -(\delta/L)V_aB_0$. If we define $M_a = V_0/V_a$ as the Alfvénic Mach number of the asymptotic inflow velocity V_0 corresponding to the electric field $E_0 = -V_0B_0/c$, it follows that $M_a = \delta/L$ with $\delta = \nu_m/V_0$, the resistive scale length. The Alfvénic Mach number may be related to a magnetic Reynold's number, $R_m = \nu_m/V_aL$, defined through the Alfvén velocity V_a and the scale L, by $M_a = 1/\sqrt{R_m}$.

The set of non-linear ordinary differential equations (5–7) is solved numerically with the boundary conditions

$$f(0) = f''(0) = \phi'(0) = \varphi'(0) = 0 \tag{8}$$

$$f(\infty) = \phi'(\infty) = 1, \tag{9}$$

sucht that f, ϕ and φ assume constant values at infinity for $y/\delta \to \infty$. The solutions for $\alpha = 0.1$ (low viscosity) and $\alpha = 1$ (high viscosity) and constant total pressure, $\mathrm{d}P/\mathrm{d}x = 0$, are displayed in Figures 1 and 2.

3. Results

The top panels of Figures 1 and 2 show that there exist steady state solutions which assume constant values at infinity (the edge of the boundary layer). The corresponding stream- and field lines (ψ = const, A = const) are given in the bottom panels and show a quarter of a configuration which is expected for magnetic reconnection. The inward convected field lines are reconnected at the origin and set up a stagnation-type plasma motion. Remarkebly, the plasma is accelerated solely by the acting Lorentz force, i. e. the tension in the reconnected field.

Enhancement of viscosity broadens the boundary layer but gives essentially to the same result. The solution is limeted in the x-direction by

$$\left|\frac{1}{2}\left(\frac{x}{L}\right)^2\left(\varphi'^2 - \varphi\varphi''\right)\right| \ll \max\left\{\left|\alpha f''' - f'^2 + f f''\right|, \left|\phi'\varphi' - \phi''\varphi\right|\right\}, \tag{10}$$

because we neglected the left-hand term in equation (5). This conditions gives values of $x/L \simeq 0.2$. At these values the outflow velocity is approximately half of the inflow Alfvén velocity and it is expected that Petschek's mechanism takes over where large amplitude mhd waves become important.

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