# PROBE CHARACTERISTIC INTERPRETATION THROUGH TIKHONOV'S REGULARIZATION METHOD

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**Abstract**

Tikhonov's regularization method for the solution of the integral equation of the Langmuir probe current is discussed. The application of this method in plasma measurement experiments allows to restore the real electron distribution function in a wide range of experimental conditions. The solution of this problem permits to get an improved measurement accuracy in plasma diagnostics.

## 1 Introduction

Plasma diagnostic by Langmuir probes untill our days remains one of the most used methods. With the increase, in the last years, of cold plasma technological applications, Langmuir probes have become a common tool. This diagnostic is an easy and accurate way to make local measurements, in contrast with the theoretical dificulties of data interpretation. In order to solve this problem some authors have proposed methods for the automation of acquisition data.

In [1] a computer program is proposed, in which the computations are made in base to the ion theory of the Langmuir probe. In order to get the plasma parameters a maxwellian electron distribution function is suposed, which could be an error source in some cases. A more general method would consider the real distribution function.

In this paper we describe a method which permits to obtain the real plasma particle distribution function directly from the experimentaly measured probe characteristic.

We shall asumme that the distribution function in the non-perturbed region, is isotropic and homogeneus. In this case the distribution function presents only energy dependence,

$$f(\mathbf{r}, v) = n_e f\left(\frac{mv^2}{2} + eV\right) ,$$

where $V$ is the probe's potential [2]. Using the last expression to obtain the probe's current density

$$j_e = 2\pi e \int_0^{\pi/2} \sin\theta d\theta \int_0^{\infty} v \cos\theta f\left(\frac{mv^2}{2} + eV\right) v^2 dv ,$$

then

$$j_e = \frac{2\pi e n_0}{m^2} \int_{eV}^{\infty} (\epsilon - eV) f(\epsilon) d\epsilon \,, \tag{1}$$

here $\theta$ is the angle between the electron's velocity (**v**) and the inner normal to the probe surface.

## 2 Plasma parameters

In the present section we shall discusse the way, in which is possible to determine the plasma parameters, after solving equation (1) to get $f(\epsilon)$.

For obtainning the electron temperature it uses the discribed method in [2], which consists in derivating the characteristic respect to the probe potential necessary to eliminate the ion current of the total one, measured in the experiment. The temperature is obtained from the slope of the plot $\ln(dI/dV)$ vs. $V$.

Form Eq.(1) can be calculated the charged particles density ($n_e$) substracting the ion current from the total, which can be done extrapoling the saturation ion current. In some works [3-5] has been proposed an exponential dependence of current to obtain the electron current. When the electron current density is calculated, then $f(\epsilon)$ is substituted into Eq.(1) and by this way the density is computed.

Plasma potential can be computed using the proposed method in [6], through floating potential using the next expression

$$V_p = V_f + \frac{kT_e}{2} + \frac{kT_e}{2} \ln\left(\frac{M}{m}\right)^{1/2} .$$

In this way, the design of an algorithm which permits to solve the Eq.(1) to find $f(\epsilon)$, gives more information about the plasma and simultaneusly, a better precision in the plasma parameter measurements, particularly in that relative to the density.

## 3 Numerical Algorithm

The electronic current for a repulsive field is,

$$j_e(x) = A_0 \int_{x}^{\infty} (\epsilon - x) f(\epsilon) d\epsilon \tag{2}$$

where $x = eV$, $A_0 = \frac{2\pi e q s}{m^2}$, $\epsilon = \frac{mv^2}{2} + eV$ is the total energy for electrons.

Equation (2) is a Volterra equation of the first kind, which can be casted into a Fredholm of the first kind,

$$I(x) = \int_a^b (\epsilon - x) f(\epsilon) d\epsilon \tag{3}$$

where $I(x) = j_e/A_0$ and $a$, $b$ determines the energy range to be assessed.

Since $j_e$ is given experimentally by the current-potential characteristics of the probe, we only have an approximation to the real values of $j_e$. Let $\dot{j}_e$ be the experimental value obtained from the current-potential characteristic (CPC), and assume the difference with respect to the exact value is of the order of the whole sum for all the measurement errors, which we labeled as $\delta$.

Since we wish to determine $f(\epsilon)$, we are dealing with an ill-posed problem, accordingly with J. Hadamard [7]. To handle this sort of problems, some procedures have been proposed. In this work we use the one proposed by A. N. Tikhonov [8].

Let $0 \le \epsilon \le \epsilon_0$ be the energy range. Here $\epsilon_0$ is the electron energy and it may be determined by the probe potential. Thus we have to solve,

$$\int_0^{\epsilon_0} (\epsilon - x) f(\epsilon) d\epsilon = I(x). \tag{4}$$

in this last equation we assume that $V_0 \le x \le V_T$, where $V_0$ and $V_T$ are the initial and final potentials obtained from the CPC. Using the regularization method [8], a finite difference equation with uniform grid step h is obtained,

$$\begin{aligned} &\alpha f(t_i) + (V_T - V_0)\left[t_i - \tfrac{1}{2}(V_T - V_0)\right] \textstyle\sum_{i=0}^{N} C_i s_i f(s_i) h - (V_T - V_0)\left[\tfrac{1}{2} t_i (V_T + V_0)\right. \\ &\left. - \tfrac{1}{3}(V_T^2 + V_T V_0 + V_0^2)\right] \textstyle\sum_{i=0}^{N} C_i f(s_i) h = h\left\{t_i \sum_{i=0}^{N} C_i I(x_i) - \sum_{i=0}^{N} C_i x_i I(x_i)\right\} \end{aligned} \tag{5}$$

where $0 \le s_i \le \epsilon_0$, $0 \le t_i \le \epsilon_0$ and $V_0 \le x_i \le V_T$, and the following boundary conditions,

$$f_0 = 0 \qquad\qquad f_N = \epsilon_0 \tag{6}$$

where $s_i = t_i = ih$, $C_i$ are coefficients employed to compute the integrals by means of the trapezoid rule.

The regularization parameter $\alpha$, may be computed by the method proposed in [9] where $\alpha = C\delta^2$, the constant $C > 0$ has been determined experimentally.

From Eq. (5) we observe that the original problem for solving the functions $f_i$ has been converted into a set of algebraic equations for the $f_i$. The right hand side of Eq. (5) is determined from CPC.

# 4 Conclusions

The Tikhonov's regularization method permits the obtaintion of the real electron distribution function through a set of algebraic equations (see Eq.(5)), which can easily be resolved with a PC.

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