NUMERICAL METHOD FOR THE DISPERSION RELATION OF A HOT AND INHOMOGENEOUS PLASMA WITH AN ELECTRON BEAM

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Abstract.

We developed a numerical method that is based in kinetic theory, (Vlasov-Poisson equations) in order to calculate the dispersion relation for the interaction between a hot cylindrical and electron beam in any temperature and density. The plasma-beam system is located in a strong magnetic field. We illustrate many examples showing the effect of the temperatures and densities on the dispersion relation.

1. Introduction

The oscillatory behavior on bounded plasma is the particular importance in the solution of astrophysical problem and laboratory applications e.i on plasma heating[1] or nonconventional particle accelerators[2,3,4]. Detailed numerical and analytical solutions have been studied in the case of a fully filled plasma wave guide placed in an arbitrary external magnetic field with a cylindrical symmetric configuration, but the partially filled plasma wave guide that is in interacting with an electron beam has not been studied in detail[5].

In this work we present a numerical model based on the Vlasov-Poisson system equation for a partially filled plasma wave guide that is interacting with an electron beam. We suppose the existence of a strong axially symmetric magnetic field.

2. Basic equations

The theoretical model is based on the Vlasov-Poisson equation:

$$\frac{\partial f_\mu(\mathbf{r},\mathbf{v},t)}{\partial t} + v\frac{\partial f_\mu(\mathbf{r},\mathbf{v},t)}{\partial z} - \frac{e_\mu}{m_\mu}\frac{\partial \Phi}{\partial z}\frac{\partial f_\mu(\mathbf{r},\mathbf{v},t)}{\partial v} = 0 \tag{1}$$

$$\nabla^2\Phi = -4\pi\Sigma e_\mu \eta_\mu(0)\int_{-\infty}^{\infty} dv f_\mu(\mathbf{r},\mathbf{v},t) \tag{2}$$

Where f_μ is the distribution function of axial of axial velocities associated with a particle of charge e_μ and m_μ. In equilibrium $f_\mu = g_\mu F_\mu(v)$; $\Phi(r,t)$ is the electrostatic potential, $\eta(0)$ is the particle density on the axis, Σ is the sum carried out over all species present in the system (an electronic plasma and electron beam in our case).

The linearized Vlasov-Poisson equations are:

$$\frac{\partial f'_\mu(\mathbf{r},\mathbf{v},t)}{\partial t} + v\frac{\partial f'_\mu(\mathbf{r},\mathbf{v},t)}{\partial z} - \frac{e_\mu}{m_\mu}\frac{\partial \Phi}{\partial z}g(r)\frac{\partial F_\mu(\mathbf{r},\mathbf{v},t)}{\partial v} = 0 \tag{3}$$

$$\nabla^2\Phi' = -4\pi\Sigma e_\mu \eta_\mu(0)\int_{-\infty}^{\infty} dv f'_\mu(\mathbf{r},t) \tag{4}$$

Equations (3) and (4) are solved using Fourier Bessel expansion for the radial coordinates, a Fourier series expansion for the angular component a Fourier integral for the z coordinate, and Laplace transform for time variable. Finally, we obtain the following dispersion relation

$$\frac{ka}{x_{ml}+ka}\left[C^{ll'}_{pm} I_p(k,\omega) + C^{ll'}_{Bm} I_B(k,\omega) - \Pi\right] A^{n}_{m}(k) = 0 \qquad (5)$$

Where:

$$I_{P,B} = \frac{\omega^2_{P,B}(0)}{k^2}\int_{-\infty}^{\infty} dv \frac{\frac{dF^0_\mu}{dv}}{\left(v - \frac{\omega}{k}\right)} \qquad (6)$$

Subindex P and B represent the plasma and the beam respectively, m is the azimuthal magnetic number and Π is the unit matrix, besides:

$$\omega^2_{p_\mu} = \frac{4\pi e^2_\mu \eta_\mu(0)}{m_\mu} \qquad (7)$$

is the plasma frequency, $F\mu$ is the Maxwellian distribution and

$$C^{ll'}_{\mu m} = \frac{2}{a^2 J_{m+1}(x_{ml}) J_{m+1}(x_{ml'})}\int_{\infty}^{\infty} dr\, r\, J_m(p_{ml}r)\, J_m(p_{ml'}r)\, g_\mu(r) \qquad (8)$$

The term $C^{ll'}_{\mu m}$ is directly related to the inhomogeneity of the plasma or electron beam, the function $g\mu(r)$ is for the plasma:

$$g_P(r) = \frac{1}{1 + \left(\frac{\gamma r}{a}\right)^2} \qquad \gamma = 3 \qquad 0 \le r \le a \qquad (9)$$

and the electron beam,

$$g_B(r) = \frac{1}{1 + e\left(\frac{r}{\beta}\right)\left(\frac{r}{b} - 1\right)} \qquad \beta b = .8 \qquad 0 \le r \le a \qquad (10)$$

a in this cases is 5.2 cm. If $g\mu(r)=1$, we have the homogeneous case, that is a fully filled plasma wave guide.

3. Numerical results.

The equation (5) in terms of dimensionless quantities is:

$$\frac{\bar{k}}{x_{ml}+\bar{k}}\left[\frac{C^{ll'}_{pm} Z'(\beta)}{2\bar{\lambda}^{-2}_{dp}\bar{k}^{-2}} + \frac{C^{ll'}_{Bm} Z'(\alpha)}{2\bar{\lambda}^{-2}_{dB}\bar{k}^{-2}} - \Pi\right] A^{n}_{m}(k) = 0 \qquad (11)$$

$$\bar{\lambda}_{d\mu}^{2} = \frac{k_B T_\mu}{m_\mu \omega_{p\mu}^2 a^2} \qquad\qquad \beta = \frac{\bar{\omega}_p^2}{2\bar{\lambda}_{dP}^2 \bar{k}^2}$$

$$\alpha = \frac{\bar{\omega}_B^2}{2\bar{\lambda}_{dB}^2 k^2} \cdot \left(\frac{E}{k_B T}\right)^{\frac{1}{2}} \qquad \bar{\omega} = \frac{\omega}{\omega_{P_\mu}} \qquad \bar{k} = ka$$

E is the beam energy and Z' is the derivative of the dispersion function. Numerical solutions of equation (11) are shown in fig.1 for typical values. The lower branches of the graph represent the Landau damping.

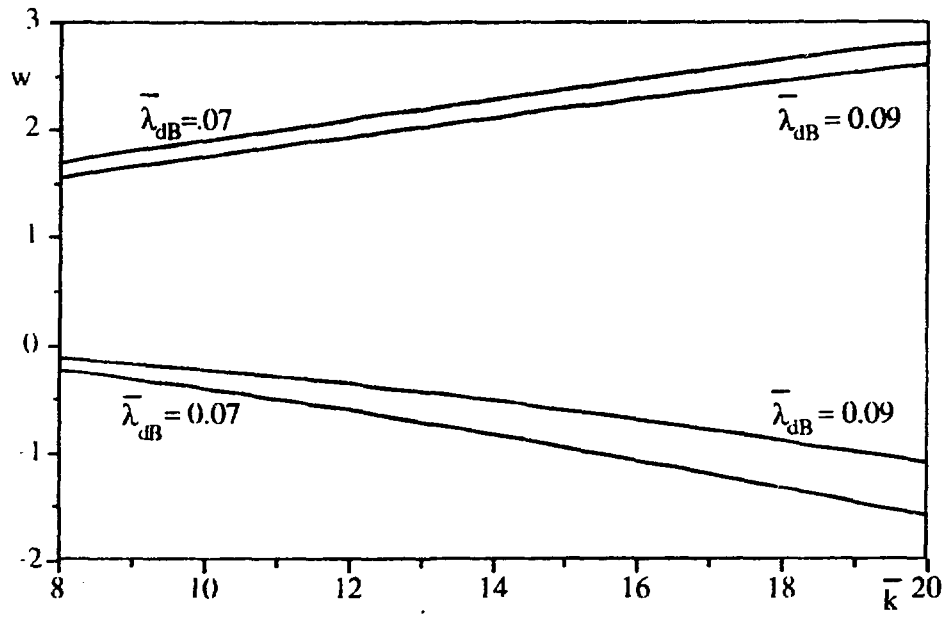

Fig1. Dispersion relation for electronic plasma interacting with electron beam. Lowest radial mode, m=0, n=1.

4. Conclusions

The dispersion relation obtained has the some form as the dispersion relation for the longitudinal waves in the infinite homogeneous case, and when the beam not is present[6], except for the term gives in (6). We can substitute the electron beam for a proton beam exchange appropriately $\bar{\lambda}_{dB}$. The waves is strongly damped for values ka >> 1.

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