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THE GENERAL GRAVITATIONAL POTENTIAL FIELD OF A HOMOGENEOUS CYLINDER ROTATING ABOUT ITS AXIS

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ABSTRACT

In the paper [1] "On the Gravitation of Moving Bodies" a new law of gravitation called the law of general gravitation was proposed. In this paper the general gravitational field of a homogeneous cylinder rotating uniformly about its axis is derived.

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1 INTRODUCTION

There are two mass equivalence principles in Physics which are called the strong equivalence principle (S.E.P) and the weak equivalence principle (W.E.P).

The strong equivalence principle asserts the equivalence of gravitational mass m_g and the rest mass m_0 for all particles in all enertial reference frames.

The weak equivalence principle on the other hand asserts the equivalence of the gravitational mass m_g and the relativistic mass

$$m = m_0(1 - \frac{v^2}{c^2})^{-\frac{1}{2}}$$

for all particles in all inertial reference frames. v being the linear speed of the particle and c is the speed of light in vacuo.

Based on the strong equivalence principle, Newton in 1686 published his popular theory of universal gravitation. Then in 1915 Einstein published his theories of Mechanics and gravitation called the general relativity(G.R) which was equally based on the strong equivalence principle and was supposed to serve as a replacement for Newton's theories of mechanics and gravitation. Unfortunately G.R does not satisfy all the demands of a physical theory. Einstein equivalence principle was formulated as follows: "... for any infinitely small world-region the coordinates can always be chosen in such a way that the gravitational field in that world-region vanishes" This erroneous statement by Einstein and G.R theory has received so many criticism from eminent scientists. One of such came from Logunov [3] who said that by formulating the equivalence principle in this way, Einstein has "consciously departed from the concept of a gravitational field as being a Faraday-Maxwell physical field, a material substratum that can never be destroyed by the choice of reference frame". Logunov further said G.R contains no classicalNewtoniaan limit and , hence it does not satisfy the correspondence principle. This implies that G.R is not only logically contradictory from the viewpoint of physics but directly contradicts the experimental data on the equality of inertial and the active gravitational mass (a consequence of law of action and reaction)

One other flaw in G.R is the fact that there is no energy-momentum conservation laws for matter and gravitational field taken together. As puts it Hilbert, 1917: "I declare that ... for the general theory of relativity, that is, in the case of general invariance of the Hamiltonian function, there are generally no energy equations that ... correspond to energy equations in orthogonal-invariant theories. I could even note this fact as being a characteristic feature of the theory". Since there is no experimental fact that directly or indirectly challenges the validity of conservation laws, the only way out is to re-examine the fundamental principles underlying the G.R, so that in the new theory of gravitation the fundamental laws of physics, the laws of conservation of energy-momentum of gravitational field and matter taken together holds true.

Contrary to general relativity S.X.K Howusu in 1991 published his theory of gravitation based on the weak equivalence principle. Although this theory deviated from the common practice, it duplicated the results of Einstein general relativity theory with additional terms.

Consequently the competitiveness posed by this theory to G.R has brought to every sharp focus the need to distinguish the equivalence principle. As pointed out by T.E. Phipps Jr. " Still data may already exist that would permit a choice between our two forms of the equivalence principle. If so we hope that this will be brought to light and its significance appreciated. If not, it seems a worthy topic of future experimental physics".

Such an experiment has not to our present knowledge been published except Howusu's theory of general gravitation. Although the weak equivalence principle is apparently not discordant with any observed facts in special relativity, it has not received any attention in the teaching textbooks of relativity. And so there is no sufficient and convincing reasons to close the study of gravitation theories because of G.R. Instead we should explore other theories based on the weak equivalence which I believe Howusu's General gravitation theory has done the latent capability that G.R could not do.

2 FORMULATION OF GENERAL GRAVITATION THEORY

Howusu postulates that in all inertial reference frames, the gravitation mass m_g of a particle is equal to its general mass m.

i.e.

$$m_g = m = m_0(1 - \frac{v^2}{c^2})^{-\frac{1}{2}}$$

According to this postulate, the motional as well as the rest part of the general mass of a body will simultaneously generate gravitational fields. Based on this, he naturally extended the Newton's law for the generation of universal gravitational potential field by replacing the rest mass with the general mass. And he defined the general gravitational potential field as follows:

Let (r',t') be an arbitraary point within a distribution of general mass particles occupying a region T' of space -time. Let ρ be the general mass density function within T'. Then there exist at a point (r,t) of space-time a gravitational potential given by

$$\Phi_g(r,t) = -G \int_{T'} \frac{\rho(r',t')}{|r - r'|} dT' \quad (1)$$

where G is the universal gravitational constant and the integration is over all points of space-time within T' such that

$$t - t' = \frac{|r - r'|}{(C_g)}$$

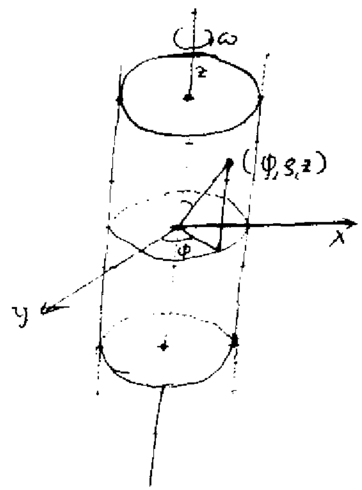
where

$$C_g$$

is the speed of gravitational effect in vacuo.

We shall apply this law of general gravitation to derive the general gravitational potential field due to a homogeneous cylinder rotating uniformly about its axis.

3 APPLICATION TO UNIFORMLY ROTATING INFINITELY LONG CYLINDER



Consider a cylinder of radius of R and rest mass density ρ_r rotating uniformly about its axis as shown in the diagram. Then in the usual cylindrical coordinates, the general gravitational potential Φ_g is given by equation (1). But since the cylinder is rotating with a uniform angular velosity ω ,

$$\rho_0(r',t') = \rho_0(r') \quad (2)$$

Also the linear velocity U' of the element of mass at the point (r',t') is given by

$$u'(r',t') = \zeta'\omega\Phi' \quad (3)$$

Hence assuming that $u' < c$ it follows by the series expansion that

$$(1 - \frac{u'^2}{c^2})^{-\frac{1}{2}} = \sum_{n=0}^{\infty} \frac{\binom{-1/2}{n}(-1)^n(\omega^{2n})(\zeta'^{2n})}{c^{2n}} \quad (4)$$

Hence from (2) and (4) the density of general mass, ρ, is given by

$$\rho(r',t') = \rho_0(r')(1 - \frac{u'^2}{c^2})^{-\frac{1}{2}}$$

$$= \sum_{n=0}^{\infty} \frac{\binom{-\frac{1}{2}}{n}(-1)^n \omega^{2n} \zeta'^{2n} \rho_0(r')}{c^{2n}} \tag{5}$$

Consequently by the law of general gravitation, the general gravitational potential Φ_g is given by

$$\Phi_g(r,t) = \sum_{n=0}^{\infty} \Phi_{gn}(r,t) \tag{6}$$

where

$$\Phi_{gn}(r,t) = D_n \int_{cyl} \frac{\rho_0(r')\zeta'^{2n} dT}{|r - r'|} \tag{7}$$

where

$$D_n = \frac{\binom{-\frac{1}{2}}{n}(-1)^{n+1}\omega^{2n} G}{c^{2n}} \tag{8}$$

The general gravitational potential corresponding to a given rest mass density function ρ_0 of (7) can be obtained explicitly for points exterior to the cylinder by direct integration, but is very laborous. Therefore we shall seek the exact expressions for the terms of the potential of equation (6) for the field points, both exterior and interior by taking the laplacian of both sides of (7). This gives

$$\nabla^2 \Phi_{gn}(r,t) = D_n \int_{cyl} \rho_0(r')\zeta'^{2n} \nabla^2 (\frac{1}{|r - r'|}) dT$$

$$= -4\pi D_n \int_{cyl} \rho_0(r')\zeta'^{2n} \delta(r - r') dT$$

or

$$\nabla^2 \Phi_{gn}(r,t) = -4\pi D_n \rho_0(r) \zeta^{2n} \tag{9}$$

As a special case consider a homogeneous cylinder having a constant rest mass density ρ_0 then

$$\rho_0 = \begin{cases} \rho_0 & : \ r \ < \ R \\ 0 & : \ r \ > \ 0 \end{cases} \tag{10}$$

Hence equation (9) becomes

$$\nabla^2 \Phi_{gn}(r) = \begin{cases} -4\pi D_n \rho_0 \zeta^{2n} & : \ r \ < \ R \\ 0 & : \ r \ > \ R \end{cases} \tag{11}$$

For n=0 equation (11) becomes

$$\nabla^2 \Phi_{g0}(r) = \begin{cases} -4\pi D_0 \rho_0 & : \ r \ < \ R \\ 0 & : \ r \ > \ R \end{cases} \tag{12}$$

For an infinitely long cylinder, the potential is independent of Z and Φ, then interior part of (12) becomes

$$\nabla^2 \Phi^i_{g0}(\zeta) = -4\pi D_0 \rho_0 \tag{13}$$

and the exterior part becomes

$$\nabla^2 \Phi^e_{g0}(\zeta) = 0 \tag{14}$$

The general solutions of (13) and (14) are as follows respectively

$$\Phi^i_{g0} = C_0 ln\zeta + F_0 - \pi D_0 \rho_0 \zeta^2 \tag{15}$$

$$\Phi^e_{g0} = A_0 ln\zeta + B_0 \tag{16}$$

where Co, Fo, Ao and Bo are constants of integration to be eliminated by boundary conditions.

At $\zeta = R$

$$\Phi^i_{g0} = \Phi^e_{g0}$$

so that

$$C_0 lnR + F_0 - \pi D_0 \rho_0 R^2 = A_0 lnR + B_0 \tag{17}$$

also

$$\frac{d\Phi^i_{g0}}{d\zeta} = \frac{d\Phi^e_{g0}}{d\zeta}$$

at s=R

so that

$$\frac{C_0}{R} - 2\pi D_0 \rho_0 R = \frac{A_0}{R} \tag{18}$$

Co and Bo in equations (17) and (18) vanishes.

So

$$F_0 - \pi D_0 \rho_0 R^2 = A_0 lnR \tag{19}$$

$$\frac{-2\pi D_0 \rho_0 R = A_0}{R} \tag{20}$$

Given

$$A_0 = -2\pi D_0 \rho_0 R^2 \tag{21}$$

and

$$F_0 = \pi D_0 \rho_0 R^2 (1 - 2lnR) \tag{22}$$

Substituting (21) and (22) into (15) and (16) gives

$$\Phi^i_{g0} = -\pi G \rho_0 R^2 (1 - 2lnR) + \pi G \rho_0 \zeta^2 \tag{23}$$

$$\Phi^e_{g0} = 2\pi G\rho_0 R^2 ln\zeta \quad (24)$$

Equations (23) and (24) are the universal gravitational fields of the body at the exterior points of the body.

Lets now consider the first connection term to the universal gravitational field in which n=1, then equation (11) becomes

$$\nabla^2\Phi_{g1}(r) = \begin{cases} -4\pi D_1\rho_0\zeta^2 & : \quad r \ < \ R \\ 0 & : \quad r \ > \ R \end{cases} \quad (25)$$

Since the cylinder is assumed to be infinitely long, the potential will be independent of ϕ and z. Therefore the interior general solution of equation (25) becomes

$$\Phi^i_{g1} = C_1 ln\zeta + F_1 - \frac{1}{4}\pi D_1\rho_0\zeta^4 \quad (26)$$

and the exterior solution becomes

$$\Phi^e_{g1} = A_1 ln\zeta + B_1 \quad (27)$$

where C1, A1, F1 and B1 are constants of integration. By chosing C1 and B1 in equations (26) and (27) to be zero we have

$$\Phi^i_{g1} = F_1 - \frac{1}{4}\pi D_1\rho_0\zeta^4 \quad (28)$$

$$\Phi^e{}_{g1} = A_1 ln\zeta \quad (29)$$

Using the boundary conditions that at $\zeta = R$

$$\Phi^i_{g1} = \Phi^e_{g1}$$

and

$$\frac{d\Phi^i_{g1}}{d\zeta} = \frac{d\Phi^e_{g1}}{d\zeta}$$

$$A_1 = -\pi D_1\rho_0 R^4$$

$$F_1 = \frac{\pi D_1\rho_0 R^4}{4}(1 - 4lnR)$$

So that equations (28) and (29) become

$$\Phi^i_{g1} = \frac{\pi D_1\rho_0 R^4}{4}(1 - 4lnR) - \frac{1}{4}\pi D_1\zeta^4$$

$$\Phi^e_{g1} = -\pi D_1\rho_0 R^4 ln\zeta$$

but $D_1 = \frac{-G\omega^2}{2c^2}$ from equation (8)

$$\Phi^i_{g1} = \frac{\pi G\rho_0\omega^2}{8c^2}(4R^4 lnR - R^4 + \zeta^4) \quad (30)$$

$$\Phi^e_{g1} = \frac{\pi G\rho_0 R^4\omega^2 ln\zeta}{2c^2} \quad (31)$$

4 SUMMARY AND CONCLUSION

From the results of equations (24) and (31) we obtain the general gravitational potential for points exterior to the infinitely long cylinder rotating uniformly about its axis to be

$$\Phi_g(r,t) = 2\pi G\rho_0 R^2 ln\zeta + \frac{\pi G\rho_0 r^4 ln\zeta}{2c^2} + ...$$

Also from equations (23) and (30) the general gravitational potential of the points interior to the infinitely long cylinder rotating uniformly about its axis is

$$\Phi_g(r,t) = -\pi G\rho_0(R^2 - 2R + \zeta) + \frac{\pi G\rho_0\omega^2}{8c^2}(4R^2 lnR - R^4 + \zeta^4) + ...$$

It can be seen that there are infinite number of the correction terms each containing the factor $\frac{1}{c^2}$ and is smaller than immediate one preceeding it. The presence of these correction fields shows that in the law of general gravitation a moving mass produces a gravitational field over and above the universal gravitation field just like a moving electric charge produces a magnetic field over and above its electrostatic field. The law of general gravitation as demonstrated here has Newtonian limit and therefore, it satisfies the correspondence principle which Einstein G.R does not have as pointed out in the introduction.

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