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DENSITY DEPENDENCE OF REACTOR PERFORMANCE WITH THERMAL CONFINEMENT SCALINGS

BY

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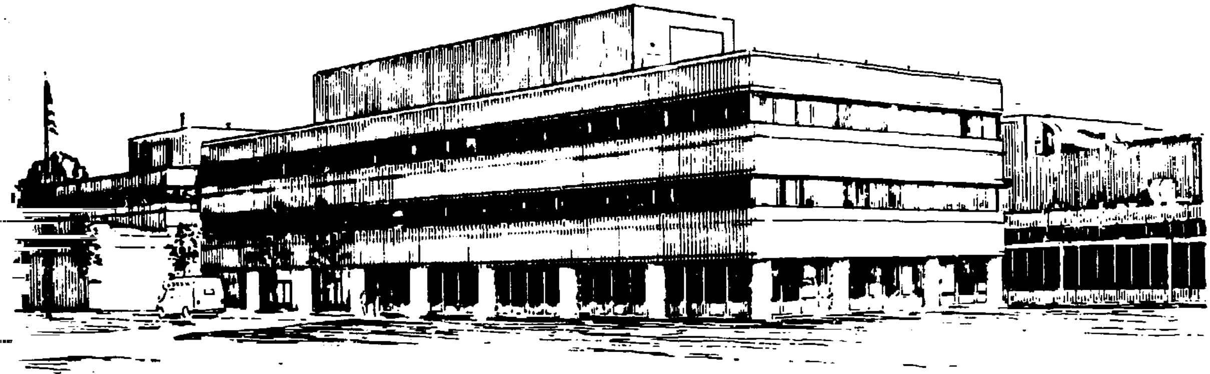

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Density Dependence of Reactor Performance with Thermal Confinement Scalings

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ABSTRACT

Energy confinement scalings for the thermal component of the plasma published thus far have a different dependence on plasma density and input power than do scalings for the total plasma energy. With such thermal scalings, reactor performance (measured by Q, the ratio of the fusion power to the sum of the ohmic and auxiliary input powers) worsens with increasing density. This dependence is the opposite of that found using scalings based on the total plasma energy, indicating that reactor operation concepts may need to be altered if this density dependence is confirmed in future research.

1. Introduction

The global analysis models used to predict the performance of proposed future reactors typically rely on experimentally derived scalings for the energy confinement time[1,2]. Calculations performed with scaling expressions constructed prior to about 1990, predict that reactor performance improves with increasing plasma density[3–9]. With improved data collection and analysis techniques, it is now possible to develop scalings for the thermal (rather than the total, i.e., thermal plus fast ion) energy confinement time[2,10]. The input power and density dependence of a number of these expressions differs substantially from that of previous scalings, and gives rise to reactor performance which decreases with increasing density.

In Sec. 2 we describe our global analysis model and derive the scaling of two measures of reactor performance with density. This result is discussed in more detail in Sec. 3; the implications for reactor operation are also presented.

2. Global Analysis Model and Ignition Margin Scaling

Global analysis codes typically solve a steady-state power balance equation similar to

$$P_\alpha + P_{OH} + P_{aux} = P_{con} + P_{rad}. \tag{1}$$

The individual terms represent the volume-integrated contributions made to the total power balance by alpha, ohmic, and auxiliary heating; thermal conduction and radiated losses are on the right-hand side. Examples of detailed expressions have been given elsewhere[8,9]. For present purposes, it is sufficient to state only their scaling with the volume averaged electron density $\langle n_e \rangle$ and density-weighted, volume-averaged temperature (assumed to be the same for electrons and ions) $\langle n_e T \rangle / \langle n_e \rangle$; for brevity, we will denote the latter by $\langle T \rangle$. Namely,

$$P_\alpha \propto \langle n_e \rangle^2 \langle T \rangle^s, \tag{2}$$

$$P_{OH} \propto \langle T \rangle^{-3/2}, \tag{3}$$

$$P_{con} \equiv \frac{W_{th}}{\tau_E} \propto \frac{\langle n_e \rangle \langle T \rangle}{\tau_E}, \tag{4}$$

and

$$P_{rad} \quad \propto \quad \langle n_e \rangle^2 \langle T \rangle^{1/2}. \tag{5}$$

The exponent s appearing in the expression for P_α is a slowly varying function of temperature[5], going from $s \sim 3$ for $\langle T \rangle \lesssim 8$ keV to $s \sim 2$ for $\langle T \rangle \gtrsim 15$ keV. The scaling for P_{rad} given in Eq. (5) is appropriate for bremsstrahlung radiation[1,5]. The conducted power P_{con} is defined as the ratio of the plasma thermal energy W_{th} to the energy confinement time τ_E. The confinement time is usually written as a function of the net input power[1,5,9]

$$P_{in} \equiv P_\alpha + P_{OH} + P_{aux} - P_{rad}. \tag{6}$$

Hence, Eq. (1) implies

$$P_{in} = W_{th}/\tau_E(P_{in}). \tag{7}$$

Given an expression for τ_E, Eq. (7) can be solved for P_{aux}, the auxiliary power required to maintain steady state in a reactor at a specified $\langle n_e \rangle$ and $\langle T \rangle$.

One measure of reactor performance is the ignition margin,

$$M_I \equiv \frac{P_\alpha}{P_{con} + P_{rad}}. \tag{8}$$

This is related to the more familiar fusion multiplication factor

$$Q \equiv \frac{5P_\alpha}{P_{aux} + P_{OH}} \tag{9}$$

by

$$Q = \frac{5M_I}{1 - M_I}. \tag{10}$$

We prefer to use the ignition margin since it is well-behaved in the ignited regime ($M_I \geq 1$).

If we define

$$\alpha \equiv \left. \frac{\partial \ln \tau_E}{\partial \ln \langle n_e \rangle} \right|_{P_{in}}, \tag{11}$$

and

$$\gamma \equiv - \left. \frac{\partial \ln \tau_E}{\partial \ln P_{in}} \right|_{\langle n_e \rangle}, \tag{12}$$

it is straightforward to show that

$$\frac{\partial M_I}{\partial \langle n_e \rangle} = \frac{M_I}{\langle n_e \rangle} \frac{P_{con}}{P_{con} + P_{rad}} \left(\frac{1 + \alpha - 2\gamma}{1 - \gamma} \right), \tag{13}$$

and

$$\frac{\partial Q}{\partial \langle n_e \rangle} = \frac{Q}{M_I} \frac{\partial M_I}{\partial \langle n_e \rangle}. \tag{14}$$

Our main result is contained within Eqs. (13) and (14). Namely, if $1 + \alpha - 2\gamma > 0$, reactor performance improves with increasing density. The predictions of a number of previous papers[3–9] have illustrated this behavior. However, Eqs. (13) and (14) indicate that if $1 + \alpha - 2\gamma < 0$ (and $1 - \gamma > 0$), reactor performance *degrades* with increasing density. That is, the dimensionless parameters M_I and Q are maximized by reducing $\langle n_e \rangle \to 0$. Of course, the fusion power produced increases with density independently of the scaling.

3. Discussion

The reason for the prevalence of the notion that reactor performance increases with density is that many of the τ_E power law scalings published up to about 1990 (i.e., L-mode scalings) yield (see, for example, Refs [11–13]), $1 + \alpha - 2\gamma > 0$. Some of the more recent scalings have $1 + \alpha - 2\gamma < 0$. This is typically the case for thermal scalings $\tau_{E,th}$, defined as the ratio of the thermal plasma energy to the net input power. That is, the earlier studies used the *total* plasma energy (including energetic particles such as those generated by neutral beam injection) in evaluating τ_E; we will designate this sort of scaling as $\tau_{E,tot}$ in order to differentiate it from a thermal scaling. Examples of both types are given in Refs [2,10].

To demonstrate the difference between these two types of confinement scalings, we examine a pair of Plasma OPeration CONtour (POPCON) plots[3,4,9]. These are contour plots of P_{aux} determined by solving Eq. (1) over a range of $\langle n_e \rangle$ and $\langle T \rangle$; contours of constant Q are included in the plots to illustrate our point.

For both cases, we employ parameters appropriate to the proposed Burning Plasma Experiment[1] (BPX). In particular we assume major and minor radii of $R = 2.59$ m and $a = 0.795$ m; a plasma elongation of $\kappa = 2$ and triangularity $\delta = 0.35$ are used. The plasma current and toroidal field are set at $I_p = 10.6$ MA and $B_T = 8.1$ T, respectively. The other parameters in our model are assigned the reference values discussed in Ref. [1].

The energy confinement time is written as

$$\tau_E = \min[\tau_{NA}, c_\tau \tau_{aux}(P_{in}, I_p, B_T, ...)], \tag{15}$$

where

$$\tau_{NA} = 7 \times 10^{-3} \overline{n}_{e,19} a R^2 q_* \text{ s} \tag{16}$$

is the neo-Alcator (ohmic) contribution, with $\overline{n}_{e,19}$ being the line-averaged electron density in units of 10^{19} m^{-3} and q_* is the cylindrical equivalent safety factor[4]

$$q_* = \frac{5a^2 B_T}{R I_p} \frac{[1 + \kappa^2(1 + 2\delta^2 - 1.2\delta^3)]}{2}.$$

The second term in Eq. (15) represents an auxiliary heated scaling. The (constant) multiplier is included to estimate H-mode performance using L-mode scalings ($c_\tau \sim 2$) or to degrade H-mode scalings ($c_\tau < 1$). By combining ohmic and auxiliary heated scalings, reasonable behavior in all regions of the POPCON plots can be obtained with a single τ_E expression[14]. Since we assume an infinitely sharp transition between the two scalings, the individual properties of each are retained within their respective regions of dominance in $\langle n_e \rangle$ and $\langle T \rangle$ space.

In generating Fig. 1, we use

$$\tau_{aux} = \tau_E^{ITER89-P} = 0.0381 I_p^{0.85} B_T^{0.2} \overline{n}_{e,19}^{0.1} P_{in}^{-0.5} \overline{A}_i^{0.5} R^{1.2} a^{0.3} \kappa^{0.5}, \tag{17}$$

where $\overline{A}_i = 2.5$ is the average ion mass. The ITER89-P scaling[11] is a power-law fit to L-mode confinement data. We multiply it by $c_\tau = 2.2$ to simulate H-mode confinement[1]. Since $\alpha = 0.1$ and $\gamma = 0.5$ for this scaling (and $\alpha = 1$ and $\gamma = 0$ for neo-Alcator), Fig. 1 exhibits a monotonic increase of Q with increasing density.

As a contrast, we show in Fig. 2 a POPCON obtained with a $\tau_{E,th}$ H-mode scaling[10],

$$\tau_{aux} = \tau_E^{ITER-Hth} = 0.034 I_p^{0.77} B_T^{0.49} \overline{n}_{e,19}^{0.3} P_{in}^{-0.71} \overline{A}_i^{0.5} R^{2.02} a^{0.26} \kappa^{0.38}. \tag{18}$$

In the regions of low density and temperature where $\tau_{NA} < \tau_E^{ITER-Hth}$, the Q contours increase with density as in Fig. 1. Elsewhere, however, the opposite is true; this behavior is exemplified by the Q contours on the right side of Fig. 2. The result is a completely enclosed ignition region. The lower $\langle n_e \rangle$ and $\langle T \rangle$ bounds on ignition are provided by the neo-Alcator scaling; the upper bounds are due to $\tau_E^{ITER-Hth}$.

To understand these results physically, we consider a power law scaling of the form

$$\tau_E(P_{in}) = f_\tau \langle n_e \rangle^\alpha P_{in}^{-\gamma}, \tag{19}$$

where f_τ is independent of power and density. Using the definitions of P_{con} and P_{in} with Eq. (1), we find

$$P_{con} \propto \langle n_e \rangle^{\frac{1-\alpha}{1-\gamma}} \langle T \rangle^{\frac{1}{1-\gamma}}. \tag{20}$$

When $1 + \alpha - 2\gamma < 0$, the density exponent in Eq. (20) is > 2. That is, the conducted losses increase *faster* with density than does the alpha power. Hence, we expect the ignition margin to fall as the density rises. Furthermore, we see that in the case of $\tau_E^{ITER-Hth}$, $P_{con} \propto \langle T \rangle^{3.45}$. Except for very low $\langle T \rangle$, this is again a stronger scaling than that of the alpha power. Hence, the losses dominate Eq. (1) at a lower temperature than that found using scalings such as Eq. (17). This is apparent when one compares Fig. 2 with Fig. 1. The important implication of this result is that stable ignited operation could be obtained below the beta limit[1,15,16] and at reasonable values for the total loss power[1]. Previous work generally predicted ignition regions which extended to higher temperatures and power levels[1,4,5,7,8].

As is apparent in Refs [2,10], typically both α and γ are larger for thermal scalings than for total energy confinement time expressions. But, it is the greater power degradation ($\gamma > 0.5$) which gives rise to the behavior noted in Fig. 2; the increase in the density exponent acts in the other direction. Although one can understand why the density scaling is stronger[2], it is not clear why the power degradation should be greater. One might speculate that it is the result of the energetic ions being better confined than their thermal counterparts. There is some evidence for this in the literature[17].

In conclusion, we have outlined how the density scaling of reactor performance, measured either by the ignition margin or the power multiplication factor Q, varies with the density and power dependence of the energy confinement time τ_E. Thermal energy confinement time scalings differ from total energy confinement expressions (see, for example, Refs [2,10]) in that they lead to a reactor performance which decreases with increasing density. Thermal scalings for τ_E are preferred in solving Eq. (1) since Eq. (4) matches the definition of $\tau_{E,th}$. If the thermal scaling trends noted here are found to be generally true, previous notions of how reactors should be operated[1,3–9] may need to be altered.

Acknowledgments

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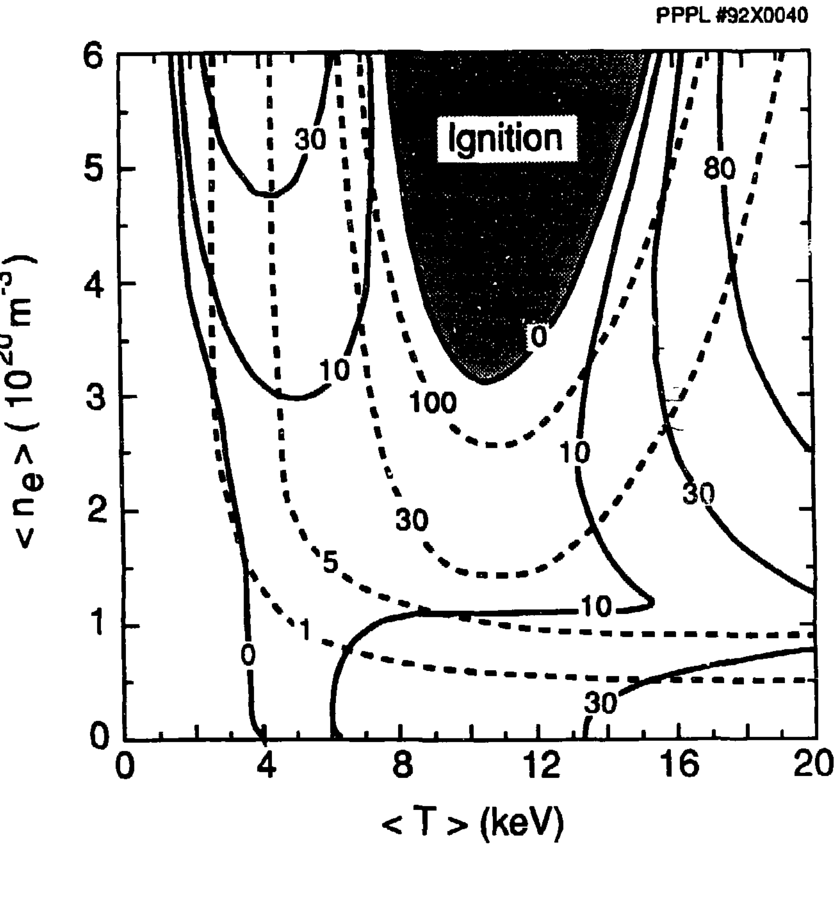
Figures

Fig. 1. Contours of constant P_{aux} in MW (solid lines) and Q (dashed lines) in $\langle n_e \rangle$ and $\langle T \rangle$ space for $\tau_E = \min(\tau_{NA}, 2.2\tau_E^{ITER89-P})$; BPX parameters and assumptions are used[1].

Fig. 2. Contours of constant P_{aux} in MW (solid lines) and Q (dashed lines) in $\langle n_e \rangle$ and $\langle T \rangle$ space for $\tau_E = \min(\tau_{NA}, \tau_E^{ITER-Hth})$; BPX parameters and assumptions are used[1].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Ignition
0
30
80
10
100
10
30
30
5
10
1
0
30
0
4
8
12
16
20
0
1
2
3
4
5
6
< T > (keV)
< n_e > (10^20 m^-3)

FIG. 1

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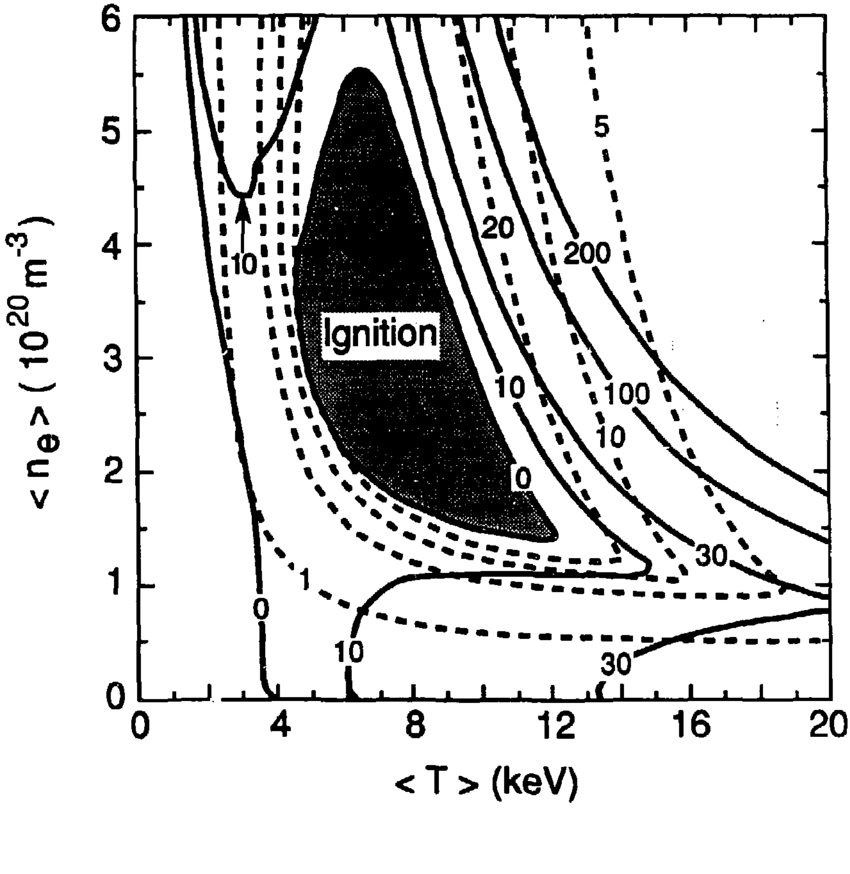

FIG. 2

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