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BUNCHED BEAM STOCHASTIC COOLING*

JIE WEI
Brookhaven National Laboratory,
Upton, New York 11973, USA

Abstract

The scaling laws for bunched-beam stochastic cooling has been derived in terms of the optimum cooling rate and the mixing condition. In the case that particles occupy the entire sinusoidal rf bucket, the optimum cooling rate of the bunched beam is shown to be similar to that predicted from the coasting-beam theory using a beam of the same average density and mixing factor. However, in the case that particles occupy only the center of the bucket, the optimum rate decrease in proportion to the ratio of the bunch area to the bucket area. The cooling efficiency can be significantly improved if the synchrotron side- band spectrum is effectively broadened, e.g. by the transverse tune spread or by using a double rf system.

1 Introduction

Since the invention of the principle in 1968, stochastic cooling has been applied extensively for the accumulation of rare particles and the preservation of particle beam quality. Recently, stochastic cooling for a bunched beam [1]–[3] has been studied to improve the luminosity and to compensate for the beam growth due to diffusion mechanisms in colliders and storage rings.

This paper summarizes the theoretical results of the bunched-beam stochastic cooling. In Sections 2 and 3, expressions of the optimum cooling rate and the mixing condition are derived for transverse and longitudinal cooling. Effects of side-band overlapping and signal suppression are briefly discussed in Section 4.

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2 Transverse Stochastic Cooling

2.1 Optimum Cooling Rate

Transverse stochastic cooling aims at reducing the transverse emittances of the particle beam. Theoretically, the reduction rate of the emittances may be obtained by averaging the [2]-[3] Fokker-Planck equation, which describes the time evolution of the transverse distribution function. For a coasting beam of N particles, the optimum (maximum) cooling rate achievable with a system of average frequency $\langle n\rangle\omega_0$ and bandwidth $\Delta n\omega_0$ is

$$\tau^{-1} = \frac{1}{\langle\epsilon_i\rangle}\frac{d\langle\epsilon_i\rangle}{dt} = \frac{\Delta n\omega_0}{\pi NM} \tag{1}$$

where $\omega_0 = 2\pi f_0$ is the angular revolution frequency, and M is the mixing factor

$$M = \begin{cases} \omega_0/\langle n\rangle\Delta\omega, & \text{if } M > 1; \\ 1, & \text{otherwise.} \end{cases} \tag{2}$$

Here, $\Delta\omega$ is the spread in particle revolution frequency. The average gain G of the system to achieve this cooling rate is

$$G = \frac{2}{NM}. \tag{3}$$

Consider the case that the mixing is not perfect within one revolution ($M > 1$), and that the system thermal noise is negligible. Eq. 1 indicates that the quantity of merit that determines the cooling efficiency is the particle density spectrum $\rho(\omega)$ in frequency, or the Schottky spectrum. Corresponding to each harmonic n of the revolution frequency are Schottky revolution bands of width $2n\Delta f$ whose intensity decrease linearly with n.

When the particles are bunched longitudinally by the rf system, side-band structure appears within each

revolution band due to synchrotron oscillation. Corresponding to the frequency spread $n\Delta\omega$ of the revolution band, the number of significant side-band is $k = n\omega_0\tau = n\Delta\omega/\Omega_0$, where τ is the average amplitude of longitudinal oscillation in time.

Since the distance between the nearby side-bands is the synchrotron-oscillation frequency Ω_0, the local density $\rho(\omega)$ in frequency becomes high if the spread $\Delta\Omega$ in synchrotron frequency is small compared with Ω_0. Consequently, stochastic cooling becomes difficult. Neglecting the side-band overlapping, the optimum cooling rate for a beam bunched by a single rf system can be shown to be

$$\tau^{-1} = \frac{2\langle n\rangle\omega_0}{\pi^{3/2}hN_bM_b}\left(\frac{J}{\hat{J}}\right)^{3/2}, \quad J < \hat{J} \tag{4}$$

where h is the harmonic number of the rf system, N_b is the number of particles in the bunch, and $J/\hat{J}$ is the ratio of the bunch area J to the rf bucket area $\hat{J}$. The mixing factor for the bunched-beam is defined as

$$M_b = \begin{cases} \omega_0/k\Omega_0, & \text{if } M_b > 1; \\ 1, & \text{otherwise.} \end{cases} \tag{5}$$

The average gain G to achieve this cooling rate is

$$G = \frac{4}{\pi^{1/2}hN_bM_b}\frac{\langle n\rangle}{\Delta n}\left(\frac{J}{\hat{J}}\right)^{3/2}. \tag{6}$$

Eq. 4 becomes invalid when the ratio $J/\hat{J}$ is close to 1 (Appendix).

A comparison between Eq. 1 and Eq. 4 shows that when the bunch area J is comparable to the bucket area $\hat{J}$, the cooling rate for the bunched beam is comparable to that of a coasting beam of the same density ($N \sim hN_b$) and mixing factor ($M \sim M_b$). On the other hand, bunched-beam cooling becomes difficult when the bunch area is small compared with the bucket area.

The difference in the Δn and $\langle n\rangle$ dependence of the cooling rates (Eqs. 4 and 1) is due to the fact that for the bunched beam, different revolution bands have the same side-band structure. Instead of being contained in each revolution band as in the case of the coasting beam, the randomness of the particle motion is now contained in each side-band.

2.2 Mixing between Pick-Up and Kicker

The effectiveness of stochastic cooling depends on the fact that during the time the particle travels from the pick-up to the kicker, the longitudinal displacement of the particle is much smaller than the characteristic sampling width of the cooling system. If $\Delta\theta$ is the azimuthal distance between the pick-up and the kicker, this condition is for the bunched beam

$$\langle n\rangle\tau\Omega_0\Delta\theta \ll 1. \tag{7}$$

Cooling with a one-turn delay between the pick-up and kicker becomes possible if Eq. 7 is true for $\Delta\theta = 2\pi$.

2.3 Cooling Power

If N_K is the total number of the kickers, and R_K is the kicker resistance, the average power required for the cooling of N_B bunches is approximately

$$P = \frac{f_0^2G'^2N_BN_b\Delta n\epsilon}{N_KR_K} \tag{8}$$

where ϵ is the average transverse emittance of the beam, and G' is the total gain of the pick-up and the amplifier.

2.4 Effect of transverse tune spread

A finite transverse tune spread effectively produces a smear and an additional overlapping in the side-band structure. In the case that the tune spread $\Delta\nu\omega_0$ is much larger than Ω_0 but, at the same time, much smaller that the width of the revolution band

$$\Omega_0/\omega_0 \ll \Delta\nu \ll \langle n\rangle\tau\Omega_0, \tag{9}$$

the cooling for the bunched beam resembles that for the coasting beam.

3 Longitudinal Stochastic Cooling

3.1 Cooling Efficiency

Longitudinal bunched-beam stochastic cooling reduces the energy spread and bunch length of the beam. In the case that the beam grows longitudinally due to intrabeam Coulomb scattering and other diffusion mechanism, [3] stochastic cooling can reduce the beam growth and particle loss through the rf separatrix.

Theoretically, although the analysis of the longitudinal cooling is complicated by the fact that the mixing factor is dependent on the longitudinal particle distribution, numerical solutions may be obtained of the non-linear Fokker-Planck equation under given boundary and initial conditions. The optimum cooling rate obtained for the bunched beam is similar to that of the transverse cooling.

In reality, however, longitudinal stochastic cooling is made more difficult by the fact that coherent signals at the revolution-frequency harmonics [1]interfere with the longitudinal Schottky spectrum. Besides, unlike the transverse cooling where transverse tune spread often broadens the side-band spectrum, the effeciency of longitudinal cooling for a bunched beam is strongly limited by the side-band splitting.

3.2 Cooling with a Double Rf System

In the case that the bunch area is small compared with the bucket area, a secondary rf system may be introduced to improve the cooling efficiency. Operating at a harmonic of the fundamental rf frequency, this system can significantly broaden the spread in synchrotron frequency for particles of small amplitudes. With the double rf system, the side-band splitting no longer exists. Instead, the "side bands" within each revolution band are all centered at the revolution-frequency harmonic. The Schottky spectrum is thus similar to that of the coasting beam. For comparison, the optimum cooling rate (Eq. 1) under a single rf system is rewritten for $M_b > 1$ as

$$\tau^{-1} = \frac{\langle n\rangle^2 C_W}{\pi^2 h^2 \rho(J)}\frac{J}{\hat{J}}, \quad C_W = \frac{h^2\omega_0^2\eta}{2E\beta^2} \qquad (10)$$

where η is the phase-slippage factor, $\rho(J)$ is the average particle density in J, E is the energy, and βc is the velocity of the particle. Using a secondary system with half the peak voltage and twice the frequency, the optimum cooling rate becomes [5]

$$\tau^{-1} = \frac{\langle n\rangle^2 C_W \pi}{8\mathrm{K}(2^{-1/2})h^2\rho(J)}, \quad J \ll \frac{\hat{J}}{4} \qquad (11)$$

where K is the complete elliptical integral of first kind. Note that the dependence on the relative bunch area $J/\hat{J}$ disappears.

4 Discussion

Qualitative estimates indicate that the conclusions in the previous sections are true even when the effect of signal suppression is considered, although a strict analysis is non trivial. On the other hand, study on signal suppression provides the condition of beam stability under the cooling process. [4], [6]

The computer program [3] recently developed is used to investigate the cooling mechanisms taking into account synchrotron side-band overlapping. In the case of a single rf system, the effect of overlapping is typically small (less than 30%). Furthermore, it has been shown that the overlapping does not change the scaling behavior of the optimum cooling rates.

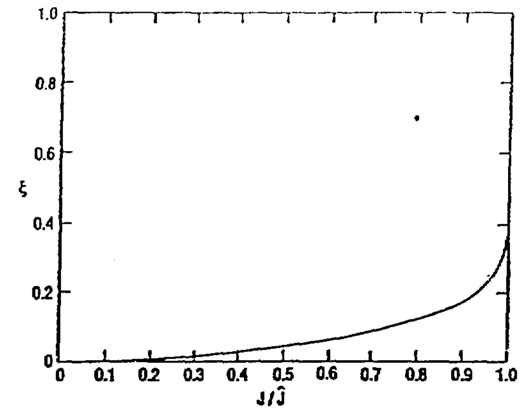

Figure 1: ξ as a function of the relative phase-space area.

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6 Appendix

The validity of Eq. 4 is based on the assumption that longitudinal motion under a sinusoidal rf voltage can be described by the harmonic oscillation of amplitude-dependent frequency. According to Ref. [3], the validity condition is

$$\xi = \exp\left[-\pi \mathrm{K}'(k)/\mathrm{K}(k)\right] \ll 1 \qquad (12)$$

with $\mathrm{K}'(k) = \mathrm{K}(\sqrt{1-k^2})$, where K is the complete elliptical integral, and $k^2 = H/\hat{H}$ is the relative Hamiltonian of the particle. As shown in Figure 1, Eq. 12 no longer holds when $J/\hat{J}$ is close to 1.