THE ALPHA-PARTICLE STRUCTURE OF ^{44}Ti

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Abstract

Some of the bound and unbound states of ^{44}Ti have a pronounced alpha-particle structure, and their energies and widths may be obtained from an alpha-^{40}Ca potential. The differential cross-sections for the elastic scattering of alpha particles by ^{40}Ca may also be described by such a potential, and some features indicate the presence of unbound states of ^{44}Ti. The attempts to unify these bound and scattering phenomena by the same potential are described, together with some new calculations using a cosh potential.

1. Introduction

There is much evidence that nucleons in the nucleus can form transient substructures that persist long enough to allow them to be described by the single-particle model. Of these substructures the alpha-particle is the most likely because of its high binding energy. We thus expect to find nuclear states that can be described as an alpha-particle moving in the potential due to the remaining nucleons.

This model has a number of observable consequences both for nuclear structure and for nuclear reactions, corresponding to the alpha-particle being in bound or in scattering states. If the alpha-particle is bound, it can be in a series of states corresponding to excited states of the nucleus. These states may be identified by their abnormally high population in alpha-transfer reactions and by enhanced gamma transitions between them. If the alpha-particle is unbound, the scattering may show effects that can be associated with the unbound states of the composite system.

In the case of nucleons, both the bound and unbound states can be described by the same energy-dependent one-body potential, with real and imaginary parts varying smoothly with energy (Hodgson, 1987). The purpose of the present work is to see whether the alpha particle data at negative and positive energies can be unified in the same way.

There are several ways in which the alpha particle analysis differs from the nucleon analysis. In addition to the effects due to the transient nature of the alpha-particle substructures, the most notable difference is that for some nuclei the imaginary part of the alpha potential is abnormally low, so that the unbound states in the continuum have observable effects on the differential elastic scattering cross-sections, in particular the anomalously high cross-sections in the backward direction.

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The nucleus ^{44}Ti is particularly suitable for analysis because it is the compound system formed when alpha-particles are elastically scattered by ^{40}Ca. The closed-shell nature of ^{40}Ca ensures that the imaginary part of the optical potential is small, and this gives rise to the anomalous large angle scattering (ALAS) that has been extensively studied. In such cases it is possible to extract information on the unbound states of ^{44}Ti from the elastic scattering data. The states of ^{44}Ti are known from gamma ray spectra and studies of alpha-transfer reactions. Additional information that can be analysed with the alpha-cluster model are the B(E2) transition rates between members of the ground state band (see Table 2.1) and the fusion cross-section as a function of incident energy (see Fig.3.2). The available experimental data on the bound and unbound states of ^{44}Ti are summarised in Fig.1.1.

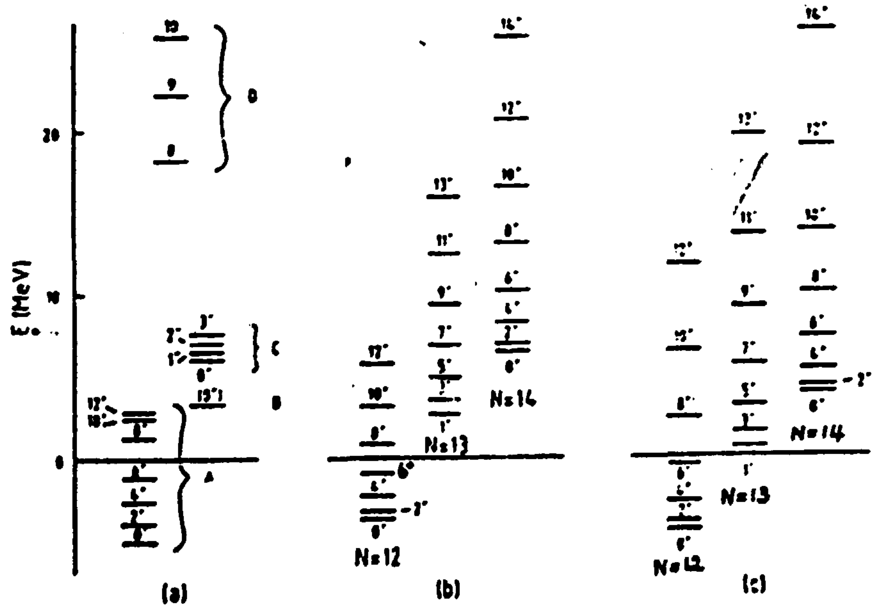

Fig.1.1.

(a) Experimental data on the possible bound and unbound alpha-cluster states of ^{44}Ti, relative to the alpha-particle - ^{40}Ca threshold at 5.127 MeV. The states are labelled to indicate the experimental methods used to detect them: (A) The ground state band (Simpson *et al*, 1975). (B) A state at 8.54 MeV relative to the ground state strongly populated in the ^{40}Ca(^{6}Li,d) reaction (Strohbusch *et al*, 1974). (C) A group of four mixed-parity states at 11.2, 11.7, 12.2 and 12.8 MeV found from (α,α) scattering (Frekers *et al*, 1976, 1983). (D) The lower members of a band of nine states found from (α,α) scattering at higher energies (Stock *et al*, 1972; Löhner *et al*, 1978).

(b) States of ^{44}Ti calculated from a phenomenological potential (Michel *et al*, 1983).

(c) States of ^{44}Ti (Present work).

The aim of this work is to unify these data by a unique alpha-particle optical potential. This should provide further information about the alpha-nucleus interaction, and may provide incentives for experimental work. For example, the model may predict states that have not been observed, perhaps because of their small width, and it would then be important to look for them.

In Section 2 we summarise the analyses of the bound and unbound states of ^{44}Ti, and in the following section the scattering of alpha-particles by ^{40}Ca.

2. The Bound and Unbound States of ^{44}Ti

The cluster model assumes that the alpha-particle is moving in a one-body potential with principal quantum number N and orbital angular momentum quantum number L. These are related to the quantum numbers n, ℓ of the constituent nucleons by the Talmi-Moshinsky relation

$$2N + L = \sum_{i=1}^{4}(2n_i + \ell_i) \tag{2.1}$$

Values of N less than 12 are excluded by the Pauli Principle.

Many analyses of bound (Buck *et al*, 1975) and scattering (Delbar *et al*, 1978) data have shown that the energies and widths of the compound states are rather sensitive to the form of the potential, and in particular that the Saxon-Woods form is inadequate. Better results have been obtained with the squared Saxon-Woods form (SW2), the cosh form and various folded potentials.

When the form of the potential has been chosen, the parameters are varied to optimise the fit to selected data. In the case of the SW2 and cosh potentials, the radius and surface diffuseness parameters are usually fixed to values similar to those of the corresponding folded potential, and then the potential depth V is adjusted to fit chosen states. This criterion is insufficient to fix the depth, because the principal quantum number is not known, so that a chosen state of known L can be fitted with several different values of V and corresponding values of N. It is thus sometimes necessary to try two or more values of V and to assess their acceptability by the goodness of fit of the corresponding potentials to a wide range of data.

The widths and energies of the bound states can be calculated from a purely real potential, but for the unbound scattering states this must be supplemented by an imaginary potential that is determined from the scattering data by the usual parameter optimisation procedure. This imaginary potential can have the Saxon-Woods, Saxon-Woods derivative or the cosh form

$$V(r) = V_c + \frac{V\{1 + ch(R/a)\}}{ch(r/a) + ch(R/a)} \tag{2.2}$$

This form was found by Buck *et al* (1975) to be better than the Saxon-Woods form for the analysis of alpha-cluster states in ^{16}O and ^{20}N, probably because it more nearly approximates the folding model potential.

Calculations of the energies and widths of states in ^{44}Ti have been made using this potential by Pál and Lovas (1980) and the results are shown in Fig.2.1 and Table 2.1. The radius R of the potential was chosen to lie in the range $r_0(A_{core}^{1/3} + A_\alpha^{1/3}) \leq R \leq r_0' A^{1/3}$, where r_0 and r_0' were found from the values of R giving the best fit in the ^{16}O region (Buck and Pilt, 1977). The values of V and a were then chosen to give the best overall fit to the low-lying alpha cluster states. These parameter values are given in Table 2.2. They also used a folded potential with a delta nucleon-nucleon interaction with the alpha and ^{40}Ca densities of Vary and Dover (1973) and of Sick (1979), and the results are also shown in Fig.2.1 and Table 2.1.

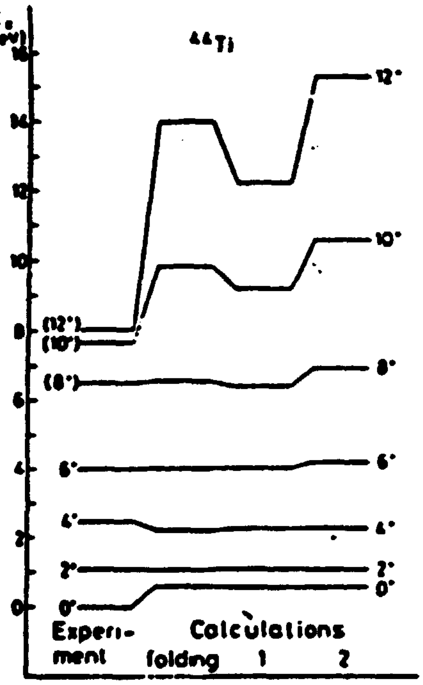

Fig.2.1. Experimental and calculated states of the $2N + L = 12$ alpha-cluster states in ^{44}Ti (Pál and Lovas, 1980).

Table 2.1. B(E2) values for transitions in ^{44}Ti in Weisskopf units.

Transition	Experiment	Folded	1	2
$2^+ \to 0^+$	13 ± 4 (1)	9.5	5.5	21.7
$4^+ \to 2^+$	30 ± 6 (1)	12.6	7.1	29.5
$6^+ \to 4^+$	17 ± 3 (2)	11.9	6.4	29.6
$8^+ \to 6^+$	> 1.5 (3)	9.8	4.8	26.5
$10^+ \to 8^+$	15 ± 3 (3)	7.6	3.0	20.7
$12^+ \to 10^+$	< 6.3 (4)	3.5	1.4	12.3

References: (1) Dixon *et al*, 1973; (2) Simpson *et al*, (1973); (3) Simpson *et al*, (1975); Kolata *et al*, (1974).

Table 2.2. Potential Parameters used by Pál and Lovas (1980).

Potential	V (MeV)	R (fm)	a (fm)
1	241.9	3	0.68
2	161.9	3	1.4

Further calculations of the structure of ^{44}Ti were made by Michel *et al* (1986a), and their results are compared with the experimental data in Fig.1.1. They used a potential found by Delbar *et al* (1978) to fit a wide range of ^{40}Ca(α,α) elastic scattering data (see Section 3). The depth of the potential was then adjusted to give the correct energy of each state relative to the $\alpha + {}^{40}$Ca threshold and the results for the B(E2) values and $< R^2 >^{1/2}$ for the $N = 12$ and $N = 13$ bands are shown in Table 2.3. It is notable that the B(E2) values for the $N = 12$ band are in good accord with the data.

The intercluster RMS radii, also given in Table 2.3, decrease from $R = 4.80$ fm

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Table 2.3.

Theoretical and experimental B(E2) values for the $J \to J-2$ transitions (in W.U.), and intercluster rms radii for the ^{44}Ti $N = 12$ and $N = 13$ states. The local potential depth given for the $N = 12$ states is that reproducing the experimental energies with respect to the $\alpha + {}^{40}$Ca threshold: for the $N = 13$ states U_0 is fixed to the average value $U_0 = 180$ MeV (Michel *et al.* 1986a).

	$N = 12$					$N = 13$	
	U_0	$B(E2)$		$< R^2 >^{1/2}$		$B(E2)$	$< R^2 >^{1/2}$
J^π	(MeV)	Theor.	Expt.	(fm)	J^π	Theor.	(fm)
0^+	184.1	-	-	4.50	1^-	-	5.35
2^+	182.5	11.6	13 ± 4	4.51	3^-	28.7	5.28
4^+	181.2	15.9	30 ± 6	4.47	5^-	30.2	5.15
6^+	180.7	15.2	17 ± 3	4.38	7^-	26.4	4.95
8^+	179.0	12.8	> 1.5	4.27	9^-	20.0	4.71
10^+	181.8	8.1	15 ± 3	4.06	11^-	12.8	4.43
12^+	186.8	3.6	< 6.3	3.83	13^-	5.9	4.14

for the ground state to $R = 3.83$ fm for the 12^+ state, showing the antistretching effect. These radii may be compared with the sum of the alpha-particle and ^{40}Ca radii, which is 5.16 fm. Since the validity of the clustering model requires rather weak overlap between the alpha-particle and the ^{40}Ca nucleus, this indicates that it is probably valid for the low-lying states but becomes progressively weaker for the higher states. The intercluster radii are much larger for the $N = 13$ band, indicating that it has a strong cluster character. These states have not been observed possibly because of their small widths, which are calculated to be less than 1 keV for the lowest lying states, and only 280 keV even for the 13^- state which terminates the band.

Horiuchi (1985) has argued that the alpha-cluster model should not be applied to the ground state band because it always puts the 1^- state at a lower energy (around 5 MeV) than the 0^+ state at 8.54 MeV. Instead, he suggested that the model should be applied to the 0^+ (8.54 MeV) and 1^- (11.7 MeV) states, but not to the ground state band. Ohkubo (1988) has investigated this using a folding model potential chosen so that the first Pauli-allowed $N = 12$, 0^+ state corresponds to the 0^+ state at 8.54 MeV. He then calculated the fusion excitation function, which agreed quite well with the data, and also the elastic scattering cross-sections for a range of energies from 18 to 100 MeV. These showed agree with the data at lower energies but failed badly at higher energies, indicating that this potential is unacceptable.

Another possibility is that, as in the proposal of Horiuchi, the analysis should omit the ground state band, and should be applied to the excited mixed parity cluster states in the group C in Fig.1.1. (Tohsaki-Suzuki and Naito, 1977, Friedrich and Langanke, 1975; Langanke, 1982; Wintgen *et al*, 1983). Ohkubo (1980) also investigated this possibility using a folded potential fitted to the 0^+ state at 11.2 MeV, and found very similar results to those just mentioned, namely that it fails to fit the higher energy scattering data.

The essential physical defect in both these potentials is that the rainbow scattering begins at too low an energy. This could perhaps be obtained by allowing the potential to become deeper as the energy increases, but this is physically unacceptable. It is not possible to improve the fit by adjusting the imaginary potential.

3. Elastic Scattering and Fusion Cross-Sections

The sequence of bound alpha-cluster states discussed in the previous section extends to positive energies and affects the corresponding elastic scattering cross-sections. As the energy increases inelastic scattering and other reactions become possible, and these also affect the elastic scattering. The influence of the non-elastic processes on the elastic scattering can be described globally by allowing the potential to become complex, and this in turn broadens the states in the continuum. Optical model analyses usually require imaginary potentials of several MeV, and the states in the continuum are broadened by a corresponding amount. Furthermore, many partial waves contribute to the cross-section. As a result, the structure that might be expected in the differential cross-sections due to the states in the continuum is usually so smeared out as to be unobservable.

There are however some exceptions to this. If the target is a doubly magic nucleus there are few low-energy excited states, and so inelastic scattering is less likely. Reactions are also inhibited if the projectile is also magic. Thus the elastic scattering of alpha-particles by magic nuclei provides very favourable circumstances for the observation of structure in the elastic scattering due to states in the continuum. Indeed such structure has been known for many years in the differential cross-section for the elastic scattering of alpha-particles by ^{40}Ca; the effects are particularly notable at large angles and are referred to as the back-angle anomaly, or anomalous large-angle scattering (ALAS). The anomaly consists in back-angle cross-sections that are one or two orders of magnitude greater than those found in scattering from neighbouring nuclei. The 'anomaly' is now well understood; in neighbouring nuclei there are very many more reaction channels open and so the imaginary potential is much larger and the structure is washed out.

There have been many optical model analyses of the differential cross-section for the elastic scattering of alpha-particles by ^{40}Ca, a particularly comprehensive one being that of Delbar *et al* (1978). They analysed data from 29–100 MeV and obtained excellent fits using a Saxon-Woods squared potential with parameters

$$\begin{aligned} U &= 198.6(1 - 0.00168E), \quad r_v = 1.37, \quad a_v = 1.39 \\ W &= 2.99 + 0.288E, \quad r_W = 1.73, \quad a_W = 1.00. \end{aligned} \qquad (3.1)$$

Some of their calculations are compared with experimental data in Fig 3.1.

The fusion excitation function for alpha-particles on ^{40}Ca has an oscillatory structure from 10–27 MeV (Eberhard *et al*, 1979), and this has been analysed by Michel *et al* (1986b) using a modified form of the potential of Delbar *et al*. It is notable from Fig.3.1 that the fits at lower energies are not so good as those at higher energies, and so Michel *et al* modified the potential of Delbar *et al* by applying a cut-off factor to the linear energy dependence of the imaginary depth: this gives a good overall fit to the excitation functions for elastic scattering at several angles from 12 to 28 MeV found by Robinson *et al* (1968). As shown in Fig.3.2a, the corresponding reaction cross-section shows little if any structure, and is larger than the measured fusion excitation function. The difference is attributed to direct reactions, and following the work of Hategai *et al* (1982) these were taken into account by reducing the radius of the imaginary potential from 6 to 4 fm. The resulting fusion cross-section shows the same structure as the experimental data. A decomposition of the fusion cross-section into the contributions of individual partial waves (Fig.3.2b) shows that it is the even partial waves of the $N = 14$ band that are responsible for the structure; the odd partial waves give a fusion cross section that shows almost no structure. Subsequently, Ohkubo and Brink (1987) showed that the oscillatory structure is due to interference between the waves reflected by the internal and external potential barriers.

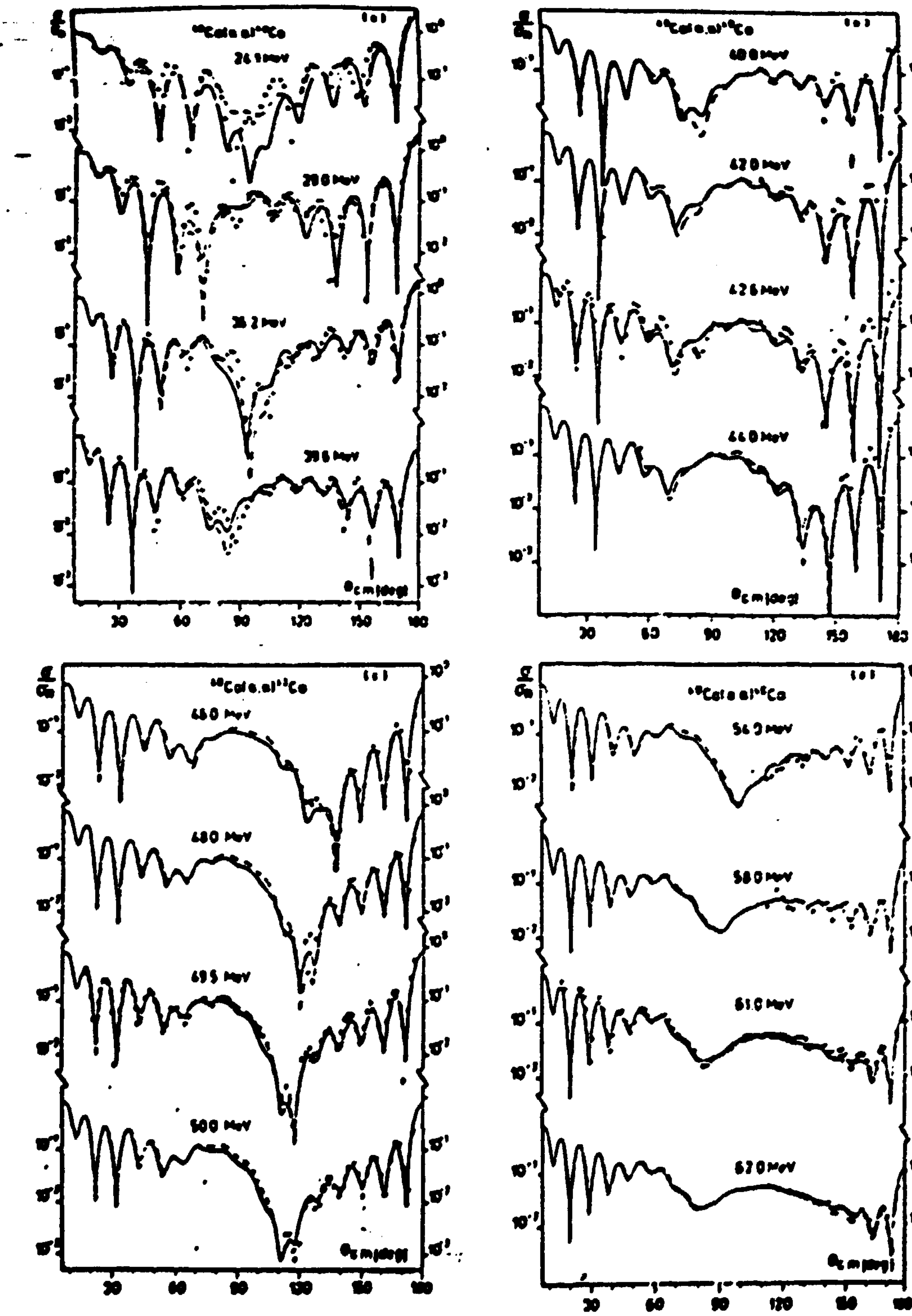

Fig.3.1. Differential cross-section for the elastic scattering of alpha-particles by ^{40}Ca compared with optical model calculations. The full curve shows the results of a calculation with the potential (3.1) and the dashed curve the result of a global search with a more flexible parametrisation (Delbar *et al*, 1978).

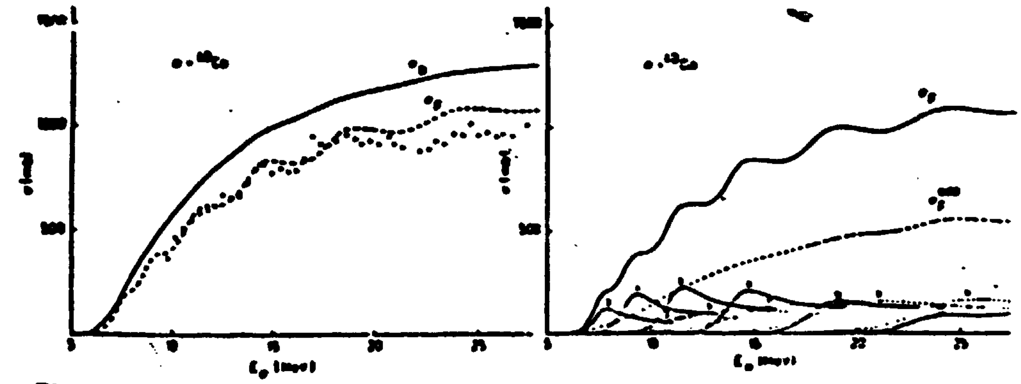

Fig.3.2.

(a) Experimental fusion cross-sections for alpha-particles on ^{40}Ca (Eberhard *et al.* 1979) compared with the reaction cross-section (full line) and fusion cross-section (dashed line) calculated from modifications of the potential of Delbar *et al* (1978) (Michel *et al*, 1986b).

(b) Calculated fusion cross-sections for alpha-particles on ^{40}Ca showing the contributions of odd and even partial waves to the observed structure (Michel *et al*, 1986b).

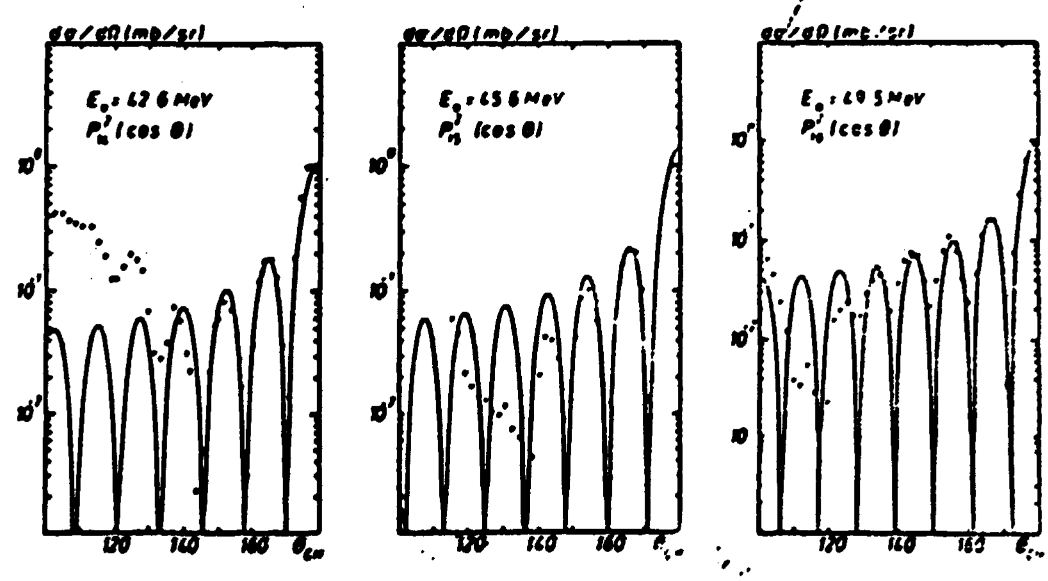

Fig.3.3. Differential cross-sections for the elastic scattering of alpha-particles by ^{40}Ca in the backward direction fitted by Legendre polynomials (Löhner *et al*, 1978).

The differential cross-sections in the backward direction were analysed by Löhner *et al* (1978) by fitting the best Legendre polynomials, as shown in Fig.3.3. The energies of these states is plotted as a function of $L(L+1)$ in Fig.3.4, and the linearity suggests a rotational band, which is supported by the extrapolation to the 1^- state at 11.7 MeV. Ohkubo (1988) however says that it is difficult to assign a simple $\alpha + {}^{40}Ca$ cluster structure to this state because its small width (40 KeV) is incompatible with its position with respect to the $L = 1$ barrier. A calculation of its width using a potential adjusted to give the correct energy for the state gives about 1 MeV (Michel *et al*, 1986a). Ohkubo (1987) has shown how the observed structure is attributable to the combined effects of the broad resonant high spin states.

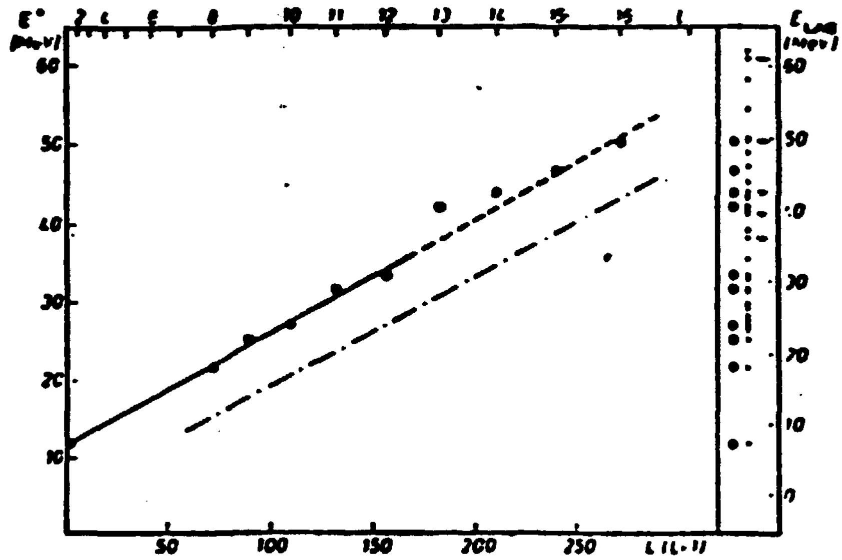

Fig.3.4. L-values extracted from a Legendre polynomial analysis of the elastic scattering of alpha-particles by ^{40}Ca as a function of energy. The dot-dash line shows the grazing L-value calculated by $L_{gr} = kR$ ($R = 1.2(A^{1/3}(\alpha) + A^{1/3}(^{40}Ca))$ (Löhner *et al.* 1978).

We have made calculations of the bound and unbound states of ^{44}Ti, and the results are compared with the experimental data in Fig.1.1. We used a potential of the cosh form (2.1) and adjusted its parameters to optimise the fit to the data. We also calculated the widths of the states and these are given in Table 3.1.

Table 3.1.

Theoretical energies and widths for the $N = 12$ and $N = 13$ bands of ^{44}Ti. A "cosh" geometry is used for the local, real potential with parameters $V_0 = 174.9$ MeV $R = 2.9$ fm and $a = 1.3$ fm. The energies are given with respect to the α-^{40}Ca breakup threshold in ^{44}Ti.

	$N = 12$			$N = 13$	
J^π	E (MeV)	Γ (MeV)	J^π	E (MeV)	Γ (MeV)
0^+	-4.61	-	1^-	0.54	$< 10^{-6}$
2^+	-4.05	-	3^-	1.47	$< 10^{-6}$
4^+	-2.71	-	5^-	3.17	$< 10^{-6}$
6^+	-0.57	-	7^-	5.69	10^{-3}
8^+	2.47	$< 10^{-6}$	9^-	9.14	0.026
10^+	6.55	2×10^{-6}	11^-	13.72	0.14
12^+	11.89	2×10^{-4}	13^-	19.83	0.28

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