Constraints on CP Violating Nucleon-Nucleon Interactions in Gauge Models from Atomic Electric Dipole Moment

**/ OZ^l/22** *\* **.** 

**Aiao-Gang He and Bruce McKellar**  *Research Center for High Energy Physics School of Physics University of Melbourne Parkville, Vic. S052 Australia* 

**In this paper CP violating nudeon-nudeon interactions are studied in several models. The experimental upper bounds on atomic electric dipole moment (EDM) are used to constrain CP violating parameters. We compare the constraints from this consideration with that obtained from the upper bound on the neutron EOM. We find that although the constraints from the fomer consideration are not yet as sensitive as the latter, in some models the constraints from both considerations are within an order of magnitude.** 

**1** 

#### **I. INTRODUCTION**

-71

ĵи

**The origin of CP violation is one of the fundamental problems of modern physics. CP violation was discovred in 1964 in neutral Kaon system[l], but no other CP violating processes have been found. In these circumstances many models have been proposed to explain the phenomenon. It is very important to find CP violation in other systems in order to isolate the source (or sources) responsible. The measurement of electric dipole moment (EMD) of fundamental particles is a very promising avenue.**  The EDM  $\vec{D}$  of a classical charge distribution  $\rho$  is given by

$$
\vec{D} = \int d^3r \vec{r} \rho \,. \tag{1}
$$

**In the case of an elementary particle, the only (pseudo) vector that characterizes its state is angular moment (spin)**  $\vec{J}$ ,  $\vec{D}$  must be proportional to  $\vec{J}$ . Therefore  $\vec{D}$  and *J* **have the same transformation properties under parity (P), charge conjugation (C)**  and time reversal  $(T)$  symmetries. The interaction of electric filed  $\vec{E}$  with the EDM of a particle is  $\sim \vec{J} \cdot \vec{E}$  which violates both P and T. If CPT is a good symmetry, T **violation implies CP violation and vice versa.** 

**Although at present no experiments have measured a non-zero EDM of a funda**mental particle, upper bound on the EDM of the neutron  $(d_n < 10^{-25}$ ecrn)[2] and of the electron  $(d_e < 10^{-26}$ ecm)<sup>[3]</sup> have put stringent constraints on CP violating **parameters in different models. Experiments have also been performed to measure EDMs of atoms,** *D<sup>A</sup> .* **Upper bounds on the EDM of several atoms have been ob-** $\text{tained: } D(^{129}Xe) = (-0.3 \pm 1.1) \times 10^{-26} e^{-0.5}$ ,  $D(^{199}Hg) = (0.7 \pm 1.5) \times 10^{-26} e^{-0.5}$ ,  $D(^{205}Tl) = (1.6 \pm 5.0) \times 10^{-24}$  ecm[3] and  $D(Cs) = (-1.8 \pm 6.7 \pm 1.8) \times 10^{-24}$  ecm[6]. **It was thought that the mesaurement of atomic EDM is difficult and not useful due to a theorem of SchifF[7] which states that the EDM of a non-relativistic atom vanishes irrespective of whether the atomic constituents have an EDM or not, if atoms con-**

**sist of non-relativistic particles which interacte only electrostatically and the EDM distribution of each atomic constituent is identical to its charge distribution. This theorem works quite well for the ground state hydrogen atom, for example. However, in many cases the conditions of the theorem are not met due to the effects of relativistic electrons, spin-orbit interaction, differences between nucleon charge and EDM density distributions, the finite size of the nucleus, and so on. All these effects can in principle give rise to an atomic EDM if CP violating interactions are present. The EDM of atoms due to these effects can be enhanced considerably compared to**  the EDM of the constituent particles. For example, in  $\frac{203}{403}$ ,  $\frac{205}{40}$ ,  $D_A$  is enhanced by **a factor of about 500 to 700 compared with the electron EDM [8]. Many CP vio**lating operators can induce EDMs for atoms: the EDM of the nucleon  $d_N[9]$ , the **EDM of the electron** *de[\$,* **10], T-odd nucleon-nucleon interactions [11-13], and T-odd**  electron-nucleon interactions<sup>[14]</sup>. In general the atomic EDM will be a linear com**bination of the contributions of several T-odd interactions to the lowest order and it can schematically be written as** 

$$
D_A = R_N d_N + R_{\rm e} d_{\rm e} + C_{N-N} + C_{\rm e-N} + \dots, \qquad (2)
$$

where  $C_{N-N}, C_{e-N}$  are contributions due to T-odd nucleon-nucleon, electron-nucleon interactions respectively. The calculated values of all the quantities  $R_i$  and  $C_i$  depende **on the models of atom, nucleus and elementary particles.** 

**Obviously if a non-zero** *DA* **for some atom should be measured, theoretical input**  would be necessary to pin down its origin. So far only  $D_A$  's consistent with zero **have been measured. It is customary to deduce upper bounds on different contributions by setting other sources to be zero, i.e. to assume that there is no accidental cancellations among different sources. It is difficult to separate these contributions in a model independent way. However, in a particular model of CP violation the** 

**strength of each interaction may be determined and therefore useful information can be obtained. There have been several recent reviews of the EDM of the neutron[15] and the electron[16]. An attempt of systematical analysis of T-odd nucleon-nucleon interactions has been made in Ref.[17] by studying some CP violating dimension 5 and 6 operators. In this paper we will study the T-odd nucleon-nucleon interactions by systematically investigating several models of CP violation and identifying the**  dominant contribtuions in each model. We then compare the constraints on the CP **violating Lagrangain obtained from this study with those obtained from the upper bound on the neutron EDM. The T-odd nucleon-nucleon interactions in which we are interested are:** 

st

 $\mathbf{b}$ 

a.

iu

aq

 $\mathbf{I}$ 

ad

 $\mathbf{v}_1$ 

 $\mathbf{b}$ 

in

 ${\bf e} {\bf i}$ 

$$
i\eta_{n\mathbf{a}}\bar{n}\gamma_{5}n\bar{n}n , i\eta_{n\mathbf{p}}\bar{n}\gamma_{5}n\bar{p}p ,
$$
  
\n
$$
i\eta_{p\mathbf{a}}\bar{p}\gamma_{5}p\bar{n}n , i\eta_{p\mathbf{p}}\bar{p}\gamma_{5}p\bar{p}p ,
$$
  
\n
$$
i\eta'\bar{p}\gamma_{5}n\bar{n}p + H.C.
$$
 (3)

**These operators will induce a P and T odd interaction of the nucleus with the atomic electron cloud[12,13] which is proportional to** 

$$
\varphi = 4\pi \vec{Q} \cdot \vec{\nabla} \rho(0) \tag{4}
$$

where  $\rho(0)$  is the electron density at the nuclear origin,  $\vec{Q}$  is called the Schiff Moment **(SM) and is given by** 

$$
\vec{Q} = \sum \frac{1}{10} e \{ \langle \mathbf{r}_p^2 \vec{r}_p \rangle - R_0^2 \langle \vec{r}_p \rangle \}, \tag{5}
$$

where  $R_0 = r_0 A^{1/3}$  is the nuclear radius (here A is the atomic number),  $r_0 = 1.15 fm$ and the summation is over all protons.  $\varphi$  will generate non-zero atomic EDM.

In the non-relativistic approximation, terms proportional to  $\eta_{ij}$  induce interactions proportional to  $\vec{\sigma} \cdot \nabla \rho(r)$ , where  $\sigma$  is the spin of a particular nucleon and  $\rho(r)$ 

**is the core neutron and proton density. The interaction generated by the term pro**portional to  $\eta'$  is of the form  $(\vec{\sigma_p} \times \vec{\sigma_n}) \cdot (\vec{p}_{fn} + \vec{p}_{in} - \vec{p}_{fp} - \vec{p}_{ip}),$  where  $\vec{p}_{(f,i)(n,p)}$  are the **neutron and proton final and initial momenta. In many cases, particularly the cases**  we will discuss, the latter term does not cause a non-zero  $\vec{Q}$  [11]. We will neglect it **in our later discussions.** 

Ņ

Calculations of  $\vec{Q}$  due to non-zero  $\eta_{ij}$  for some atoms have been carried out in **Ref.[13]. The results are given as follow** 

$$
^{129}Xe \qquad ^{199}Hg \qquad ^{203,205}Tl
$$
\n
$$
Q \quad 1.75\eta_{np} \quad -1.4\eta_{np} \quad 1.2\eta_{pp} - 1.4\eta_{pn}
$$

Here the magnitude of SM  $Q$  is given in the unit  $10^{-8} efm^3$ . In these atoms, because  $J = 1/2$ , the magnetic quadrupoles are zero, and the nuclear SM is the only nuclear **multipole that leads to the atomic EDM. Also in these atoms the term proportional**  to  $\eta'$  does not produce a non-zero  $\overline{Q}$ . There are corrections to the numbers given **above due to recoil effects. These corrections can be quite large[13], about a 30%**  effect, and also can induce a contribution to  $Q$  from  $\eta_{nn}$ . However, the corrections **are model dependent, and the order of magnitudes given above will not be changed. In our later discussions, we will use the numbers given above.** 

**Constraints on Q for several atoms have been obtained. Combining the exper**imental result for of  $D(^{129}Xe)[4]$  and theoretical calculations[18],  $Q(^{129}Xe)$  is esti**mated to be** 

$$
Q(^{129}Xe) = (-1 \pm 4) \times 10^{-9} efm^3. \tag{6}
$$

**Similarly,** *Q(l99Hg)* **is determined as** 

 $\frac{11}{11}$ 

$$
Q(^{199}Hg) = (-1.8 \pm 3.8) \times 10^{-10} efm^3 \tag{7}
$$

**from experimental data of Ref.[5] and theoretical calculations of Ref.[12j. Using**  experimental data from Ref.[19] and theoretical calculations of Ref.[20],  $Q(^{205}Tl)$ **is estimated to be** 

$$
Q(^{205}Tl) = (-1.8 \pm 3.0) \times 10^{-10} efm^3 \,. \tag{8}
$$

If one use the more recently measured bound on the EDM of  $D({}^{205}Tl)$  [3],  $Q({}^{205}Tl)$ **is constrained to be about 6 times smaller.** 

**From all of these results, we deduce** 

$$
|\eta_{np}| < 7 \times 10^{-2}, \ |\eta_{pp} - 1.17\eta_{pn}| < 2 \times 10^{-2} \ . \tag{9}
$$

**at the 90% confedence level.** 

 $\mathbf{r}$ 

o

In the rest of this paper we calculate  $\eta_{ij}$  in several models of CP violation and **compare the constraints on the CP violating parameters of the Lagrangian with those obtained from the upper bound on the neutron EDM. In sections II to VII, we estimate**   $\eta$  in the minimal standard model, in the model due to CP violating  $\theta$  term in QCD, **in multi-Higgs doublet models, in Left-Right symmetric models, in supersymmetric extension of the minimal standard model and in di-quark scalar models. We emphasise the possible new CP violating sources in addition to the minimal standard model. In section VIII we make our conclusing observations.** 

#### **II.**  $\eta$  **IN THE STANDARD MODEL**

In the  $SU(3)_C \times SU(2)_L \times U(1)_Y$  model with one Higgs doublet (the minimal **standard model), CP violation is due to the non-removable phase in the quark mixing matrix** *VKM* **of the charged current [21]. There must be at least three generations in order to have non-zero CP violating phase. The charged current interaction Lagrangian is** 

$$
L_W = \frac{g}{\sqrt{2}} \bar{U}_L \gamma_\mu V_{KM} D_L W_\mu^+ + H.C. \,, \tag{10}
$$

where W is the W-gauge boson and  $U = (u, c, t, ...)$  and  $D = (d, s, b, ...)$  are the charge 2/3 and  $-1/3$  quark fields respectively. In the three generation case,  $V_{KM}$  can **be parametrized as** 

$$
V_{KM} = \begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2s_3 - c_2c_3e^{i\delta} \end{pmatrix}
$$
 (11)

where  $c_i = cos\theta_i$  and  $s_i = sin\theta_i$ . The mixing angles are determined by the analysis of **many experimental data. The allowed range of the CP violating phase** *6* **is determined**  from the observed C" violation in  $K^0 - \tilde{K}^0$  system and is[22]

$$
2 \times 10^{-4} < s_2 s_3 s_\delta < 10^{-3} \tag{12}
$$

**when one varies the top quark mass** *m<sup>t</sup>*  **from 90 GeV to 200 GeV with the maximum**  being reached for small values of  $m_t$ .

The quantity  $\eta_{ij}$  has been studied before by two groups.  $\eta_{ij}$  was found to be of order  $10^{-8}$  in Ref.[11]. In the evaluation of  $\eta$ , Ref.[11] omitted some diagrams which **cancel out the leading terms. This was corrected in the calculation of Ref.[23]. It was found that the dominant contribution is from baryon pole diagrams and the result is**  smaller by a factor of 25 than in Ref.[11]. In Ref.[23] only  $\eta_{pn}$  was evaluated. In this paper we report a calculation for  $\eta_{ij}$  using different method. The set of diagrams we will evaluate are shown in Fig.1. We will use the chiral Lagrangain for the  $\pi - K$ **transition** 

$$
L_{\pi-K} = h e^{i\theta} Tr(\lambda_+ D^{\mu} M D_{\mu} M) + H.C. \,, \tag{13}
$$

where  $M$  is the pesudo-scalar octet of flavour  $SU(3)$ ,

 $\hat{\mathcal{A}}$ 

$$
\lambda_{+} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix},
$$

 $\overline{7}$ 

$$
D_{\mu}=\partial_{\mu}M+ieA_{\mu}[Q,M]\ ,
$$

**with** 

$$
Q = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix},
$$

is the covariant derivetive. This Lagrangain will guarantee the  $k^2$  dependence in the  $\pi$  – *K* transition which was missed in Ref.[11] and was partly corrected in Ref.[23]. *h* and the CP violating phase  $\theta$  are obtained by relating *h* and  $\theta$  to  $\lt \pi^+\pi^-|H_W|K>$ **using PCAC,** 

$$
hm_K^2 = i\sqrt{2}f_\pi < \pi^+\pi^-|H_W||K_s>,\\
\theta \approx \frac{Im A_0}{Re A_0} \tag{14}
$$

i,

We obtian,  $h = 1.49 \times 10^{-7}, \theta = -3.2ImC_s$  where  $ImC_s$  is approximately  $-0.1s_2s_3s_6[24]$  and a full evaluation for large  $m_t$  can be found in Ref.[25].

**For the strong interacting vertices, we use** 

$$
L_{\bullet} = -\sqrt{2}g_{\pi NN}[Tr(\bar{B}i\gamma_{5}MB) + (2\alpha - 1)Tr(\bar{B}i\gamma_{5}BM)], \qquad (15)
$$

where  $\alpha = 0.64$ ,  $g_{\pi NN}^2/4\pi = 14$  and B is the matrix of baryon octet fields.

**Neglecting small terms, the relevant weak parity violating** *BBM* **interaction Lagrangain can be written as** 

$$
L_W = \sqrt{2} f_3 [e^{i\phi_3} Tr(BM\lambda_+ \bar{B} + e^{-i\phi_3} Tr(BM\lambda_- \bar{B})]
$$
  
+  $f_4 [e^{i\phi_4} Tr(\bar{B}M\lambda_+ B) + e^{-i\phi_4} Tr(\bar{B}M\lambda_- B)]$  }, (16)

where  $\lambda_{-} = \lambda_{+}^{\dagger}$ . The parameter  $f_3$  and  $f_4$  are related to the S-wave hyperon decay **amplitudes**  $A(\Sigma^+ \to p\pi^0)$  and  $A(\Lambda^0 \to n\pi^-)$ . Using the experimental values, one

**and** 

obtains,  $f_3 = -3.2 \times 10^{-7}$ ,  $f_4 = 1.18 \times 10^{-7}$ . The phases  $\phi_3$  and  $\phi_4$  are similarly obtained from the calculated CP violating amplitudes for hyperon decays [26],  $\phi_3 =$  $-0.29ImC_5$ ,  $\phi_4 = -0.61ImC_5$ .

**For the baryon-baryon transition amplitude, we use the SU(3) parametrization,** 

$$
L_{BB} = -G_F m_\pi^2 (f f_{i6j} + dd_{i6j}), \qquad (17)
$$

where  $f_{i6j}$  and  $d_{i6j}$  are the antisymmetric and symmetric structure constant of SU(3) group. By fitting P-wave hyperon decay data,  $f$  and  $d$  are determined to be  $f =$  $-0.57GeV$  and  $d = 0.65GeV$ . The amplitudes  $a_{An}$ ,  $a_{\Sigma^0 n}$  and  $a_{\Sigma^+ p}$  of  $\Lambda^0 - n$ ,  $\Sigma^0 - n$ and  $\Sigma^+$  – p transitions are given by  $-(3f + d)G_F m_\pi^2/\sqrt{6}$ ,  $-(d - f)G_F m_\pi^2/\sqrt{2}$  and  $(d-f)G_Fm_\pi^2$ , respectively. Their phases  $\phi_{\Lambda^0\pi}$ ,  $\phi_{\Sigma^+\rho} = \phi_{\Sigma^0\pi}$  are calculated to be  $\phi_{\Lambda^0\pi} = 0$ *ImC*<sub>5</sub> and  $\phi_{\Sigma^+ p} = \phi_{\Sigma^0 n} = -0.36$ *ImC*<sub>5</sub> respectively by using MIT bag model[27].

**The contribution of Fig. la to the T-odd nucleon-nudeon interaction is** 

ĥ

$$
H = 2\sqrt{2}f_{kn}hg_{\pi NN}sin(\theta - \phi_{kn})\frac{k^2}{(k^2 - m_k^2)(k^2 - m_\pi^2)}i(\bar{n}\gamma_5 n - \bar{p}\gamma_5 p)\bar{n}n
$$
  
+2\sqrt{2}f\_3hg\_{\pi NN}sin(\theta - \phi\_3)\frac{k^2}{(k^2 - m\_k^2)(k^2 - m\_\pi^2)}i(\bar{n}\gamma\_5 n - \bar{p}\gamma\_5 p)\bar{p}p. (18)

**The baryon pole contributions (Fig. lb,lc) to the T-odd nucleon-nucleon interaction are** 

$$
H = \frac{4f_{kn}}{k^2 - m_K^2} \left( \frac{a_{\Sigma^0 n} g_{\Sigma^0 n} \kappa^0 \sin(\phi_{kn} - \phi_{\Sigma^0 n})}{m_N - m_\Sigma} + \frac{a_{kn} g_{kn} \kappa^0 \sin(\phi_{kn} - \phi_{kn})}{m_N - m_\Lambda} \right) i \bar{n} \gamma_5 n \bar{n} n + \frac{4f_3}{k^2 - m_K^2} \left( \frac{a_{\Sigma^0 n} g_{\Sigma^0 n} \kappa^0 \sin(\phi_3 - \phi_{\Sigma^0 n})}{m_N - m_\Sigma} + \frac{a_{kn} g_{kn} \kappa^0 \sin(\phi_3 - \phi_{kn})}{m_N - m_\Lambda} \right) i \bar{n} \gamma_5 n \bar{p} p + \frac{4f_{kn}}{k^2 - m_K^2} \frac{a_{\Sigma^+ p} g_{\Sigma^+ p} \kappa^0 \sin(\phi_{kn} - \phi_{\Sigma^+ p})}{m_N - m_\Sigma} i \bar{p} \gamma_5 p \bar{n} n + \frac{4f_3}{k^2 - m_K^2} \frac{a_{\Sigma^+ p} g_{\Sigma^+ p} \kappa^0 \sin(\phi_3 - \phi_{\Sigma^+ p})}{m_N - m_\Sigma} i \bar{p} \gamma_5 p \bar{p} p , \tag{19}
$$

 $\mathbf{where} \ g_{\Sigma^+ pK^0} = -\sqrt{2}g_{\Sigma^0 nK^0} = \sqrt{2}g_{\pi NN}(2\alpha - 1), \ g_{\Lambda nK^0} = -g_{\pi NN}(3 - 2\alpha)/\sqrt{6}, \text{ and}$  $f_{kn}e^{ip_{kn}} = f_3e^{ip_3} - f_4e^{ip_4}.$ 

**Notice that our results are invariant under redefinition of the s-quark phase, as was emphasised in Ref.[28] this is an important consistency check in the construction of models of CP violation. Inserting the numerical values of all quantities, we find the dominant contributions are from baryon poles. The contributions from Fig. la are much smaller than the baryon pole contributions and we can neglect them. The results are** 

4

$$
|\eta_{nn}| = 2.2 \times 10^{-5} |ImC_5| \ , \ |\eta_{np}| = 1.7 \times 10^{-5} |ImC_5| \ . \tag{20}
$$

The numerical values for  $\eta_{pn}$  and  $\eta_{pp}$  are of order  $10^{-7}ImC_5$  and thus are much smaller.  $\eta_{nn}$  and  $\eta_{np}$  can be as large as  $2 \times 10^{-9}$ . In our estimates of the finite range effect for non-zero  $k^2$  we have made the approximation that  $k^2 \approx -m_\pi^2$  as a mean momentum **transfer in the nucleus. If the KM mechanism is the only source for CP violation,** *Xe*  **and** *Hg* **would be better places to looking for** *D<sup>A</sup>*  **than** *Tl.* **Of cousre these numbers are still very small compared with the present experimental upper bounds.** 

## **HI.**  $\eta$  DUE TO THE  $\theta$  TERM IN QCD

In this section, we study  $\eta$  due to the  $\theta$  term in QCD. It has long been realized **that due to instanton effects, in non-abelian gauge theory, the total divergence term** 

$$
\tilde{G}_{\alpha\beta}G^{\alpha\beta} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}G_{\alpha\beta}G^{\mu\nu} \tag{21}
$$

constructed from the field strength  $G^{\mu\nu}$  has nonvanishing physical effects. In the case **of QCD,** *G,w* **is the gluon field strength. The full QCD Lagrangain is then** 

$$
L_{QCD} = -\frac{1}{4}G^{\mu\nu}G_{\mu\nu} + \bar{q}(D_{\mu}\gamma^{\mu} - m)q - \theta \frac{g_{\rho}^{2}}{32\pi^{2}}\tilde{G}_{\mu\nu}G^{\mu\nu} , \qquad (22)
$$

where q is the quark field, m is the quark mass and  $D_{\mu}$  is the covariant derivative and  $\theta$  is a constant.

The last term in  $L_{QCD}$  violates P and CP. The physical effects of a non-zero  $\theta$ **have been extensively studied[29,30]. This Lagrangian will generate CP violating meson-nucleon couplings. In this paper we use the result from the chiral Lagrangian to leading 1/N order, where N is the number of colour[30]. One obtains** 

$$
L_{\pi NN} = -\sqrt{2}\bar{N}\vec{\tau} \cdot \vec{\pi}(i\gamma_5 g_{\pi NN} + f_{\pi NN})N \qquad (23)
$$

**with** 

$$
f_{\pi NN} = 0.027\theta \,. \tag{24}
$$

From the above effective Lagrangian, we obtain CP violating nucleon-nucleon **interactions by exchange of a neutral pion,** 

$$
H_{eff} = i \frac{g_{\pi NN} f_{\pi NN}}{m_{\pi}^2 - k^2} \dot{N} \tau_3 \gamma_5 N \dot{N} \tau_3 N \ . \tag{25}
$$

Here we only need to evaluate contributions due to exchange of  $\pi^0$  because exchange **of charged pious will generate a term proportional to** *i\'* **which can be neglected in our case. This interaction is of finite range because the dependence on the momentum exchanged by pion. For an order of magnitude estimate we use our previous approxi**mation that  $k^2 \approx -m_\pi^2$ . We obtain  $\eta_{nn} = -\eta_{np} = -\eta_{pn} = \eta_{pp} = g_{\pi NN}f_{\pi NN}/\sqrt{2}G_F m_\pi^2$ .

*0* **has been constrained to be less than 10~<sup>9</sup> [29] from the upper bound on the neutron EDM. This implies that**  $|\eta_{ij}| = 10^6 |\theta| < 10^{-3}$ **. This is about an order of magnitude below the experimental bound.** 

# **IV.**  $\eta$  **IN MULTI-HIGGS DOUBLET MODELS**

In this section we study  $\eta$  in multi-Higgs doublet models. In multi-Higgs doublet **models, it is possible to have CP violation in the Higgs sector. With more than one** 

**Higgs doublet, in general there will be flavour changing neutral cunents induced by neutral Higgs particles at the tree level. Such dangerous neutral flavour changing current can be prevented by imposing certain discrete symmetries. These symmetries will eliminate some terms in the Higgs potential and also eliminate CP violation in Higgs sector in some cases. In order to have neutral flavour current conservation at the tree level and CP violation in the Higgs sector at least three Higgs doublets are needed[31]. With three Higgs doublets it is also possible to have spontaneous CP violation. It has been shown recently that if CP violation is due only to spontaneous symmetry breaking, the three Higgs doublet model and many other models are ruled out because they have an unacceptablely large**  $\theta$  **term (** $\gg 10^{-9}$ **)[32, 33]. In the following we will discuss models in which CP is broken explicitly such that the large** *6*  **term can always be cancelled out by tuning relevant parameters.** 

**The interactions of Higgs particles with quarks are given by** 

$$
L_{H^{+}} = (2\sqrt{2}G_{F})^{1/2}(\alpha_{i}\bar{U}_{L}V_{KM}M_{D}D_{R} + \beta_{i}\bar{U}_{R}M_{U}V_{KM}D_{L})H_{i}^{+}, \qquad (26)
$$

**and** 

 $\begin{array}{c} \begin{array}{c} \begin{array}{c} \begin{array}{c} \end{array} \\ \begin{array}{c} \end{array} \\ \begin{array}{c} \end{array} \\ \begin{array}{c} \end{array} \end{array} \end{array} \end{array}$ 

 $\begin{array}{c} \begin{array}{c} \begin{array}{c} \begin{array}{c} \end{array}\\ \begin{array}{c}$ 

$$
L_{H^0} = (2\sqrt{2}G_F)^{1/2} (\alpha_{di}\bar{D}M_D D + \beta_{di}\bar{D}iM_D\gamma_5 D
$$
  
+
$$
\alpha_{ui}\bar{U}M_U U + \beta_{ui}\bar{U}iM_U\gamma_5 U)H_i^0 ,
$$
 (27)

where  $M_U$  and  $M_D$  are the diagonalized quark mass matrices, and  $H_i^+$  and  $H_i^0$  are mass **eigenstates of charged Higgs and neutral Higgs particles respectively. In three Higgs doublet models, there are three physical charged and five neutral Higgs particles. If in the weak eigenstate of Higgs particles only one of the Higgs doublets couples to up**  and down quarks,  $\alpha_i = \beta_i$ ,  $\alpha_{di} = \alpha_{ui}$  and  $\beta_{di} = -\beta_{ui}$ . In this case CP violation due **to exchange of the charged Higgs is soly from KM-matrix. If up and down quarks couple to different Higgs doublets, CP violation will occur in both the Higgs sector** 

**and the KM-sector with exchange of charged Higgs particles. In all cases, exchange**  of neutral Higgs particles can violate CP. In the literature some times  $\alpha$  and  $\beta$  are **parametrized as[34]** 

$$
Im Z = 2Im \alpha \beta^*, Im Z_{qq'} = 2\alpha_q \beta_{q'}.
$$
 (28)

**In our later discussions, we will assume that the effect of Higgs exchange is dominated by a single Higgs particle.** 

As we have already seen in section II that  $\eta$  aue to KM-mechanism is extremely **small, we will study possible large contributions from exchange of Higgs particles. There are several CP violating operators which may have large contributions. Here we study the following ones** 

$$
Q_{gq} = i\frac{g_o}{2}\bar{q}\sigma_{\mu\nu}\gamma_5 \frac{\lambda^a}{2}G^{a\mu\nu}q\ ,\ \ Q_g = -\frac{g_o^3}{3}f^{abc}\tilde{G}^a_{\mu\nu}G^b_{\mu\alpha}G^c_{\nu\alpha}\ ,\ \ Q_q = i\bar{q}\gamma_5q\bar{q}'q'\ .\tag{29}
$$

**We write the effective Lagrangian as** 

$$
L_{eff} = C_{\rho q} Q_{gq} + C_q Q_q + C_g Q_g \,. \tag{30}
$$

The coefficients  $C_i$  need to be calculated in the model. It turns out that the dominant contributions to  $C_{gq}$ [35] and  $C_g$ [34, 36] are from two loop diagrams while  $C_q$  is **dominated by exchange charged and neutral Higgs particles at tree level. We have** 

$$
C_g = \frac{G_F}{\sqrt{2}(4\pi)^4} [Im Z_{\text{ws}} \xi_N h(\frac{m_t^2}{m_H^2}) + Im Z \xi_N h'(\frac{m_t^2}{m_H^2})],
$$
  
\n
$$
C_{gq} = -\frac{G_F}{16\sqrt{2}\pi^3} m_q \alpha_s(\mu) \left(\frac{g_s(m_H)}{g_s(\mu)}\right)^{74/23} G(m_t^2/m_H^2; q),
$$
\n(31)

**where** 

$$
\xi_N = \left(\frac{g_s(m_b)}{g_s(m_t)}\right)^{-108/23} \left(\frac{g_s(m_c)}{g_s(m_b)}\right)^{-108/25} \left(\frac{g_s(\mu)}{g_s(m_c)}\right)^{-108/27}.
$$
  

$$
\xi'_N = \left(\frac{g_s(m_b)}{g_s(m_t)}\right)^{-28/23} \left(\frac{g_s(m_c)}{g_s(m_b)}\right)^{-108/25} \left(\frac{g_s(\mu)}{g_s(m_c)}\right)^{-108/27}.
$$

The functions *G(z;* g), and *h, h'* are given by

$$
G(z; u) = [f(z) + g(z)]Im Z_{uu}
$$
  

$$
G(z; d) = f(z)Im Z_{ud} + g(z)Im Z_{du}
$$
,

$$
h(z) = \frac{1}{2}z^2 \int_0^1 dx \int_0^1 dy \frac{x^3 y^3 (1-x)}{(zx(1-xy)+(1-x)(1-y))^2},
$$
  

$$
h'(z) = \frac{1}{2} \frac{z}{(1-z)^3} (-\ln z - \frac{3}{2} + 2z - \frac{z^2}{2}).
$$

and

Th

an

n.

W

W

$$
f(z) = \frac{1}{2}z \int_0^1 dz \frac{1 - 2x(1 - z)}{x(1 - z) - z} ln \frac{x(1 - z)}{z},
$$
  

$$
g(z) = \frac{1}{2}z \int_0^1 dz \frac{1}{x(1 - z) - z} ln \frac{x(1 - z)}{z}.
$$

In multi-Higgs doublet models *CqQ<sup>q</sup>* receives contributions from both charged and neutral Higgs particles. For the charged Higgs contribution, we have

$$
C_{\mathbf{q}}Q_{\mathbf{q}} = -i\frac{\sqrt{2}G_F}{12m_H^2} \{ImZm_{\mathbf{u}}m_{\mathbf{d}}|V_{\mathbf{ud}}|^2(\bar{u}\gamma_5 u\bar{d}d + \bar{u}u\bar{d}\gamma_5 d) ImZm_{\mathbf{u}}m_{\mathbf{s}}|V_{\mathbf{u}\mathbf{s}}|^2\bar{u}\gamma_5 u\bar{s}s + ... \},
$$
 (32)

where a Fierz rearrangement has been made and we have singled out the terms which will give dominant contributions to  $\eta$  when use the vacuum saturation and factorization approximation. For the neutral Higgs contribution, we have

$$
C_{q}Q_{q} = i\frac{2\sqrt{2}G_{F}}{m_{H}^{2}} \{Im Z_{dd}m_{d}\bar{D}M_{D}D\bar{d}\gamma_{5}d + Im Z_{du}m_{u}\bar{D}M_{D}D\bar{u}\gamma_{5}u + Im Z_{ud}m_{d}\bar{U}M_{U}U\bar{d}\gamma_{5}d + Im_{uu}m_{u}\bar{U}M_{U}U\bar{u}\gamma_{5}u\}.
$$
 (33)

To evaluate  $\eta$  we need to calculate several hadronic matrix elements. We will use the pion pole dominance approximation. We first calculate the CP violating  $\pi^0 NN$ verterx



$$
\langle \pi N | Q_i | N \rangle = B_i \,. \tag{34}
$$

**and then x° exchange to obtain** 

$$
H_{eff} = i g_{\pi NN} \frac{B_i C_i}{m_{\pi}^2 - k^2} \bar{N} \tau_3 \gamma_5 N \bar{N} \tau_3 N . \qquad (35)
$$

CP violating  $\pi NN$  vertices due to  $Q_{qq}$  and  $Q_g$  have been carried out in Ref.[37]. Using  $\eta^0$  meson dominance, the parameters  $B_i$  are estimated to be

$$
B_i \approx -\frac{8\bar{m}a_1f_i}{4F_\pi^2}\,,\tag{36}
$$

where  $\bar{m} = (m_u + m_d)/2$ ,  $a_1 = (m_{\tilde{z}} - m_{\tilde{z}})/2(m_s - \bar{m})$  and

$$
B_{gg} = -0.22 GeV, B_g = 0.54 GeV^2. \tag{37}
$$

The contributions to  $\eta$  from  $Q_g$  and  $Q_{gq}$  are

$$
|\eta(Q_g)| = 1.1 \times 10^{-3} [0.005 Im Z_{uu} h(m_t^2/m_H^2) + Im Zh'(m_t^2/m_H^2)],
$$
  

$$
|\eta(Q_{gq})| = \begin{cases} 1.4 \times 10^{-5} |G(m_t^2/m_H^2; u)|, & \text{for } q = u \\ 2.8 \times 10^{-5} |G(m_t^2/m_H^2; d)|, & \text{for } q = d. \end{cases}
$$
(38)

We will use the vacuum saturation and factorization approximation to estimate  $\langle \pi N | Q_q | N \rangle$ . Within this framework,

$$
\langle \pi N | Q_u | N \rangle = \langle \pi | \bar{u} \gamma_5 u | 0 \rangle \langle N | \bar{q} q | N \rangle \,, \tag{39}
$$

and  $\langle \pi N|Q_d|N\rangle$  is obtained by making the obvious substitution  $u \to d$ .

We use

 $\frac{1}{1}$ 

$$
2m_u < \pi^0 |\bar{u}\gamma_5 u|0\rangle = -2m_d < \pi^0 |\bar{d}\gamma_5 d|0\rangle = -i F_\pi m_\pi^2 \,. \tag{40}
$$

To evaluate  $\langle N|\bar{q}q|N\rangle$ , we use the pion-nucleon  $\sigma_{\pi N}$ -term,  $\sigma_{\pi N} = \bar{m} \langle N|\bar{u}u + \bar{u}u\rangle$  $\bar{d}d|N> = 45 MeV$  extracted from experiments and relations of nucleon mass shift due to SU(3) breaking quark masses. For heavy quarks, we use

$$
\langle N | m_h \bar{h} h | N \rangle = - \langle N | \frac{\alpha_s}{12\pi} G_{\mu\nu} G^{\mu\nu} | N \rangle + O(\frac{\mu^2}{m_h^2}). \tag{41}
$$

The second term in the above equation will be neglected. Using  $m_u = 4.2 MeV$ ,  $m_d = 7.5MeV$  and  $m_s = 150MeV$ , we obtain

$$
m_u < p|\bar{u}u|p\rangle = 18MeV \,, \quad m_d < p|\bar{d}d|p\rangle = 26MeV \,,
$$
\n
$$
m_u < n|\bar{u}u|n\rangle = 14MeV \,, \quad m_d < n|\bar{d}d|n\rangle = 32MeV \,, \tag{42}
$$
\n
$$
m_s < N|\bar{s}s|N\rangle = 247MeV \,, \quad < N| - \frac{\alpha_s}{12\pi} G_{\mu\nu}G^{\mu\nu}|N\rangle = 48MeV \,,
$$

where  $N = n, p$ . Combining all the information gathered above, we have

$$
\langle \pi n | C_q^{H^+} Q_q | n \rangle = \frac{G_F}{\sqrt{2}} Im Z \frac{m_{\pi}^2}{m_H^2} \times 2.2 \times 10^{-4} (GeV^2) ,
$$
  

$$
\langle \pi p | C_q^{H^+} Q_q | p \rangle = \frac{G_F}{\sqrt{2}} Im Z \frac{m_{\pi}^2}{m_H^2} \times 1.5 \times 10^{-4} (GeV^2) ,
$$
(43)  

$$
\langle \pi N | C_q^{H^0} Q_q | N \rangle = \frac{G_F}{\sqrt{2}} \frac{m_{\pi}^2}{m_H^2} \times [0.06 (GeV^2)(Im Z_{dd} - Im Z_{du}) + 0.02 (GeV^2)(Im Z_{uu} - Im Z_{du}) ].
$$

Here q is summed over u and d quarks. From these we obtain the contribution to  $\eta$ from the operator *Q<sup>q</sup>*

$$
|\eta_{ni}| = 1.5 \times 10^{-7} |Im Z| \frac{(100 GeV)^2}{m_H^2},
$$
  
\n
$$
|\eta_{pi}| = 10^{-7} |Im Z| \frac{(100 GeV)^2}{m_H^2},
$$
  
\n
$$
|\eta| = 4 \times 10^{-5} |(Im Z_{dd} - Im Z_{du})
$$
  
\n
$$
+ 0.3 (Im Z_{ud} - Im Z_{uu}) \left| \frac{(100 GeV)^2}{m_H^2} \right|.
$$
 (44)

Here i indicates n or p. Using a similar method, in Ref.[38] a calculation of the charged pion coupling to nucleon has been done. It was found that its contribution to neutron EDM is small compared with that from other operators.

**17 can also be calculated by first evaluating the Higgs-neucleon couplings and then exchanging Higgs particle between nucleons. The Higgs-nucleon couplings have been**  calculated by several groups[33, 39]. Using the values for  $m_q < N|\bar{q}q|N >$  from eq(42), **and the values** 

$$
m_u < n|\bar{u}i\gamma_5|u|n\rangle = -419MeV, \quad m_d < n|\bar{d}i\gamma_5 d|n\rangle = 772MeV,
$$
\n
$$
m_u < p|\bar{u}i\gamma_5 u|p\rangle = 432Mev, \quad m_d < p|\bar{d}i\gamma_5 d|p\rangle = -748MeV, \quad (45)
$$
\n
$$
m_s < N|\bar{s}i\gamma_5 s|N\rangle = -165MeV, \quad m_h < N|\bar{h}i\gamma_5 h|N\rangle = -63MeV,
$$

**determined from EMC data and others[33], we obtain** 

$$
|\eta| = 4.3 \times 10^{-5} |Im Z_{qq}| \frac{100^2 GeV^2}{m_{H^0}^2} \,. \tag{46}
$$

**Comparing the contributions discussed before, we see that the charged Higgs**  contribution via the operator  $Q_g$  may dominate if  $Im Z_{g}$  and  $Im Z$  have the same **order of magnitude.** 

## **V.** *v* **IN LEFT-RIGHT SYMMETRIC MODELS**

**In this section we study** *t)* **in Left-Right symmetric models. Left-Right symmetric**  <code>models</code> are based on the gauge group  $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} [40]$  with **quarks and leptons being assigned as** 

$$
Q_L: (3, 2, 1, 1/3), Q_R: (3, 1, 2, 1/3),
$$
  

$$
L_L: (1, 2, 1, -1), L_R: (1, 1, 2, -1).
$$
 (47)

**There are new CP violating interactions in the charged current due to the existence of the right handed gauge boson** *WR* **of** *SU(2)<sup>R</sup> .* **The interactions of mass**  eigenstates of charged gauge bosons  $W_{1,2}$  with quarks have the form

$$
L_{Wq} = \frac{1}{\sqrt{2}} [g_L \bar{U}_L \gamma_\mu V_L D_L \cos\zeta + g_R \bar{U}_R \gamma_\mu V_R D_R \sin\zeta] W_1^\mu
$$
  
+ 
$$
\frac{1}{\sqrt{2}} [-g_L \bar{U}_L \gamma_\mu V_L D_L \sin\zeta + g_R \bar{U}_R \gamma_\mu V_R D_R \cos\zeta] W_2^\mu .
$$
(48)

ţ.

where all fields are in their mass eigenstates, and  $\zeta$  is the mixing angle between  $W_L$ of  $SU(2)_L$  and  $W_R$  of  $SU(2)_R$ .  $V_L$  is the KM matrix for the left-handed charged **current and** *VR* **is an analogous KM matrix involving the right-handed current. If we parametrize** *VL* **in the usual KM way, then for n-generations of quarks, there are**   $(n-1)(n-2)/2$  CP violating phases in  $V_L$ .  $V_R$  can have a different number of phases **depending on the possible models. In the manifest Left-Right symmetric models,**   $V_L = V_R$ . In pseudo-manifest Left-Right symmetric models, there are 2n-1 additional **phases in**  $V_R$  compared with  $V_L$ . If there is no relation between  $V_L$  and  $V_R$ , there are **n(n+l)/2 phases in VR. It is no longer necessary to have three generations of quarks in order to have CP violation. To see how the new CP violating phases may generate**  non-zero  $\eta$  with large values, we consider the case, for simplicity, of two generations and  $g_L = g_R$ . In this case,  $V_L$  and  $V_R$  can be parametrized as

$$
V_L = \begin{pmatrix} \cos\theta_L & \sin\theta_L \\ -\sin\theta_L \cos\theta_L \end{pmatrix},
$$
  

$$
V_R = e^{i\gamma} \begin{pmatrix} e^{-i\delta_2} \cos\theta_R & e^{-i\delta_1} \sin\theta_R \\ -e^{i\delta_1} \sin\theta_R & e^{i\delta_2} \cos\theta_R \end{pmatrix}.
$$
 (49)

We find that the operator which dominates the contribution to  $\eta$  is due to exchange of *W\* **at tree level. The four quark effective Lagrangian which contains the CP violating interaction is** 

$$
L_{LR} = 4 \frac{G_F}{\sqrt{2}} \cos\zeta \sin\zeta [\bar{U}_L \gamma_\mu V_L D_L \bar{D}_R \gamma_\mu V_R^+ U_R \bar{U}_R \gamma_\mu V_R D_R \bar{D}_L \gamma_\mu V_L^+ U_L]. \tag{50}
$$

**We again use the vacuum saturation and factorization approximation to evaluate**  *rj* **from the above Lagrangian. After a Fierz rearrangement, we obtain the operator**  which gives the dominant contribution to  $\eta$ 

$$
L = -i\frac{4G_F}{3\sqrt{2}}cos\zeta sin\zeta [Im(V_{Lud}V_{Rud}^*)(\bar{u}\gamma_5 u\bar{d}d - \bar{u}u\bar{d}\gamma_5 d) + Im(V_{Lus}V_{Rus}^*)\bar{u}\gamma_5 u\bar{s}s].
$$

**The CP violating** *rNN* **vertex is** 

$$
f_{\pi NN} = \langle \pi N | L_{LR} | N \rangle = -i \frac{4}{3} \frac{G_F}{\sqrt{2}} \cos \zeta \sin \zeta
$$
  
 
$$
\times [Im(V_{Lud} V_{Rud}^*) \langle \pi^0 | 2m_u \bar{u} \gamma_5 u | 0 \rangle \langle N | \frac{\bar{u}u}{2m_u} + \frac{\bar{d}d}{2m_d} | N \rangle \qquad (52)
$$
  
 
$$
+ Im(V_{Lus} V_{Rud}^*) \langle \pi^0 | \bar{u} \gamma_5 u | 0 \rangle \langle N | \bar{s} s | N \rangle
$$
  
 
$$
\approx -90 \frac{G_F}{\sqrt{2}} m_{\pi}^2 \cos \zeta \sin \zeta Im(V_{Lud} V_{Rud}^*) .
$$

**Inserting the known values into the above equation, we obtain** 

$$
|\eta| \approx 0.6 \times 10^3 |cos\zeta sin\zeta sin(\gamma - \delta_2)|. \tag{53}
$$

There are also some other contributions to  $\eta$ , for example, the colour dipole **moment contribution. We have evaluated this contribution and found it is smaller by several orders of magnitude compared to the operator discussed above.** 

### VI. η IN SUSY MODELS

**In supersymmetric extensions of the standard model, there are new sources of**  CP violation. In this section we study the contribution to  $\eta$  due to a CP violating **quark-squark-gluino interaction. This interaction can be parametrized as** 

$$
L_{\tilde{g}\tilde{d}d} = i\sqrt{2}g_{\tilde{e}}\tilde{d}^+\tilde{G}_a \frac{1}{2}\lambda^a (\Gamma_L \frac{1-\gamma_5}{2} + \Gamma_R \frac{1+\gamma_5}{2})d\;, \tag{54}
$$

where  $\tilde{d} = (\tilde{d}_L, \tilde{d}_R)$  are the squarks,  $\tilde{g}$  is the gluino, and the coupling matrices  $\Gamma_{L,R}$ **are each 6x 3 matrices which are related to the squark mass matrix and contain new CP violating phases.** 

**N The above Lagrangian will generate a colour dipole moment of quarks at the one loop level. The down quark colour dipole moment is give by{41]** 

$$
f_d = \frac{\alpha_s}{4\pi m_{\tilde{g}}} Im(\Gamma_L^{id} \Gamma_R^{id}) (3E(z_i) - \frac{8}{3} D(z_i)) . \qquad (55)
$$

Here  $D(z)$  and  $E(z)$  are given by

le:

NÌ.

:ha

 $ar$ 

list

the

**SC** 

 $nc$ 

 $a_{i1}$ 

Ł,

ar.

wl

**co** 

T

 $\mathbf{r}$ 

$$
D(z) = \frac{1}{2(z-1)^2} (1 + z + \frac{2z}{1-z} ln z),
$$
  

$$
E(z) = \frac{1}{(1-z)^2} (1 - z + z ln z),
$$

where  $z = \tilde{m}_i^2 / \tilde{m}_{\tilde{g}}^2$ ,  $\tilde{m}_i$  and  $m_{\tilde{g}}$  are the squark and gluino masses respectively. We find that the colour dipole moment induces a value of  $\eta$  given by  $|\eta| = 0.9 \times 10^7 |f_d|$ .

#### **VII.**  $\eta$  **DUE TO DI-QUARK SCALARS**

**In this section we study contributions to 17 from di-quark scalars. Di-quark scalars are potential sources of large CP violation in neutral flavour conserving processes. A list of possible di-quark scalars which couple to standard model quarks and some of their phenomenological implications can be found in Ref.[42]. There are two di-quark scalars which can induce CP violation at the tree level. These are** *H<sup>9</sup>*  **and** *Hto* **in the notation of Ref.[42]. They transform under the standard model group as (3,1, —2/3)**  and  $(\bar{6}, 1, -2/3)$  respectively. In the following we consider the contribution to  $\eta$  from  $H_9$ . The contribution of  $H_{10}$  can be similarly worked out. The couplings of  $H_9$  to up **and down quarks are** 

$$
L = (\lambda_9 \bar{u}_{Ri}^c d_{Rj} + \lambda_9' \bar{u}_{Li}^c d_{Lj}) \epsilon^{ijk} H_{9k} + H.C. , \qquad (56)
$$

where  $i, j, k$  are colour indices, c indicates charge conjugation, and  $\lambda_9$  and  $\lambda'_9$  are **complex numbers. Exchange of** *H9* **will generate a four quark interaction** 

$$
L_{int} = -\frac{1}{m_H^2} (\lambda_9 \bar{u}_{Ri}^c d_{Rj} + \lambda_9' \bar{u}_{Li}^c d_{Lj})
$$
  
×  $(\lambda_9^* \bar{d}_{Rj'} u_{Ri'}^c + \lambda_9' \bar{d}_{Lj'} u_{Li'}^c) \epsilon^{ijk} \epsilon^{i'j'k}$ . (57)

*l i* 

This effective Lagrangian will induce a CP-odd  $\pi NN$  coupling and then will generate a non-zero *η*. The calculations are similar to that in section V. After a Fierz rearrangement, we obtain the dominant term which contributes to  $\eta$ 

$$
L_{int} = \frac{i}{m_H^2} \frac{2}{3} Im(\lambda_9 \lambda_9') [\bar{u}u \bar{d}\gamma_5 d + \bar{d}d\bar{u}\gamma_5 u] + \dots
$$
 (58)

**From this we compute** 

$$
|\eta| = \frac{1}{\sqrt{2}G_F} g_{\pi NN} \frac{1}{m_H^2} \frac{2}{3}
$$
  
 
$$
\times |Im(\lambda_9 \lambda_9^*) \frac{F_{\pi}}{2m_{\pi} m_d} (m_u < N|\bar{u}u|N > -m_d < N|\bar{d}d|N >)|
$$
  
\n
$$
= \begin{cases} 0.7 \times 10^7 (GeV^2) |Im(\lambda_9 \lambda_9^*)| / m_H^2, & N = p, \\ 1.4 \times 10^7 (GeV^2) |Im(\lambda_9 \lambda_9^*)| / m_H^2, & N = n. \end{cases} \tag{59}
$$

### **VIII. DISCUSSIONS AND CONCLUSIONS**

**In the previous sections, we have calculated CP odd nucleon-nucleon interactions in several models. From these calculations, we can put constraints on CP violating parameters in the different models. It is useful to check whether these constraints are really meaningful when other constraints have been taken into account. For this purpose, let us compare these constraints with the ones obtained from the upper**  bound on the neutron EDM. In the standard model,  $d_n = 10^{-31} \sim 10^{-33}$  [28, 15] and  $\eta = 2 \times 10^{-9} \sim 10^{-10}$  are both very small compared with the experimental upper **bounds. No significant information can be extracted from experiments at this stage. One only hopes that future experimental sensitive will reach the region of theoretical predictions and provide us with useful information. However, should experiments**  measure  $d_n$  or  $D_A$  at the level much larger than the standard model predictions, we **have to go beyond the minimal standard model to explain the results. The studies of other models in the previous sections are a first step in this direction. The upper bound** on the strong CP violating  $\theta$  term of QCD from  $\eta$  is about 10<sup>-8</sup> which is **weaker by one order of magnitude compared with the one obtained from the neutron EDM constraint.** 

**In multi-Higgs doublet models, the constraints for the CP violating parameters from** *DA* **at the present are weaker than the ones from the upper bound on the neutron EDM by two orders of magnitude[34-36,43]. The reason is because that the operators**  which dominate the contributions to  $\eta$  also dominate  $d_n$ . For example, the colour dipole moment  $f_d$  of the down quark contribute to both  $d_n$  and  $\eta$ . The contribution **to**  $d_n$  is given by  $d_n(f_d) = 4ef_d/9[15]$  and to  $\eta$  is given by  $\eta \approx g_{\pi NN}B_{gd}f_d/\sqrt{2G_F m_{\pi}^2}$ . If we require  $d_n(f_d)$  to satisfy the upper bound  $d_n < 10^{-25}$  ecm, we would have  $\eta$ less than 10<sup>-4</sup> which is about two orders of magnitude below the direct experimental constraint on  $\eta$ . Our result for  $\eta$  is about two orders of magnitude smaller than that obtained in Ref.[17]. The main difference is in the evaluation of  $B_{\text{gr}}$ . If we use the **result of Ref.**[17], the constraint on  $f_d$  from  $\eta$  is comparable to the one from the upper **bound on the neutron EDM.** 

 $Q = R$   $R = 12$   $R = 0$ 

In Left-Right symmetric models, the constraint from  $\eta$  on the CP violating parameter ( $|\sin\zeta\cos\zeta\sin(\gamma - \delta_2)| < 4 \times 10^{-5}$ ) is stronger than the existing constraint **from the upper bound on the neutron EDM by one order of magnitude[44,15]. How**ever, the same  $\pi^0 NN$  CP violating vertex will also generate  $d_{\pi}$  at one loop level by exchange  $\pi^0$  and n in the loop[45]. Let photon couple to neutron through its **anomalous magnetic dipole moment, we have** 

$$
d_n = \frac{eg_{\pi NN}f_{\pi NN}}{8\pi^2} \frac{\kappa_n}{2m_n} F_n(m_\pi^2) , \qquad (60)
$$

where  $\kappa_n$  is the anomalous magnetic dipole moment of neutron and  $F_n(m_\pi^2)$  is from **loop integral which can be found in Ref.[15,45]. Requiering** *dn* **to be less than the**  experimental upper bound, we find that the constraint on  $f_{\pi NN}$  is about a factor of **10 better than the one from n.** 

In the SUSY model, the dominant contribution to  $\eta$  is from the colour dipole **moment of the down quark. The constraint is weaker than that obtained from the**  **neutron EDM, as pointed out above. Using our result of CP-odd** *x°NN* **coupling,**   $\eta$  due to CP violation in quark-squark-gluino interaction discussed in section VI, is **predicted to be less than 10 - 4 .** 

In the di-quark scalar model, we obtain  $Im(\lambda_9\lambda_9^{'\bullet})/m_H^2 < 2 \times 10^{-9}GeV^{-2}$  from experimental constraint on  $\eta$ . The situation is similar to that for Left-Right symmetric **models, this constraint is about a factor of 10 less stringent than that obtained from the upper bound on the neutron EDM.** 

**From the above discussion we see that atomic EDM measurements give interesting constraints on CP violating parameters. These constraints should not be underestimated. In some cases these constraints are within an order of magnitude of those obtained from the upper bound on the neutron EDM. With improved accuracy in the measurement, information extracted from atomic EDM will play more important roles. More experiments should be performed.** 

## **ACKNOWLEDGMENTS**

**Part of the work of XGH was done at CERN while he was a fellow of the Theory Division. He thanks Dr. Valencia for some useful discussions. This work is supported in part by Australian Research Council.** 

#### REFERENCES

[1] J.H. Christenson, J. Cronin, V.L. Fitch and R. Turley, Phys. Rev. Lett.13, 138(1964).

Ŷ,

- {2] K.F. Smith et al., Phys. Lett. B234, 234(1990).
- [3] K. Abdullah et al., Phys. Rev. Lett.65, 2347(1990).
- [4] T.G. Void et al., Phys. Rev. Lett.52, 2229(1984).
- [5] S.K. Lamoreaux et al., Phys. Rev. Lett.59, 2275(1987).
- [6] S.A. Murthy, D. Krause, Z.L. Li and L.R. Hunter, Phys. Rev. Lett. 63,965(1989).
- [7] L.I. Schiff, Phys. Rev. **132,** 2194(1963).
- (8) P.G.H. Sandars and R.M. Sternheimer, Phys. Rev. **All,** 473(1975); V.V. Flambaum, Sov. Nucl. Phys.24, 199(1976); W.R. Johnson, D.S. Guo, M. Idrees and J. Sapirstein, Phys. Rev. **A34,**1043(1986).
- [9] LB. Khriplovich, Sov. Phys. JETP 44, 25(1976); P.V. Coveney c.d P.G.H. Sandars, J. Phys. B16, 3727(1983); V. Dzuba, V.V. Flambaum and P. Silvestrov, Phys. Lett. **B154,** 93(1985); V.V. Flambaum and I.B. Khriplovich, Sov. Phys. JETP 62, 872(1985).
- (10] P.G.H. Satdars, Phys. Lett. 14, 194(1965); 22, 290(1966); R.M. Sterheimer, Phys. Rev. **183,** 112(1969); V. Ignatouich, Sov. Phys. JETP 29, 1084(1969); M.A. Player and P.G.H. Sandars, J. Phys. B2, 1620(1970).
- [11] O.P. Sushov, V.V. Flambaum and I.B. Khriplovich, Sov. Phys. JETP 60,

873(1984).

- [12] V.V. Flambaum, LB. Khriplovich and O.P. Sushkov, Phys. Lett. B162, 213(1985).
- [13] V.V. Flambaum, I.B. Khriplovich and O.P. Sushkov, Nucl. Phys. **A449,**  750(1986).

V

- [14] V.V. Flambauni and LB. Khriplovich, Ref.(9); V.A. Dzuba, V.V. Flambaum and P.G. Silvestrov, Ref(9); C. Bouchiat, Phys. Lett. B57, 284(1975); E.A. Hinds, C.E. Loving and G.P.H. Sandars, Phys. Lett. B62, 97(1976); Ann-Marie Martensson-Pendrill, Phys. Rev. Lett. 54, 1153(1985).
- [15] X.-G. He, B. McKellar and S. Pakvasa, Int. J. Mod. Phys. A 4, 5011(1989); Errata A6, 1063(1991); See also S.M. Barr and W. Marciano, in CP violation, Edited by C. Jarlsko, (World Scientific Pub., Singapore, 1989).
- [16] W. Berareuther and M. Suzuki, Rev. Mod. Phys. 63, 313(1991); See also S.M. Barr and W. Marciano, Ref(15).
- [17] V.M. Khatsymovsky, I.B. Khriplovich and A.S. Yelkhovsky, Novosibirsk, Preprint 87-28(1987).
- [18] V. Dzuba, V.V. Flambaum and P. Silvestrov, Ref(9).
- [19] P. Cho, K. Sangster and E.A. Hinds, Phys. Rev. Lett.63, 2559(1989).
- [20] P.V. Coveney and P.G. H. Sandars, Ref(9).
- [21] M. Kobayashi and T. Maskawa, Prog. Thor. Phys. 49, 652(1973).
- [22] Y.-L. Wu, MZ-TH-91-14, May 1991.
- [23] J.F. Donoghue, B. Holstein and M. Musolf, Phys. Lett. B196, 196(1987).
- (24] F. Gilman and M. Wise, Phys. Rev. **D27,** 1128(1983); R.D.C. Miller and B. McKellar, Phys. Rep. **106,** 171(1983).
- (25] G. Buchalla, A.B. Buras and M. Harlander, Nud. Phys. B337, 313(1990); E.A. Paschos and Y.-L. Wu, Mod. Phys. Lett. A6, 93(1991).
- (26] J.F. Donoghue, X.-G. He and S. Pakvasa, Phys. Rev. **D34,** 833(19S6).
- [27] J.F. Donoghue, E. Golowich, W. Ponce and B. Holstein, Phys. Rev. **D21,**  186(1980).
- (28] B. McKellar, S.R Choudhury, X.-G. He and S. Pakvasa, Phys. Lett. B197, 556(1987).
- (29] R. Crewther, P. Divecchia, G. Veneziano and E. Witten, Phys. Lett. **B88,**  123(1979); Errata, **B91,** 487(1980); S. Aoki and T. Hatsuda, Preprint, CERN-TH.5808/90; H.-Y. Cheng, Preprint, IP-ASTP-04-91-REV, Jul. 1991; A. Pich and E. de Rafael, Preprint, CERN-TH.6071/91.
- (30] P. Divecchia, Acta. Phys. Austriaca 22, 341(1980).
- [31] S. Weinberg, Phys. Rev. Lett. 37, 657(1976).
- (32] X.-G. He, Talk at the 14th Int. Warsaw Meeting on Elementary Particle Physics: Puzzles on the Electroweak Scale. Warsaw, Poland, May 27-31, 1991. CERN-Th.6177/91.
- [33] K.-Y. Cheng, Preprint, IP-ASTP-03-91.
- [34] S. Weinberg, Phys. Rev. Lett.63,2333(1989); Phys. Rev. **D42,** 860(1990).
- [35] J. Gunion and D. Wyler, Phys. Lett. B248, 170(1990); D. Chang, W.Y. Keung and T.C. Yuan, Phys. Lett. B251, 608(1990).
- **[36] D.A. Dicus, Phys. Rev. D41, 999(1990).**
- **[37] M. Chemtob, Preprint, SPh-T/91-085.**
- **[38] G. Valencia, Phys. Rev. D41, 1562(1990).**
- **[39] A.A. Anselm, V. Bunakov, V. Gudkov and N. Uraltsev, Phys. Lett. B152, 116(1985); T.P. Cheng and L.F. Li, Phys. Lett. B234, 165(1990).**
- **[40] J. Pati and A. Salam, Phys. Rev. DIO, 275(1974); G. Senjanovic and R. Mohapatra, Phys. Rev. D12,1502(1975).**
- **[41] J.M. Gerard, W. Grimus and A. Raychaudhuri, Phys. Lett. B145, 400(1984).**
- **[42] X.-G. He and A. Davies, Phys. Rev. D43, 225(1991).**
- **[43] S. Barr and A. Zee, Phys. Rev. Lett. 65, 21(1990).**
- **[44] X.-G. He, B. McKellar and S. Pakvasa, Phys. Rev. Lett. 61, 1267(1988).**
- **[45] X.-G. He and B. McKellar, Preprint, UM-P-92/01, OZ-92/01.**

 $\mathbf{I}$ 



**R** 

 $\ddot{\phantom{a}}$ 

Ŕ.

 $\hat{\mathcal{A}}$ 

 $\frac{1}{4}$ 

 $\begin{array}{c} \begin{array}{c} \begin{array}{c} \begin{array}{c} \end{array} \\ \begin{array}{c} \end{array} \end{array} \end{array} \end{array}$ 

 $\left\vert \cdot \right\vert$ 

÷

Fig. 1. Diagrams contributing to  $\eta$  in the standard model.  $B'$  indicates **the intermediate baryon and x indicates a CP violating vertex.** 

**28**