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ENERGY PRINCIPLE WITH INDUCED SURFACE CURRENTS

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ABSTRACT

An extended analysis is presented of the earlier investigated induced surface current effects which arise in a magnetized plasma when the confining field has an imposed inhomogeneous part being generated by currents in external conductors. The electromagnetic induction law requires a corresponding contribution from these effects to be added to the formulation of the energy principle for plasma MHD stability. For a plasma with a free boundary, conventional theory therefore holds only in the case of a homogeneous external magnetic field.

When the characteristic length of the imposed field is comparable to or smaller than that of the field generated by the plasma currents, the induced surface current effects on electromagnetic free-boundary modes become at least as important as any other effect due to the plasma forces. This holds both in the case of small (linear) and of large (non-linear) plasma displacements. Any non-uniform displacement which includes a translatory component of the plasma motion across the imposed magnetic field then leads to a restoring partial force and a positive contribution to the change in potential energy.

An illustration is given by two-dimensional straight Extrap geometry with a peaked current distribution, being subject to a combined translational and ballooning-like displacement. In this case there are strong restoring forces, leading to stable oscillations around the plasma equilibrium position. A peaked profile also becomes consistent with an efficient power production of a thermonuclear plasma core.

1. INTRODUCTION

The motion of a plasma across a magnetic field which has a strongly inhomogeneous part due to currents in external conductors is expected to induce strong currents which influence the dynamics of the plasma (LEHNERT, 1974). A detailed analysis of induced surface current effects has recently been presented (LEHNERT, 1991), thereby showing that such effects have to be included in the energy principle earlier formulated by LUNDQUIST (1951) and BERNSTEIN et al. (1958).

The induced surface current effects are important to free-boundary modes, both in the linear case of small plasma displacements and in the non-linear case of large displacements; there is in principle no difference between these two cases with respect to the corresponding electromagnetic induction process.

In plasma equilibrium configurations such as Extrap (LEHNERT, 1989), a high-beta plasma is confined in a magnetic field which consists of one part $\mathbf{B}_{jo}$ generated by the plasma current density, and one part $\mathbf{B}_{co}$ produced by currents in external rigid conductors. An essential primary feature of the Extrap configuration is the inhomogeneity of the field $\mathbf{B}_{co}$ which has two secondary consequences of special importance:

- The formation of a non-circular plasma cross-section and a magnetic separatrix in an equilibrium state.
- The occurrence of induced surface current effects which influence the stability of free-boundary modes in a perturbed state.

A relevant stability theory has to take both these consequences into account. The non-circularity of the plasma cross-section alone can impair the stability of a Z-pinch-like configuration, but such a behaviour is modified by the simultaneous appearance of induced surface current effects.

The present paper extends earlier investigations on these questions within a number of areas:

- The difference between the basic formulations of the energy principle with and without induced surface currents is demonstrated to some detail, in particular by separating the translational part of any non-uniform plasma displacement from its remainder.
- A concrete example on the effect of induced surface currents is given by a two-dimensional high-beta case with a strongly peaked plasma current distribution.

2. BASIC FORMULATION OF THE ENERGY PRINCIPLE

A quasi-neutral plasma is considered, having the electron and ion densities n, the temperature T, the pressure $p = 2knT$ and being confined in a magnetic field $\mathbf{B} = \text{curl } \mathbf{A}$. A static equilibrium is assumed to exist, and small deviations from this state are studied for which every field quantity can be written as $\mathbf{Q} = \mathbf{Q}_0 + \tilde{\mathbf{Q}}$ with $\mathbf{Q}_0$ standing for the equilibrium value, $\tilde{\mathbf{Q}}$ for the corresponding perturbation, and where $|\tilde{\mathbf{Q}}| << |\mathbf{Q}_0|$ The unperturbed magnetic field $\mathbf{B}_0 = \mathbf{B}_{j0} + \mathbf{B}_{c0}$ consists of one part $\mathbf{B}_{j0}$ being due to the unperturbed plasma current density $\mathbf{j}_0 = \text{curl } \mathbf{B}_{j0}/\mu_0$ and one part $\mathbf{B}_{c0}$ being generated by currents in solid conductors situated outside of the plasma body, i.e. curl $\mathbf{B}_{c0} = 0$ inside the plasma and at its boundary. The pressure balance of the equilibrium state is given by

$$\mathbf{j}_0 \times \mathbf{B}_0 = \nabla p_0 \qquad (1)$$

The present analysis is restricted to equilibria where there is no pressure discontinuity at the plasma boundary, i.e. there are no pressure-driven surface currents.

The analysis is further restricted to electromagnetic perturbations (curl $\tilde{\mathbf{E}} \neq 0$) of a "frozen-in" plasma where the electric field $\tilde{\mathbf{E}} = -\partial\tilde{\mathbf{A}}/\partial t$. Electrostatic perturbations (curl $\tilde{\mathbf{E}} = 0$) due to an electric potential $\tilde{\phi}$ and a drift velocity $\mathbf{B}_0 \times \nabla\tilde{\phi} / B_0^2$, such as in the flute-type mode, are excluded. This drift does not contribute to the magnetic field perturbation $\tilde{\mathbf{B}}$.

As in the earlier conventional linearized analysis by LUNDQUIST (1951) and BERNSTEIN et al. (1958) we define the plasma displacement $\tilde{\xi}(\mathbf{r},t)$ of the perturbed state where $\mathbf{r} = (x, y, z.)$, the plasma fluid velocity $\mathbf{v} \equiv \tilde{\mathbf{v}} = \partial\tilde{\xi}/\partial t$, and the perturbation of the magnetic vector potential $\tilde{\mathbf{A}} = \tilde{\xi} \times \mathbf{B}_0$ when choosing a gauge for which $\tilde{\mathbf{E}} = 0$ and $\tilde{\mathbf{A}} = 0$ when $\tilde{\xi} = 0$. Here $\tilde{\xi}(\mathbf{r}_0, t) \cong \tilde{\xi}(\mathbf{r}, t)$ for small displcements where $\mathbf{r}_0$ stands for the initial location of a fluid element. The momentum balance of the plasma volume forces of the conventional theory is then given by

$$n_0 m(\partial^2\tilde{\xi}/\partial t^2) = \tilde{\mathbf{F}}_V = \tilde{\mathbf{j}} \times \mathbf{B}_0 + \mathbf{j}_0 \times \tilde{\mathbf{B}} - \nabla\tilde{p} \qquad (2)$$

where $\tilde{\mathbf{B}} = \text{curl } \tilde{\mathbf{A}}$, $\tilde{\mathbf{j}} = \text{curl}^2\tilde{\mathbf{A}}/\mu_0$ and

$$\tilde{p} = -\gamma p_0 \text{div } \tilde{\xi} - \tilde{\xi} \cdot \nabla p_0 \qquad (3)$$

with γ standing for the ratio between the specific heats. In eq. (2) the forces from electromagnetically induced surface currents at a free plasma boundary have been neglected.

In a recent analysis (LEHNERT, 1991) the effect of induced surface currents due to the inhomogeneity of the external field $\mathbf{B}_{co}$ has been taken into account and is included in a modified formulation of the energy principle. This surface current contribution is determined from the volume force $\mathbf{j} \times \mathbf{B}$ in a boundary layer when approaching the limit of zero layer thickness. The corresponding infinite volume current density $\mathbf{j}$ is then converted into a finite surface current density $\mathbf{K}$. According to this analysis the total change in potential energy becomes

$$\delta W = \delta W_V + \delta W_S \tag{4}$$

where

$$\delta W_V = -\frac{1}{2} \iiint_V \tilde{\boldsymbol{\xi}} \cdot \tilde{\mathbf{F}}_V dV \equiv (1/2\,\mu_0) \iiint_V w_V dV \tag{5}$$

is the contribution due to the work of the force $\tilde{\mathbf{F}}_V$ of eq. (2) within the plasma volume V, and δW_S stands for an additional contribution due to the equivalent work arising from electromagnetically induced surface currents at the plasma boundary. This latter contribution can be cast into various forms as given by

$$\delta W_S = \frac{1}{2} \iint_S \tilde{\mathbf{K}}_B \cdot \tilde{\mathbf{A}} dS = (1/2\mu_0) \iint_S W_S dS = (1/2\mu_0) \iiint_V w_S dV \tag{6}$$

where $\tilde{\mathbf{K}}_B$ is the induced surface current density.

$$W_S = \tilde{\boldsymbol{\xi}} \cdot \hat{\mathbf{n}} \left\{ \mathbf{B}_0 \cdot \left[(\tilde{\boldsymbol{\xi}} \cdot \nabla) \mathbf{B}_{c0} \right] \right\} = (\hat{\mathbf{n}} \cdot \tilde{\boldsymbol{\xi}})(\tilde{\mathbf{B}}_c \cdot \mathbf{B}_0) \tag{7}$$

$$\tilde{\mathbf{B}}_c = - \left[(\tilde{\boldsymbol{\xi}} \cdot \nabla) \mathbf{B}_{c0} \right] \tag{8}$$

$$w_S = \mathrm{div}(\tilde{\mathbf{A}} \times \tilde{\mathbf{B}}_c) \tag{9}$$

and $\hat{\mathbf{n}}$ is the outward directed normal of the closed plasma surface S. Here the transition from the second to the third member of eq. (7) has been performed by means of the vector identity

$$\mathbf{a} \cdot (\mathbf{b} \cdot \nabla)\mathbf{c} = \mathbf{b} \cdot (\mathbf{a} \cdot \nabla)\mathbf{c} + \mathbf{b} \cdot (\mathbf{a} \times \mathrm{curl}\ \mathbf{c}) \tag{10}$$

and the conditions curl $\mathbf{B}_{c0} = 0$ and $\hat{\mathbf{n}} \cdot \mathbf{B}_0 = 0$. The magnetic field perturbation $\tilde{\mathbf{B}}_c$ of eq. (8) arises from the motion of the plasma boundary across the inhomogeneous field $\mathbf{B}_{c0}$. The displacement $\tilde{\boldsymbol{\xi}}$ can be assumed to grow slowly from $\tilde{\boldsymbol{\xi}} = 0$ at time t = 0 to its final value. This should occur on a time scale which is long as compared to the relaxation time of the plasma being associated with MHD signals which traverse the plasma body at the Alfvén velocity. As has been explained earlier in detail (LEHNERT, 1991), the induced surface current effect is then transformed into a volume force effect given by w_S in eq. (9), and by the volume integral in the last member of eq. (6). This further implies that the boundary conditions of the conventional theory, including those of the pressure balance, still apply to the present modified analysis of eq. (4) which can then be handled in the form

$$\delta W = (1/2\,\mu_0)\iiint_V w\,dV \qquad w = w_V + w_S \qquad (11)$$

In some cases the effects of induced surface currents are easier to handle by means of the surface integral of eqs. (6) and (7), but the result is, of course, the same as that obtained from the equivalent volume force formulation of eqs. (6), (9) and (11).

We finally observe that the contribution w_S of eq. (9) vanishes in the case of electrostatic modes, because these modes do not induce any perturbations $\tilde{\mathbf{B}}$ and $\tilde{\mathbf{A}}$.

It has here to be pointed out that the volume force contribution δW_V of eq. (5) is the basis of the theory by BERNSTEIN et al. (1958), and that all reformulations of the integral (5) into parts containing surface integrals and volume integrals in vacuo are due to analytic deductions which contain no additional assumptions or constraints except the imposed necessary boundary conditions. In fact, the original form expressed by eqs. (2) and (5) is also that which represents the real physical forces which act on the plasma fluid elements, due to the local current density, magnetic field and pressure gradient.

The contributions w_V and w_S to the changes in potential energy can become stabilizing or destabilizing, depending on the special equilibrium plasma state and perturbation to be considered. These contributions are identified as follows:

- The first part of w_V originates from the term $\tilde{\mathbf{j}} \times \mathbf{B}_0$ in eq. (2). It is due to the bending of the magnetic field lines within the plasma volume. It contributes to the potential energy, also in regions where $\mathbf{j}_0$ and ∇p_0 become negligible.

- The second part of w_V originates from the term $\mathbf{j}_0 \times \tilde{\mathbf{B}}$ in eq. (2). It is due to displacements of the unperturbed current pattern $\mathbf{j}_0$ across the magnetic field $\mathbf{B}_0$.

- The third part of w_V originates from the term $-\nabla \tilde{p}$ in eq. (2) being produced by a pressure perturbation which arises both for incompressible and compressible displacements. It depends on the unperturbed pressure p_0 and pressure gradient $\nabla p_0 = \mathbf{j}_0 \times \mathbf{B}_0$.

- The contribution w_S from induced surface current effects, as given by eqs. (6) and (7), has two parts being due to an interaction between the current density $\tilde{\mathbf{K}}_B$ on one hand, and the fields $\mathbf{B}_{i0}$ and $\mathbf{B}_{e0}$ from the volume and external conductor currents on the other.

3. GENERAL FEATURES OF THE PRESENT MODIFIED ENERGY PRINCIPLE

In this section some general features of the present modified energy principle will be studied and compared to those of the earlier conventional formulation.

3.1. Reformulation leading to an Equivalent Plasma Volume Force

Starting from eq. (9), the equivalent volume contribution from the induced surface current effect can be written as

$$w_S = \tilde{\mathbf{B}}_c \cdot \mathrm{curl}\,\tilde{\mathbf{A}} - \tilde{\mathbf{A}} \cdot \mathrm{curl}\,\tilde{\mathbf{B}}_c \qquad (12)$$

Using the vector identity (10), this expression is cast into the form

$$w_S = -\tilde{\boldsymbol{\xi}} \cdot \tilde{\mathbf{F}}_S \qquad \tilde{\mathbf{F}}_S = \left[\mathrm{curl}\left[(\tilde{\boldsymbol{\xi}} \cdot \nabla)\mathbf{B}_{c0}\right]\right] \times \mathbf{B}_0 + \left\{\left[\mathrm{curl}\,(\tilde{\boldsymbol{\xi}} \times \mathbf{B}_0)\right] \cdot \nabla\right\} \mathbf{B}_{c0} \qquad (13)$$

This implies that eqs. (2), (9) and (11) can be combined into

$$\delta W = -(1/2\mu_0) \iiint_V \tilde{\boldsymbol{\xi}} \cdot \tilde{\mathbf{F}}\, dV \qquad \tilde{\mathbf{F}} = \tilde{\mathbf{F}}_V + \tilde{\mathbf{F}}_S = n_0 m (\partial^2 \tilde{\boldsymbol{\xi}} / \partial t^2) \qquad (14)$$

where the equivalent force $\tilde{\mathbf{F}}$ now has one volume force contribution $\tilde{\mathbf{F}}_V$ defined by eq. (2), and one equivalent surface force contribution $\tilde{\mathbf{F}}_S$ defined by eq. (13). When using the total force $\tilde{\mathbf{F}}$, this reformulation of the last one of eqs. (14) should make it possible to study eigenmodes also for configurations where there is an inhomogeneous imposed magnetic field $\mathbf{B}_{c0}$ leading to induced surface current effects.

3.2. Rigid Component Separation of the Conventional Plasma Volume Force

Any arbitrary displacement $\tilde{\boldsymbol{\xi}}$ which includes a motion of the centre-of-mass of the plasma body can be written as

$$\tilde{\boldsymbol{\xi}}(\mathbf{r}, t) = \tilde{\boldsymbol{\xi}}_0(t) + \tilde{\boldsymbol{\eta}}(\mathbf{r}, t) \qquad (15)$$

where $\tilde{\boldsymbol{\xi}}_0$ = <u>const</u>. represents a spatially homogeneous (rigid) translation of the plasma body and $\tilde{\boldsymbol{\eta}}$ is a remaining superimposed "fine-structure" of $\tilde{\boldsymbol{\xi}}$ representing various types of non-uniform deformations of the plasma shape.

Introducing the operator

$$\tilde{\mathbf{D}} = \left[(\partial \tilde{\boldsymbol{\eta}}/\partial x) \cdot \nabla,\ (\partial \tilde{\boldsymbol{\eta}}/\partial y) \cdot \nabla,\ (\partial \tilde{\boldsymbol{\eta}}/\partial z) \cdot \nabla\right] \qquad (16)$$

the volume force of eq. (2) can be written on a form where its rigid and non-rigid contributions are separated. i.e.

$$\tilde{\mathbf{F}}_V(\tilde{\xi}) = \tilde{\mathbf{F}}_V(\tilde{\xi}_0 + \tilde{\eta}) = (\tilde{\xi}_0 + \tilde{\eta}) \cdot \nabla\, (\nabla p_0 - \mathbf{j}_0 \times \mathbf{B}_0) + \\ + \operatorname{curl}(\mathbf{B}_0 \cdot \nabla \tilde{\eta} - \mathbf{B}_0 \operatorname{div} \tilde{\eta} - \tilde{\mathbf{D}} \times \mathbf{B}_0) \times \mathbf{B}_0/\mu_0 + \\ + \mathbf{j}_0 \times (\mathbf{B}_0 \cdot \nabla \tilde{\eta} - \mathbf{B}_0 \operatorname{div} \tilde{\eta}) + \nabla(\gamma p_0 \operatorname{div} \tilde{\eta}) + \tilde{\mathbf{D}} p_0 \tag{17}$$

Here the first term of the right-hand member vanishes according to the equilibrium condition (1). i.e.

$$\tilde{\mathbf{F}}_V(\tilde{\xi}_0) = 0; \qquad w_V(\tilde{\xi}_0) = 0 \tag{18}$$

This result is expected. A rigid displacement moves the entire plasma body such as to produce no internal work. i.e. work by stretching the field lines inside the plasma body. by differential motion between the current-carrying plasma layers. and by compression or expansion which produces a work by the pressure force. The only remaining work is then due to the induced surface currents.

The form (15) for the plasma displacement can also be applied to the equivalent force (13) of the induced surface currents. but a detailed deduction will not be presented here. A special example on the separation of the rigid contribution to the induced surface current effects is presented later in Section 4.2.

The results of eqs. (17) and (18) finally imply that the total equivalent force on the plasma can be written as

$$\tilde{\mathbf{F}}(\tilde{\xi}) = \tilde{\mathbf{F}}_V(\tilde{\eta}) + \tilde{\mathbf{F}}_S(\tilde{\xi}_0 + \tilde{\eta}) \tag{19}$$

and the total change in potential energy is determined by

$$w = -\tilde{\xi} \cdot \tilde{\mathbf{F}} = -(\tilde{\xi}_0 + \tilde{\eta})\left[\tilde{\mathbf{F}}_V(\tilde{\eta}) + \tilde{\mathbf{F}}_S(\tilde{\xi}_0 + \tilde{\eta})\right] \tag{20}$$

where

$$-\tilde{\xi}_0 \cdot \tilde{\mathbf{F}}_S(\tilde{\xi}_0) = \left[(\tilde{\xi}_0 \cdot \nabla)\mathbf{B}_{c0}\right]^2 + \left[(\tilde{\xi}_0 \cdot \nabla)\mathbf{B}_{j0}\right] \cdot \left[(\tilde{\xi}_0 \cdot \nabla)\mathbf{B}_{c0}\right] \equiv w_{S0} \tag{21}$$

The integrated contribution to δW_V from $\tilde{\xi}_0 \cdot \tilde{\mathbf{F}}_V(\tilde{\eta})$ vanishes when the volume integral of the force $\tilde{\mathbf{F}}_V(\tilde{\eta})$ has no component in the direction of $\tilde{\xi}_0$.

At this stage attention should be called to a number of special but important features which follow from the present deductions:

- Any displacement which includes a translation $\tilde{\boldsymbol{\xi}}_0$ of the plasma body across an inhomogeneous external magnetic field becomes subject to a restoring partial force and a positive contribution to the potential energy originating from the induced surface current effects. This has earlier been demonstrated in a special case of incompressible kink motion (LEHNERT, 1991) and will be further illustrated in this paper for two-dimensional geometry.

- When the characteristic length $L_{Be} = |\mathbf{B}_{e0}|/|\nabla \mathbf{B}_{e0}|$ of the externally imposed magnetic field is comparable to or smaller than the length $L_{Bi} = |\mathbf{B}_{i0}|/|\nabla \mathbf{B}_{i0}|$ of the field generated by the plasma current density, it is obvious from eqs (17), (19) and (21) that the induced surface current effects become at least as important to free-boundary modes as any other effect due to the plasma forces.

- In the case of low beta values where $\mathbf{B}_{i0}$ is much smaller than $\mathbf{B}_{e0}$, and when the departure $\tilde{\boldsymbol{\eta}}$ from rigid motion $\tilde{\boldsymbol{\xi}}_0$ is small, expressions (17), (20) and (21) yield the result $w \cong w_{S0} > 0$. Then there is a strong stabilizing effect from the induced surface currents for any displacement in a strongly inhomogeneous externally applied magnetic field $\mathbf{B}_{e0}$. For high-beta systems there is a similar tendency, but the situation becomes more complicated and there can also arise local destabilizing contributions from the induced surface current effects, as will be demonstrated by a simple example in Section 4.2.1.

- For strongly peaked current distributions where $\mathbf{j}_0$, p_0 and $\nabla p_0 = \mathbf{j}_0 \times \mathbf{B}_0$ can be neglected within the outer plasma layers, and for non-uniform parts $\tilde{\boldsymbol{\eta}}$ of the displacement $\tilde{\boldsymbol{\xi}}$ being localized to these layers, the contributions from $\mathbf{j}_0$, p_0 and ∇p_0 to the force $\tilde{\mathbf{F}}_V(\tilde{\boldsymbol{\xi}})$ of eq. (17) can be neglected. This implies that the destabilizing volume forces of free-boundary modes which can arise from these contributions are diminished when there is a reduced pressure gradient at the plasma boundary. A stabilizing tendency of this kind, leading to reduced growth rates on account of reduced plasma volume forces, is consistent with earlier results by DALHED and HELLSTEN (1982), HELLSTEN (1982), BRYNOLF (1985) and WAHLBERG (1985, 1988).

3.3. Comparison with the Conventional Energy Principle

The case of rigid plasma motion in a magnetic bottle of closed field line geometry provides a clear demonstration of the difference between the conventional and present formulations of the energy principle.

It is obvious from the law of electromagnetic induction that such a rigid motion across a strong externally applied inhomogeneous field $\mathbf{B}_{e0}$ generates strong induced plasma currents and forces, as expressed by eqs. (20) and (21). In both a low- and a high-beta case this leads to $w = w_{S_0} \neq 0$ and $\delta W \neq 0$, on account of the contribution w_S included in the present theory. In a low-beta case, where the first term of the intermediate member of eq. (21) is dominating, we further have $w_{S_0} > 0$ and $\delta W > 0$. This corresponds to a restoring force and a stabilizing effect which become substantial when there is a strong inhomogeneity of the external magnetic field $\mathbf{B}_{e0}$.

Using instead the conventional energy principle as expressed by $\delta W = \delta W_V$, i.e. where the contribution δW_S of eq. (4) is neglected, one arrives at the result $w = 0$ and $\delta W = 0$, as given by eqs. (14) and (18). This would imply that the externally imposed inhomogeneous field leaves the plasma unaffected, thereby contradicting the electromagnetic induction law. Thus, the conventional energy principle holds only in the case of a homogeneous externally imposed magnetic field.

4. AN EXAMPLE OF TWO-DIMENSIONAL GEOMETRY

To obtain a somewhat better understanding of the behaviour of a high-beta system, we shall now study a special type of two-dimensional geometry. For this purpose a linear Extrap configuration (LEHNERT, 1989) is chosen where a Z-pinch is immersed in a magnetic octupole field generated by currents in external solid conductor rods (Figs. 1a and 1b). The displacements ξ are localized to the xy plane which is perpendicular to the axis of the configuration, and they are uniform along z, such as in the example of Fig. 2. Additional contributions to δW will, of course, arise in the three-dimensional case of sausage and kink disturbances, but these will not be treated here.

The plasma parameters are assumed to be chosen such as to place the plasma boundary close to the magnetic separatrix of Fig. 1b. The non-circular plasma cross-section and an arbitrary current density profile can roughly be simulated by a plasma line current model, not to be used in the present deductions but merely applied in a simple qualitative demonstration. This model consists of one line current J_{p0} at the axis and four line currents J_{p1}, J_{p2}, J_{p3}, J_{p4} in off-axis positions.

4.1. Peaked Current Profiles

In this paper we shall for several later clarified reasons pay special attention to current profiles which are peaked towards the axis. In the main parts of the plasma volume, except in the near-axis region, the magnetic field $\mathbf{B}_{j0}$ can then be treated as if it would be generated only by one line current located at the axis.The off-axis line currents in Fig. 1b are thereby neglected. We introduce the total plasma current $J_p = J_{p0}$, the constant currents J_c in the external rod conductors being antiparallel to J_p, the axial conductor distance a_c, and the axial distance a_s of the x points which determine the magnetic separatrix. With these definitions the octupole field generated by the external conductor currents becomes

$$\mathbf{B}_{c0} = B_{0c}\left[\lambda^3 - 3\rho^2\lambda, - \rho^3 + 3\rho\lambda^2, 0\right] \tag{22}$$

with good approximation when $a_c > 2a_s$. The field generated by the plasma currents is

$$\mathbf{B}_{j0} = B_{0p}\left[-\lambda/(\rho^2+\lambda^2), \ \rho/(\rho^2+\lambda^2), 0\right] \tag{23}$$

where $\rho = x/a_s$, $\lambda = y/a_s$,

$$B_{0c} = (2\mu_0 J_c/\pi a_s)(a_s/a_c)^4 \tag{24}$$

and the near-axis regions are excluded when applying expression (23). The corresponding total magnetic vector potential is $\mathbf{A}_0 = (0, 0, A_0)$ where

$$A_0 = (B_{0c} a_s/4)a_0; \quad a_0 = \rho^4 + \lambda^4 - 6\rho^2\lambda^2 - 2\,ln\,(\rho^2 + \lambda^2) \tag{25}$$

It can easily be demonstrated that the separatrix given by $a_0(\rho, \lambda) = 1$ deviates only by a few percent from a square shape represented by

$$\lambda = 1-\rho \tag{26}$$

This expression will be used as an approximation of the separatrix in the following sections.

In self-consistent deductions of Extrap plasma equilibria by SCHEFFEL (1984) it has further been shown that the deviation of the magnetic surfaces from a circular shape becomes very small at axial distances $(\rho^2+\lambda^2)^{1/2} < 1/2$, i.e. half-way out to the separatrix. Since the current density j_0 is constant along a magnetic surface, this implies that the near-axis distribution of a peaked current density also becomes circular with good approximation. For this reason we can use the ordinary circular Z-pinch as a first approximation when studying the profile of the pressure distribution which results from such a current distribution. Having ρ as radial coordinate of the circular Z-pinch, a current density profile of the form

$$j_0 = j_{00}(1-\rho^{\alpha}) \tag{27}$$

is adopted where j_{00} is the value at the axis $\rho = 0$, and $j_0 = 0$ at the boundary $\rho = 1$ where $p_0(1) = 0$. Integration of eq. (1) yields a pressure profile

$$p_0/p_{00} = 1-\left[(1+\alpha)(2+\alpha)^2\rho^2 - 2(1+\alpha)(4+\alpha)\rho^{2+\alpha} + 2(2+\alpha)\rho^{2+2\alpha}\right] / \alpha^2(3+\alpha) \tag{28}$$

$$p_{00} = \mu_0\alpha^2(3+\alpha)(j_{00}a_s)^2/4(1+\alpha)(2+\alpha)^2 \tag{29}$$

The normalized profiles of j_0 and p_0 are demonstrated in Fig. 3 for a number of decreasing values of α corresponding to an increased peakening of the current profile. The following important features should be observed:

- For values of α decreasing below $\alpha = 1/2$ there are only minor changes in the shape of the pressure profile.

- For decreasing values of α there are outer plasma layers of increasing size within which p_0 and ∇p_0 become quite small.

- The pressure ratio p_0/p_{00} is substantial at $\rho = 1/2$, even for strongly peaked current distributions.

- Increased peaking of the current distribution leads to an enhanced concentration of the plasma current to the near-axis regions, i.e. to decreasing ratios between the equivalent off-axis and axial line currents in Fig. 1b.

4.2. Translational and Ballooning-like Displacements for Peaked Profiles

The deductions of this section make no claim to present a complete stability analysis but should merely be taken as a demonstration of the effects which enter such an analysis in a two-dimensional case, in particular the induced surface current effects. For this purpose we study a translational displacement, with a special superimposed ballooning-like plasma perturbation in the x-direction, i.e. in the spacing between the external conductor rods as shown by Fig 2. The total displacement is

$$\tilde{\xi} = (\tilde{\xi}_0 + \tilde{\eta}, 0, 0,); \quad \tilde{\xi}_0 = \text{const.} \geq 0 \tag{30}$$

where

$$\tilde{\eta} = \begin{cases} \tilde{\eta}_0(\rho - b) \text{ for } \rho \geq b; & \tilde{\eta}_0 = \text{const.} \geq 0 \\ 0 \qquad\qquad \text{for } \rho < b \end{cases} \tag{31}$$

and $0 < b = x_b/a_s < 1$ with x_b = const. This does, of course, not represent an eigenmode of the plasma.

A strongly peaked current profile is further adopted. This implies that j_0, p_0 and ∇p_0 should become negligible within the outer parts of the plasma volume, and among these parts also in the region $x \geq x_b$ of Fig. 1b where $\tilde{\eta} \neq 0$ according to eq. (31). As shown in the last paragraph of Section 3.2., this further implies that the induced surface current effects and the effects due to internal magnetic field deformation within the plasma volume provide the only contributions to the change in potential energy for the adopted two-dimensional perturbation defined by expressions (30) and (31). One of the consequences of this is that the equivalent line current J_{pl} of Fig. 2 and its displacement $\tilde{\xi}_0 + \tilde{\eta}$ can also be neglected.

4.2.1. *The Contributions from Induced Surface Current Effects*

To evaluate the contributions from the induced surface current effects we now use the surface integral of expression (6). Integration of the quantity W_s given by eqs. (7), (8) and the magnetic fields (22) and (23) is performed along the magnetic separatrix being approximated by relation (26). For an axial length L of the configuration of Figs. 1 and 2 the change (6) in potential energy is then determined by

$$(\mu_0/LB_{00}^2)\delta W_S = 3\int_{-1}^{1}(\tilde{\xi}_0+\tilde{\eta})^2 G_S(\rho)d\rho \qquad (32)$$

where $\tilde{\eta}$ is given by eq. (31) and

$$G_S = \rho[(2\rho^2-2\rho+1)^{-2}+(2\rho^2-6\rho+3)/(2\rho^2-2\rho+1)] \equiv G_{Sc}+G_{Sj} \qquad (33)$$

The contributions G_{Sc} and G_{Sj} originate from $\tilde{\mathbf{B}}_c \cdot \mathbf{B}_{c0}$ and $\tilde{\mathbf{B}}_c \cdot \mathbf{B}_{j0}$ in expression (7) and are thus due to the interactions between the induced surface current density $\tilde{\mathbf{K}}_B$ with the external conductor field $\mathbf{B}_{c0}$ and with the field $\mathbf{B}_{j0}$ due to the plasma current density. The contribution from G_{Sc} is always positive, whereas the contribution from G_{Sj} becomes negative in regions near the x-point at $\rho = 1$ in Fig. 1b, i.e. where $\tilde{\mathbf{K}}_B \times \mathbf{B}_{c0}$ and $\tilde{\mathbf{K}}_B \times \mathbf{B}_{j0}$ are antiparallel and point in the negative and positive x directions of Figs. 1a and 1b, respectively.

Introducing the definitions (31) of $\tilde{\eta}$ into eq. (32), the corresponding integral is evaluated by elementary methods. After some lengthy deductions the result becomes

$$(\mu_0/LB_{00}^2)\delta W_S = \tilde{\xi}_0^2(S_{0c}+S_{0j}) + 2\tilde{\xi}_0\tilde{\eta}_0(S_{1c}+S_{1j}) + \tilde{\eta}_0^2(S_{2c}+S_{2j}) \qquad (34)$$

where

$$S_{0c} = 7/5 \qquad S_{0j} = 3(\pi-3) \qquad (35)$$

$$S_{1c}/3 = (4/7)(1-b^7) - (2/3)(2+b)(1-b^6) + (8/5)(1+b)(1-b^5) + $$
$$- (1+2b)(1-b^4) + (1/3)(1+4b)(1-b^3) - (b/2)(1-b^2) \qquad (36)$$

$$S_{1j}/3 = (1/3)(1-b^3) - (1/2)(2+b)(1-b^2) - (1-2b)(1-b) +$$
$$+ (b/2)\, ln\,(1-2b+2b^2)^{-1} + (1-b)[(\pi/4)+\mathrm{arctg}(1-2b)] \qquad (37)$$

$$S_{2c}/3 = (1/2)(1-b^8) - (8/7)(1+b)(1-b^7) + (2/3)(2+4b+b^2)(1-b^6) +$$
$$- (4/5)(1+4b+2b^2)(1-b^5) + (1/4)(1+8b+8b^2)(1-b^4) +$$
$$- (2/3)b(1+2b)(1-b^3) + (b^2/2)(1-b^2) \qquad (38)$$

$$S_{2j}/3 = (1/4)(1-b^4) - (1/3)(1+b)(1-b^3) - (1/2)(1-4b-b^2)(1-b^2) +$$
$$+ 2b(1-b)^2 + (1/4)(1-2b^2)\, ln\,(1-2b+2b^2)^{-1} +$$
$$+ (1/2)(1-4b+2b^2)\,[(\pi/4)+\mathrm{arctg}(1-2b)] \qquad (39)$$

In the particular case of $b = 1/2$ we obtain $S_{oc}+ S_{oj} \cong 1.8?$, $2(S_{1c}+ S_{1j}) \cong -0.0984$, and $S_{2c}+ S_{2j} \cong -0.0207$. When $\tilde{\eta}_0 \le \tilde{\xi}_0$ the total change δW_S in potential energy is clearly positive, due to the contribution $S_{oc}+ S_{oj}$ from the translational part $\tilde{\xi}_0$ of the displacement (30). The superimposed ballooning-like part η results in slightly negative contributions $2(S_{1c}+ S_{1j})$ and $S_{2c}+S_{2j}$ which are due to the fact that the negative parts S_{1j} and S_{2j} originating from the field $\mathbf{B}_{jo}$ near the x-point slightly outbalance the positive parts S_{1c} and S_{2c} originating from the field $\mathbf{B}_{co}$.

4.2.2. *The Contribution from the Internal Magnetic Field Deformation*

We now consider the contribution from the internal magnetic field deformation, here denoted by δW_{VM} and being due to the term $\tilde{\mathbf{j}} \times \mathbf{B}_0$ in eq. (2) and to the second term of the right-hand member of eq. (17). Combining relations (22)-(24), (30) and (31), this contribution becomes determined by

$$(\mu_0/LB_{oo}^2)\delta W_{VM} =$$
$$= 8\tilde{\eta}_0 \int_0^1\!\!\int_0^{1-\rho} \left[\tilde{\xi}_0+ \tilde{\eta}_0(\rho-b)\right]\rho(\rho^2-\lambda^2)\left[3+(\rho^2+\lambda^2)^{-2}\right]\left[3\lambda^2-\rho^2+(\rho^2+\lambda^2)^{-1}\right]d\lambda\, d\rho \qquad (40)$$

where integration is performed over the part of the plasma cross section being to the right of the plane $x = x_b$. As it stands, the integral (40) does not lead to elementary functions. However, when observing that $\rho^2+ \lambda^2$ has a maximum value equal to unity inside the separatrix represented by eq. (26), a minimum value of the contribution δW_{VM} can be obtained, when $b \ge 1/2$ and the integrand of eq. (40) is positive within the corresponding ranges of ρ and λ. This minimum value is given by

$$(\mu_0/LB_{oo}^2)\delta W_{VMmin} = 8\,\tilde{\eta}_0\,(\tilde{\xi}_0 M_0 + \tilde{\eta}_0 M_1) \qquad (41)$$

where

$$M_0 = -(7/15)(1-b^2) + (4/3)(1-b^3) - (71/60)(1-b^4) + \\ + (7/25)(1-b^5) + (4/105)(1-b^7) \quad (42)$$

and

$$M_1 = (4/15)b(1-b^2) - (4/3)[(2/15) + b](1-b^3) + \\ + [1+(7/6)b](1-b^4) - (2/5)(7+2b)(1-b^5) + (2/9)(1-b^6) + \\ + (4/105)b(1-b^7) + (1/30)(1-b^8) \quad (43)$$

In the particular case of $b = 1/2$ we obtain $8M_0 \cong 0.130$ and $8M_1 \cong 0.362$. The stabilizing parts $8\tilde{\xi}_0\tilde{\eta}_0 M_0$ and $8\tilde{\eta}_0^2 M_1$ thus readily outbalance the destabilizing parts $2\tilde{\xi}_0\tilde{\eta}_0(S_{1c}+S_{1j})$ and $\tilde{\eta}_0^2(S_{2c}+S_{2j})$ of the induced surface current effect in Section 4.2.1.

4.2.3. *Stability*

In the present two-dimensional case of strongly peaked current profiles the following results are obtained with respect to stability:

- For pure translation of the plasma body we have $\tilde{\xi}_0 \neq 0$ and $\tilde{\eta}_0 = 0$. Then there is a strong stabilizing (restoring) force represented by $S_{0c}+S_{0j}$ and being due to the induced surface current effects.

- For a ballooning-like displacement without translation we have $\tilde{\xi}_0 = 0$ and $\tilde{\eta}_0 \neq 0$. Then there is in the present case a weak destabilizing force represented by $S_{2c}+S_{2j}$ and being due to the induced surface current effects, plus a strong stabilizing (restoring) force represented by M_1 and being due to the deformation (stretching) of the magnetic field lines.

- For a combined translation and a ballooning-like displacement such as in Fig. 2 both $\tilde{\xi}_0$ and $\tilde{\eta}_0$ differ from zero. Then there is one restoring force which tends to push the centre-of-mass of the plasma body back to its equilibrium position, at the same time as there is another restoring force which tends to press back the bumpy ballooning-like deformation to the original plasma equilibrium shape. If the plasma body is given an impulse of momentum at time $t = 0$ in the positive x direction, it will oscillate back and forth around its equilibrium position in Fig. 1b. Such a behaviour seems to be consistent with that recently computed for a plane Extrap configuration by BENDA and BONDESON (1991).

4.3. Rigid Body Oscillations

The restoring partial force which is associated with the translational part $\tilde{\xi}_0$ of any displacement is thus expected to give rise to macroscopic oscillations of the plasma body around its equilibrium position. For such small-amplitude oscillations, and when we in a first approximation assume the non-rigid part $\tilde{\eta}$ of the displacement to be small as compared to $\tilde{\xi}_0$, the total equivalent force and momentum balane equation reduces to

$$n_0 m(\partial^2 \tilde{\xi}_0 / \partial t^2) \cong \tilde{\mathbf{F}}_S(\tilde{\xi}_0) \tag{44}$$

We now assume the rigid displacement to perform oscillations of the form

$$\tilde{\xi}_0 = \tilde{\xi}_0(t) = \tilde{\xi}_{00} \cdot \exp(i\,\omega\,t) \tag{45}$$

where $\tilde{\xi}_{00}$ is a homogeneous constant amplitude. Eq. (44) is multiplied by $\tilde{\xi}_0(t)$ and integrated over the plasma volume V. After combination with eqs. (13), (6), (32) and (34) the result becomes

$$\omega^2 \cong (2B_{\varphi 0}^2/\mu_0 m N_0)(S_{0c} + S_{0j}) \tag{46}$$

where N_0 is the line density of the configuration in Fig. 1b.

The linear oscillation of the plasma body at the frequency ω is due to the induced surface current effect. It does not result from conventional theory which would lead to $\omega = 0$.

As a numerical illustration we choose a hydrogen plasma with $J_c = 4 \times 10^4$ A, $a_c = 2a_s = 0.06$ m, and $N_0 = 5 \times 10^{18}$ m^{-1}. This yields $\omega \cong 1.24 \times 10^6$ s^{-1}.

In a more detailed and rigorous description of the dynamics of the plasma, the finite time of propagation of MHD signals across the plasma body has to be taken into account, as well as the deviation $\tilde{\eta}$ from a homogeneous displacement $\tilde{\xi}_0$. The induced surface current effects are then expected to generate MHD wave trains propagating from the surface into the interior of the plasma during the oscillations, at the same time as the plasma body becomes subject to non-uniform deformations.

4.4. Reactor Technological Consequences of Peaked Profiles in Extrap

At a first glance a strongly peaked current profile may appear to reduce the relative size of the hot plasma core of a fusion reactor, thereby lowering the thermonuclear efficiency of the system. However, even for an extreme peaking of the current profile given by very small values of α in eq. (27), a considerable plasma pressure $p_0 = 2kn_0T_0$ can still be sustained within a substantial fraction of the plasma volume as shown by Fig. 3.

In the particular case of the deuterium-tritium (DT) reaction at high temperatures T_0 the corresponding reaction rate ρ_{DT} becomes a rather slow function of T_0. For a relatively constant plasma density n_0 within the thermonuclear plasma core, this implies that the rate ρ_{DT} can be kept near its maximum within a comparatively large core volume. A crude numerical illustration can be given here by assuming $\alpha = 1/8$ which corresponds to $p_0(\rho)/p_{00} > 0.4$ for $\rho < 0.4$ according to Fig. 3b and eq. (28). With a core region defined by $\rho \leq 0.4$ we imagine a nearly constant density $n_0 \cong n_{00}$ and a radial temperature profile $T_0(\rho)$ which decreases from the value T_{00} at the axis to $T_0(\rho = 0.4) \cong 0.4\ T_{00}$ at the edge of the core. Then, there is a nearly constant thermonuclear power

$$\pi_{DT} = (1/4)\, Q_{DT}\, n_0^2\, \rho_{DT} \qquad (Q_{DT} \doteq 28.2 \times 10^{-13}\,\mathrm{J}) \tag{47}$$

per unit volume within the core, i.e. $\rho_{DT} \cong 10^{-24}\ \mathrm{m^3/s}$ and $\pi_{DT} \cong 7 \times 10^{-34}\ n_{00}^2\ \mathrm{W/m^3}$ for $\rho \leq 0.4$ when choosing $T_{00} = T_0(\rho = 0) \cong 9 \times 10^8$ K. We shall adopt this value of π_{DT} and neglect the thermonuclear power produced in the region $\rho > 0.4$, thus obtaining a lower limit of the estimated power production.

A question of special interest to Extrap configurations is that of the relative power loss of normally conducting external coils. Here the discussion is limited to pure Extrap geometry and a plasma bounded by the magnetic separatrix. Reference is further made to an earlier reactor technological discussion (LEHNERT, 1989) as follows. We introduce the average minor radius $\bar{a} = a_s(2/\pi)^{1/2}$ and the major radius R of a toroidal configuration with the aspect ratio $f_R = R/\bar{a}$, the total fusion power P_{DT}, the electric power conversion efficiency f_e, the resistivity η_c of the external coil conductors, the fraction g_c of coil area occupied by the conductors, and the dimensionless geometrical factor

$$F_c = (1 - \rho_c^8) / \rho_c^4 f_c \tag{48}$$

where $\rho_c = a_s/a_c$ and $f_0 = S_c^{1/2}/a$ with S_c standing for the cross-sectional area of one external conductor. Under the present conditions the ratio θ_{ce} between the ohmic power loss in the coils and the net electrical power output becomes

$$\theta_{ce} = (5/8\pi)\,(\eta_c/f_e g_c)\,(f_R/Q_{DT}\rho_{DT})^{1/2}\,(kT_0/\mu_0)\left[(1+\alpha)/(3+\alpha)\right]F_c^2\,(\bar{a}\,P_{DT})^{-1/2} \tag{49}$$

Taking a minimum blanket thickness of 0.7 m into account, a value of F_c near its minimum is obtained for $f_c \cong 0.2$ and $F_c \cong 38$. With $\alpha = 0.125$, $f_c = 0.3$, $g_c = 0.8$, $f_R = 4$, and $\eta_c = 2.7 \times 10^{-8}\ \Omega \cdot m$ for aluminium coils at room temperature, we then obtain $\theta_{ce}\ (aP_{DT})^{1/2} \cong 4330\ (W \cdot m)^{-1}$. Putting $\bar{a} = 1$ m and $P_{DT} = 5 \times 10^8$ W finally yields $\theta_{ce} \cong 0.2$.

Provided that a pure Extrap geometry can be used, this example shows that even normally conducting aluminium coils could be used in a compact reactor of moderate total power output, also when there is a strongly peaked current density profile. Needless to say, the question then arises, how to establish such a profile.

5. RIGID BODY MOTION OF ARBITRARY AMPLITUDE

The general features of the present analysis are expected to be the same also for arbitrary perturbation amplitudes ξ, as long as the plasma body does not hit the external conductors or other external surfaces. A simple demonstration is given by rigid arbitrarily large displacements

$$\xi = \mathbf{v}_0 \cdot t \qquad \mathbf{v} = \mathbf{v}_0 = \text{const.} \tag{50}$$

for which the plasma passes its equilibrium state given by eq. (1) at t = 0. The non-linear volume force then becomes

$$\mathbf{F}_V = \mathbf{j} \times \mathbf{B} - \nabla p \qquad \mathbf{F}_V(t = 0) = 0 \tag{51}$$

with the total derivative $d/dt = \partial/\partial t + \mathbf{v}_0 \cdot \nabla$, and since $\mathbf{v}_0 \cdot \nabla$ commutes with $\partial/\partial t$, it can then easily be shown by means of Maxwell's equations that $d\mathbf{B}/dt$ and $d\mathbf{j}/dt$ vanish. A further consequence of the rigid motion (50) is that the pressure profile becomes unaffected for an observer following the motion, i.e. dp/dt and $d(\nabla p) / dt$ also vanish. This implies that the volume force $\mathbf{F}_V$ vanishes for all times t and for any inhomogeneous external field $\mathbf{B}_{c0}$, in analogy with the result (18) of the linearized case.

The interaction between the rigidly moving plasma body and the inhomogeneous field $\mathbf{B}_{c0}$ is then due to the effects of an induced surface current having the density

$$\mathbf{K}_B = (1/\mu_0)\, \hat{\mathbf{n}} \times [\mathbf{B}_{c0}(\mathbf{r}) - \mathbf{B}_{c0}(\mathbf{r} - \mathbf{v}_0 \cdot t)] \tag{52}$$

where $\mathbf{r}$ stands for the position of the displaced plasma boundary and $\hat{\mathbf{n}}$ for its normal.

6. CONCLUSIONS

The conclusions of this paper are as follows:

- Free-boundary electromagnetic modes of a magnetized plasma become subject to induced surface current effects when the magnetic field has an inhomogeneous part generated by currents in external conductors. These effects are important to the dynamics and stability of the plasma, both in the case of linear (small-amplitude) and non-linear (large amplitude) displacements.

- When the characteristic length of the externally imposed magnetic field is comparable to or smaller than the corresponding length of the field from the plasma currents, these surface current effects become at least as important to free-boundary modes as any other effect due to the plasma forces.

- Any non-uniform displacement, which includes a component representing a translation of the plasma body across an externally imposed inhomogeneous magnetic field, will be subject to a corresponding restoring force and to a positive contribution to the change in potential energy, as produced by the induced surface current effects.

- The earlier deduced conventional form of the energy principle does not take induced surface currents into account. It is therefore not reconcilable with the electromagnetic induction law of plasma displacements in an inhomogeneous externally imposed magnetic field.

- For free-boundary modes the momentum balance equation (2) of the conventional theory becomes valid only when the externally imposed magnetic field is homogeneous in space. When the latter field is inhomogeneous, ideal MHD theory has instead to be based on eq. (14) when treating electromagnetic eigenmodes, growth rates and potential energy changes. Electrostatic modes do not generate any induced magnetic field and associated induced surface currents.

- Current density profiles which are peaked towards the plasma core promote stability of free-boundary modes, in the sense that the plasma volume forces due to displacements of the current density pattern and due to the pressure perturbation are suppressed in the outer plasma layers. Such profiles are still able to sustain a high pressure within a substantial part of the inner plasma layers. This also makes a high power production possible within a thermonuclear plasma core.

- In the simple special case of straight two-dimensional Extrap geometry, an example has been given on translational and ballooning-like displacements of a plasma with a strongly peaked current density profile. In this example there are strong restoring forces due to which the plasma should perform stable linear oscillations around its equilibrium position.

Stockholm, February 22, 1992

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FIGURE CAPTIONS

Fig. 1 An example of straight two-dimensional Extrap geometry.

(a) Outline of an octupole field $\mathbf{B}_{c0}$ which is generated by currents J_c in four external conductor rods.

(b) A Z-pinch is immersed in the octupole field, having the same axis of symmetry at x = y = 0 and a current density $\mathbf{j}_0$ being antiparallel to the currents J_c. The current density generates a field $\mathbf{B}_{j0}$ and the resulting total magnetic field $\mathbf{B}_0 = \mathbf{B}_{j0} + \mathbf{B}_{c0}$ has a magnetic separatrix with four x-points. A neutral gas blanket has been left out in this model where the plasma is bounded by a vacuum region.

Fig. 2 A two-dimensional type of plasma displacement $\tilde{\xi} = \tilde{\xi}_0 + \tilde{\eta}$ in the x direction between the conductor rods. It consists of a translatory part $\tilde{\xi}_0$ = <u>const.</u> and a superimposed ballooning-like non-uniform part $\tilde{\eta}$ within the region $a_s - x_b \leq x \leq a_s$ of the plasma body in Fig. 1b.

Fig. 3 Normalized profiles of a Z-pinch with circular cross section.

(a) Current density profiles for different values of the parameter α which determines the degree of peaking.

(b) Corresponding profiles of the plasma pressure.

Fig. 1a

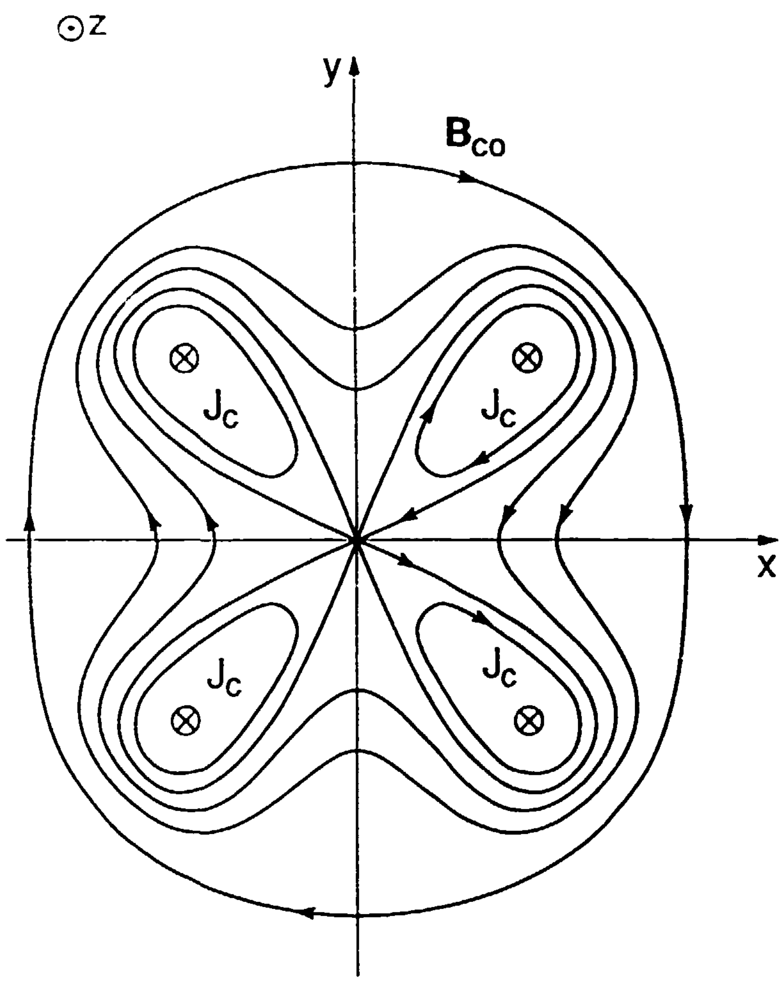
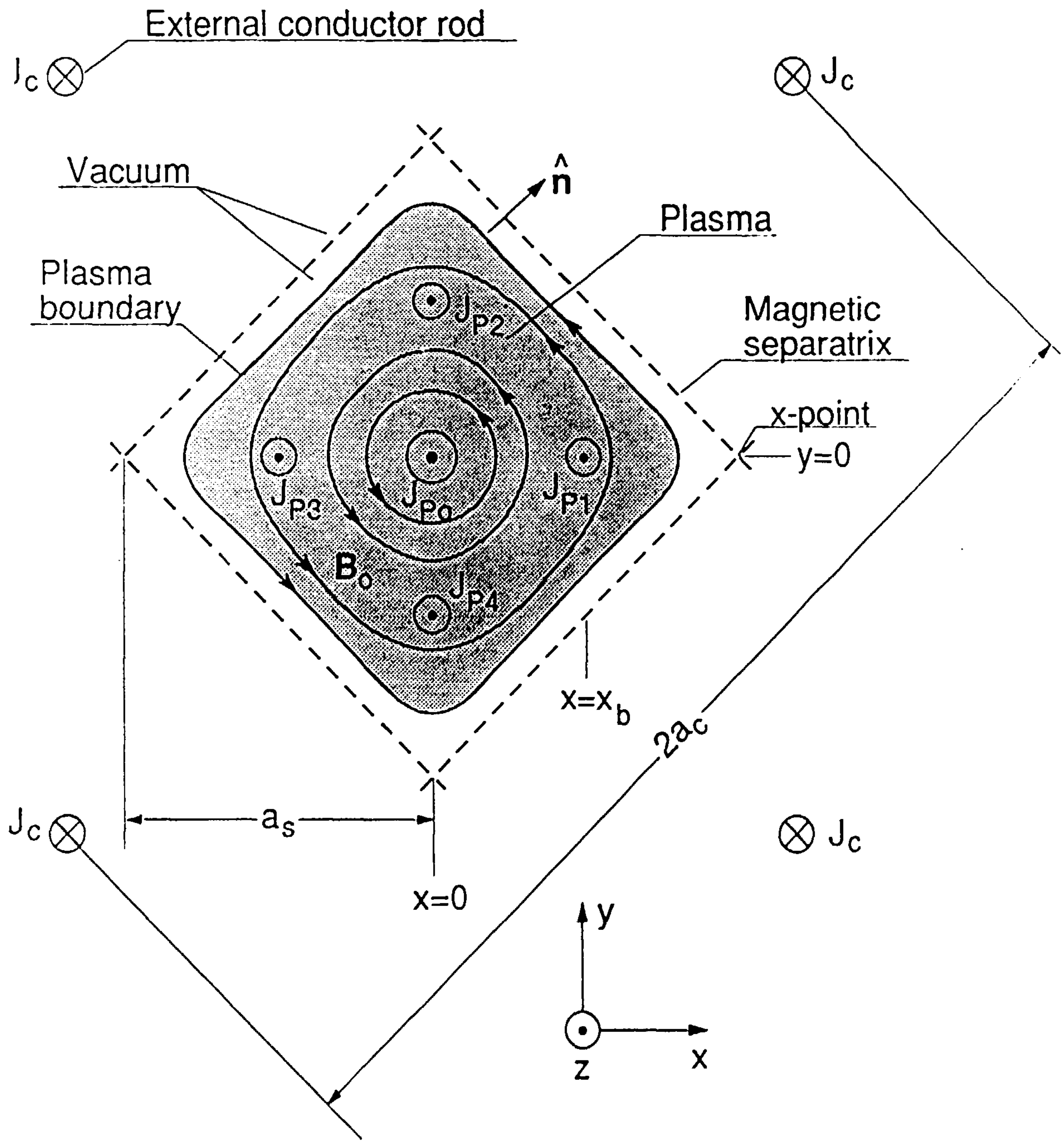
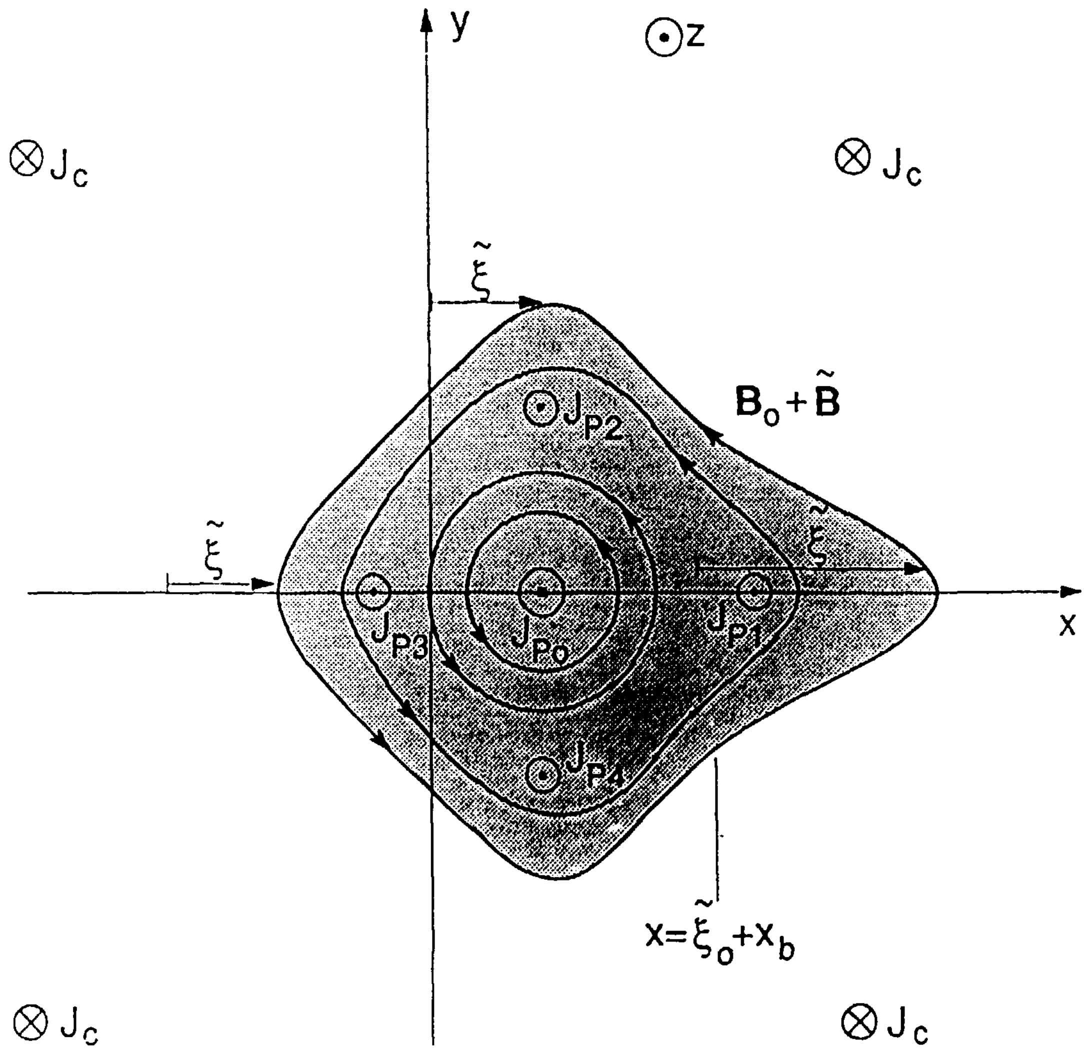


Fig. 1 b



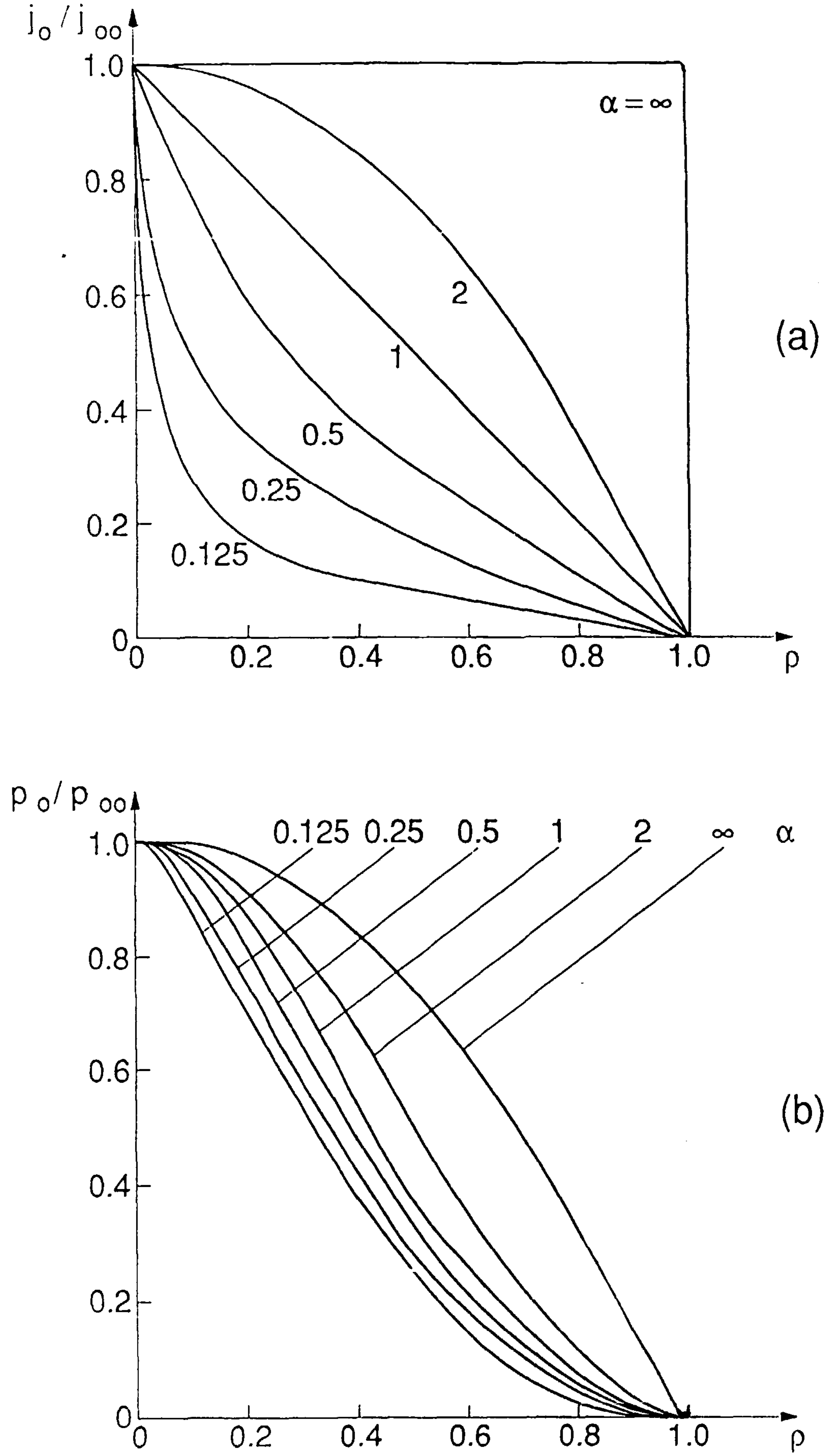
External conductor rod
Jc
Jc
Vacuum
n
Plasma
Plasma
boundary
JP2
Magnetic
separatrix
x-point
y=0
JP3
JPo
JP1
Bo
JP4
x=xb
2ac
Jc
as
Jc
x=0
y
z
x

Fig. 2



y
⊙z
⊗ J_c
⊗ J_c
$\tilde{\xi}$
J_{P2}
$B_o+\tilde{B}$
$\tilde{\xi}$
$\tilde{\xi}$
J_{P3}
J_{Po}
J_{P1}
x
J_{P4}
$x=\tilde{\xi}_o+x_b$
⊗ J_c
⊗ J_c

Fig 3



j o / j oo
1.0
0.8
0.6
0.4
0.2
0
α = ∞
2
1
0.5
0.25
0.125
(a)
0
0.2
0.4
0.6
0.8
1.0
ρ
p o / p oo
0.125
0.25
0.5
1
2
∞
α
(b)

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ENERGY PRINCIPLE WITH INDUCED SURFACE CURRENTS

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Abstract

An extended analysis is presented of the earlier investigated induced surface current effects which arise in a magnetized plasma when the confining field has an imposed inhomogeneous part being generated by currents in external conductors. The electromagnetic induction law requires a corresponding contribution from these effects to be added to the formulation of the energy principle for plasma MHD stability. For a plasma with a free boundary, conventional theory therefore holds only in the case of a homogeneous external magnetic field.

When the characteristic length of the imposed field is comparable to or smaller than that of the field generated by the plasma currents, the induced surface current effects on electromagnetic free-boundary modes become at least as important as any other effect due to the plasma forces. This holds both in the case of small (linear) and of large (non-linear) plasma displacements. Any non-uniform displacement which includes a translatory component of the plasma motion across the imposed magnetic field then leads to a restoring partial force and a positive contribution to the change in potential energy.

An illustration is given by two-dimensional straight Extrap geometry with a peaked current distribution, being subject to a combined translational and ballooning-like displacement. In this case there are strong restoring forces, leading to stable oscillations around the plasma equilibrium position. A peaked profile also becomes consistent with an efficient power production of a thermonuclear plasma core.

Key words: Plasma MHD stability, energy principle, induced surface currents, Extrap.