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ABSTRACT

The statistical approach of maximal randomness of the velocity field is extended for the case of decaying turbulence with spontaneous parity violation. The set of self-consistent equations in one–loop approximation is obtained. It is without the infrared and ultraviolet divergences and has a scaling solution which leads to Kolmogorov spectrum in inertial range of wave numbers k and gives the well–known time–dependence laws for integral turbulence scale $r_c(t) \sim t^{2/5}$ and turbulent energy per mass $\epsilon(t) \sim t^{-6/5}$. The set of equations for scaling functions of energy and helicity spectral density, depending only on dimensionless parameter kr_c, is presented.

1 Introduction

In the present paper we continue our investigations of developed homogeneous isotropical turbulence of incompressible fluid based on the principle of maximal randomness of the velocity field. In a previous paper [1] we have considered the stationary case and our principal conclusion was the following; the Kolmogorov spectrum of velocity pair correlation function is valid in the inertial range and it is accompanied by spontaneous parity violation with large gyrotropic coefficient. It would be interesting to know if this effect also shows itself in the energy - containing range of wave numbers (between the inertial range and the range of energy source). That is why we will try to extend our approach to decaying developed turbulence. It will allow us to introduce in a natural way a turbulent integral scale and to determine its decay law as well as the decay law of turbulence intensity.

2 Basic assumptions and self-consistent equations

We start again from the equation for velocity pair correlation function [1]:

$$\partial_t < E_{ij}(\mathbf{k}) > /2 =< \dot{E}_{ij}(\mathbf{k}) > +d_{ij}(\mathbf{k}) \tag{1}$$

which follows from stochastic Navier-Stokes equation for the incompressible fluid. Here

$$\begin{aligned} E_{ij}(\mathbf{k}) &= v_i(\mathbf{k})v_j(-\mathbf{k}), \\ \dot{E}_{ij}(\mathbf{k}) &= -\nu k^2 E_{ij}(\mathbf{k}) - T_{ij}(\mathbf{k}), \\ T_{ij}(\mathbf{k}) &= [v_i(\mathbf{k})(v_s\partial_s v_j)(-\mathbf{k}) + (v_s\partial_s v_i)(\mathbf{k})v_j(-\mathbf{k})]/2\,, \end{aligned} \tag{2}$$

ν is the kinematic viscosity coefficient, $\mathbf{v}(\mathbf{k})$ - Fourier component of transversal velocity field $\mathbf{v}(\mathbf{k}) = \int d\mathbf{x} exp(-i\mathbf{k}\mathbf{x})\mathbf{v}(\mathbf{x})$, ($\partial_i v_i = 0$ - incompressibility), d_{ij} - an external energy source. Hereafter, the traces over the repeated vector indexes are implied. In the isotropical case with spontaneous parity violation the equation (1) is equivalent to two scalar equations for spectral densities of the conserved (in inviscid fluid) quantities - the energy $E(\mathbf{k}) = E_{jj}(\mathbf{k})$ and the helicity $\Psi(\mathbf{k}) = i\varepsilon_{jst}k_j E_{st}(\mathbf{k})$. All tensors in the considered case contain two mutually commuting parts: the symmetric one which is proportional to the transversal projector $\mathcal{P}_{ij}(\mathbf{k}) = \delta_{ij} - k_i k_j k^{-2}$ and the antisymmetrical one which is proportional to the transversal pseudoprojector $Q_{ij} = \varepsilon_{ijs}k_s/k$ $(k \equiv |\mathbf{k}|)$.

In [1] we have considered the stationary turbulence $\partial_t < E_{ij}(\mathbf{k}) >= 0$. The velocity distribution function $\rho(\mathbf{v}(\mathbf{x}))$ has been obtained from the demand of maximum of information entropy $\sigma = - < ln\rho >= - \int D\mathbf{v}\rho(\mathbf{v})ln\rho(\mathbf{v})$, ($\int D\mathbf{v}$ denotes the functional integration) with the equation of condition (1). Its form is

$$\rho(\mathbf{v}) = Q^{-1}exp[(2\pi)^{-3}\int d\mathbf{k}\lambda_{ij}(\mathbf{k})\dot{E}_{ji}(\mathbf{k})]\,, \tag{3}$$

where Q is a normalization factor, $\lambda_{ij}(\mathbf{k})$ are Lagrange multipliers which ensures the realization of the equation of condition (1).

We consider now the situation when the external source d_{ij} is turned off at some time t_0. Our essential supposition is the following; the decay of energy and helicity is relatively small during some time so that the pulsation distribution function preserves the same form (3) with slowly varying parameters $\lambda_{ij}(\mathbf{k})$ at any time. The time-dependence of this decay must be found from equation (1) (with $d_{ij} = 0$). The similar treatment is possible for turbulent flow behind the grid with constant mean velocity V along any axis z. The system is stationary in this case and the distance z from the grid plays the role of the time t. Such turbulence may be considered as local homogeneous if the statistical quantities of the velocity field slowly vary along z in the range of the integral turbulent scale r_c (correlation length). Then the equation for the velocity pair correlation function may be written in form (1) replacing ∂_t for $V\partial_z$. The assumed supposition will be justified when z-dependence of r_c is found.

In the considered case the left-hand side of equation (1) plays the role of 'energy source' which gives quasistationary spectrum flux of energy. It is localized in energy-containing wave number range where the small value of time-derivative ∂_t is compensated by large value of $< E(\mathbf{k}) >$. The equation of total energy balance has the form:

$$\partial_t e(t) = -W(t) \tag{4}$$

where $e(t) = (2\pi)^{-3}\int d\mathbf{k} < E(\mathbf{k}) >$ and $W(t) = (2\pi)^{-3}\int d\mathbf{k}2\nu k^2 < E(\mathbf{k}) >$ are the full energy of pulsations and dissipation rate per mass, respectively. The quantity $W(t)$ defines the second (in addition to r_c) typical scale of the system - the Kolmogorov microscale $k_d^{-1} = (\nu^3/W)^{1/4}$. If the turbulence is fully developed these scales are very different $k_d r_c \gg 1$ and the inertial interval of wave-numbers $r_c^{-1} \ll k \ll k_d$ exists.

The relation (1) is an equation for $\lambda_{ij}(\mathbf{k}, t)$ if the mean values of $E_{ij}(\mathbf{k})$ and $\dot{E}_{ij}(\mathbf{k})$ are calculated with distribution function (3). This complicated task has been solved by means of a renormalization group approach and ϵ-expansion [2]. However, the significance of that analysis is depreciated because the real value of $\epsilon = 2$ is too large. In the present paper we have used another way without ϵ-expansion and have obtained a set of self-consistent equations for full propagator $G_{ij}^{vv}(\mathbf{k}, t) \equiv< v_i(\mathbf{k})v_j(-\mathbf{k}) >=< E_{ij}(\mathbf{k}) >$ and Lagrange multiplier $\lambda_{ij}(\mathbf{k}, t)$ with the aid of Dyson matrix equation in one loop-approximation. For this purpose it is convenient to introduce auxiliary transversal fields $\mathbf{u}$, $\mathbf{w}$ and in this way one obtains the distribution function with a local interaction [1]:

$$\rho(\mathbf{v}, \mathbf{u}, \mathbf{w}) = Q^{-1}\exp S\,, \tag{5}$$

$$\begin{aligned} S = &-\varphi_0(\partial_j v_i)(\partial_j v_i)/2 - \partial_j v_n(i\psi_0\varepsilon_{njs}\partial_s)\partial_l v_j/2 - \\ &\nu(\partial_j u_i)(\partial_j v_i) + (\partial_j u_i)v_i v_j - w_i\tilde{\lambda}_{ij}v_j + w_i u_i. \end{aligned} \tag{6}$$

Now the mean values must be calculated by functional integration over all fields $\mathbf{v}, \mathbf{u}, \mathbf{w}$

The action S is written in coordinate representation and the integration over $\mathbf{x}$ is implied. We have separated the additional terms in Lagrange multiplier λ_{js}

$$\lambda_{js}(\mathbf{k}, t) = \mathcal{P}_{js}(\mathbf{k})\varphi_0(t)/2\nu + i\varepsilon_{jsl}k_l\psi_0(t)/2\nu + \tilde{\lambda}_{js}(\mathbf{k}, t)\,. \tag{7}$$

The first two terms in action (6) are necessary to cancel the ultraviolet (UV) divergences of diagrams. It is important that the analogous terms do not appear in vertex terms after substitution (7) to (3) what is the consequence of the energy and helicity conservation laws.

Using Schwinger equation $< u_i \delta S/\delta u_j >= 0$ of the theory (6) we rewrite (1) (with $d_{ij} = 0$)to the form

$$\partial_t < E_{ij}(\mathbf{k}) >= - < v_i(\mathbf{k}) w_j(-\mathbf{k}) > . \qquad (8)$$

The propagators G^{vu}_{ij} and $G^{vw}_{ij} =< v_i w_j >$ which we are interested in are expressed with the aid of the self-energy matrix Σ_{ij} of the theory (6) by relation [1]:

$$(G^{vv})^{-1}_{sj} = \varphi_0 k^2 \mathcal{P}_{sj} + i\varepsilon_{sjl} k_l k^2 \psi_0 + (2\nu k^2 \tilde{\lambda} - \Sigma^{vv} - \tilde{\lambda}^2 \Sigma^{uu} - \tilde{\lambda}\Sigma^{uv} - \tilde{\lambda}\Sigma^{vu})_{sj}, \qquad (9)$$

$$G^{vw}_{sj} = [G^{vv}(\nu k^2 - \Sigma^{uv} - \tilde{\lambda}\Sigma^{uu})]_{sj}, \qquad (10)$$

and the propagators G^{vu}, G^{uu} are related to G^{vv};

$$G^{vu}_{sj} = (\tilde{\lambda} G^{vv})_{sj}, G^{uu}_{sj} = (\tilde{\lambda}^2 G^{vv})_{sj}. \qquad (11)$$

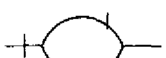
The terms which include viscosity ν in (9), (10) may be neglected in the wave number range $k \ll k_d$. Then it yields from (8)-(10)

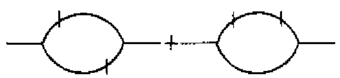
$$\partial_t G^{vv}_{sj} = [G^{vv}(\Sigma^{uv} + \tilde{\lambda}\Sigma^{uu})]_{sj}, \qquad (12)$$

$$(G^{vv})^{-1}_{sj} = -(\Sigma^{vv} + \tilde{\lambda}^2\Sigma^{uu} + \tilde{\lambda}\Sigma^{uv} + \tilde{\lambda}\Sigma^{vu})_{sj}, \qquad (13)$$

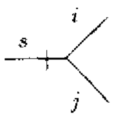
where we omitted for brevity the first and the second terms in (9) which play the role of counterterms for the UV divergences cancelation of Σ^{vv} [1] (see also Sec.4).

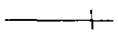
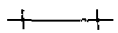
For self-energy elements of Σ of the theory (6) in one-loop approximation we have [1]:

$\Sigma^{uu} = \frac{1}{2}$  , $\Sigma^{uv} =$  ,

$\Sigma^{vv} = \Sigma^{vv}_1 + \Sigma^{vv}_2 =$  , (14)

where graphic representations are introduced for the symmetrized vertex of interaction and full propagators:

 $= \frac{1}{2} v_i v_j (\partial_i \delta_{js} + \partial_j \delta_{is}) u_s$, $=< vv >\equiv G^{vv}$,

 $=< vu >\equiv G^{vu}$,  $=< uu >\equiv G^{uu}$.

3 Scaling solution and time-dependence laws

The relations (12), (13) represent the full set of equations for $G^{vv}_{sj}(\mathbf{k}, t)$ and $\tilde{\lambda}_{sj}(\mathbf{k}, t)$. We will not try to find its solutions corresponding to some given initial conditions at a time t_0, but we will search for the scaling solution of the form

$$G^{vv}_{sj}(\mathbf{k}, t) = R^{2/3}(t) k^{-2\gamma} g_{sj}(k r_c(t)), \qquad (15)$$

$$\tilde{\lambda}_{sj}(\mathbf{k}, t) = R^{-1}(t) k^{3\gamma - 5/2} f_{sj}(k r_c(t)). \qquad (16)$$

This solution parametrically depends on t through the correlation length $r_c(t)$ and amplitude $R(t)$. The exponents in (15), (16) are chosen to eliminate $R(t)$ and $r_c(t)$ in equation (13). Indeed, by introducing a dimensionless variable $y \equiv k r_c$ and taking into account (14) we have

$$g^{-1}_{sj}(y) = -[\Sigma^{vv}(y) + f^2(y)\Sigma^{uu}(y) + f(y)\Sigma^{uv}(y) + f(y)\Sigma^{vu}(y)]_{sj}, \qquad (17)$$

Argument y in elements of Σ-matrix denotes that in diagram representations (14) it is necessary to take an external moment $\mathbf{k}$ being $|\mathbf{k}|= 1$ and for internal lines it must be substituted $R \to 1$, $r_c \to y$. Thus, the time t does not appear explicitly in (17). It means that this equation is only for the dimensionless scaling functions g_{sj} and f_{sj}.

Using the dimensionless variable y also in equations (12), (13) we obtain

$$\frac{1}{3}\partial_t ln R(t) g_{sj}(y) + \frac{1}{2}\partial_t ln r_c(t) y \partial_y g_{sj}(y) =$$
$$R^{1/3}(t) r_c^{\gamma - 5/2}(t) y^{5/2-\gamma} \{g(y)[\Sigma^{uv}(y) + f(y)\Sigma^{uu}(y)]\}_{sj}. \qquad (18)$$

The time-dependence of $R(t)$ and $r_c(t)$ may be found analysing the asymptotic behaviour of this equation in large and small y ranges. In the first case we assume that the scaling functions $g_{sj}(y)$ and $f_{sj}(y)$ are finite when $y \to \infty$. After that the first term in the left-hand side is dominant and the variables t, y are separable:

$$\frac{1}{3} R^{-4/3}(t) \partial_t R(t) r_c^{5/2-\gamma}(t) \mathcal{P}_{sj} = y^{5/2-\gamma}[\Sigma^{uv}(y) + f(y)\Sigma^{uu}(y)]_{sj} = C_1 \mathcal{P}_{sj}, \qquad (19)$$

where C_1 is some separation constant. We conclude from (19) that for $\gamma < 5/2$ it must be

$$[\Sigma^{uv}(y) + f(y)\Sigma^{uu}(y)]_{sj} = 0, \quad y \gg 1. \qquad (20)$$

For constant values of f_{sj}, g_{sj} this equation coincides with the one of those considered in [1]. As it has been shown in [1] that equation with traced indices turns into identity if the exponent γ has the Kolmogorov value $2\gamma = 11/3$. Further on we use this value of γ. Thus, for time-dependent terms of (19) we have the equation

$$R^{-4/3}(t) \partial_t R(t) = 3 C_1 r_c^{-2/3}(t). \qquad (21)$$

Let us consider now the range $y \ll 1$. For this case it can be easily seen from (14) that both $\Sigma^{vu}(y)$ and $\Sigma^{uu}(y)$ vanish therefore we may neglect the right-hand side of equation (18) in this limit:

$$\frac{1}{3}\partial_t ln R(t) g_{sj}(y) + \frac{1}{2}\partial_t ln r_c(t) y \partial_y g_{sj}(y) = 0, \quad y \ll 1. \qquad (22)$$

The variables y and t are separable in this equation, too. In particular, for traced vector indices we have

$$\partial_y ln g_{ss}(y) = \frac{-2\partial_t ln R(t)}{3\partial_t ln r_c(t)} = C_2 , \tag{23}$$

where C_2 is a new separation constant. From (23) it follows

$$\begin{aligned} g_{ss}(y) &= A y^{c_2}, \quad y \ll 1 , & (24) \\ R^2(t) r_c^{3C_2}(t) &= B , & (25) \end{aligned}$$

where A, B are some integration constants. The last relation together with equation (21) lead to

$$\begin{aligned} R(t) &= (at+b)^{-9C_2/(4+3C_2)} , & (26) \\ r_c(t) &= C(at+b)^{6/(4+3C_2)} & (27) \end{aligned}$$

with some constants a, b, C. For turbulence intensity $e(t) = (2\pi)^{-3} \int d\mathbf{k} G^{vv}_{ss}(\mathbf{k}, t)$ using (15) we obtain

$$e(t) = C^{2/3}(at+b)^{(4-6C_2)/(4+3C_2)}(2\pi)^{-3} \int dy y^{-11/3} g_{ss}(y) . \tag{28}$$

Then the relation (4) gives

$$W(t) = aC^{2/3}(4-6C_2)/(4+3C_2)(at+b)^{-9C_2/(4+3C_2)}(2\pi)^{-3} \int dy y^{-11/3} g_{ss}(y) . \tag{29}$$

It can be seen from (26), (29) that the amplitude $R(t)$ is proportional to the dissipation rate per unit mass $W(t)$.

The power law (24) determines asymptotic behaviour at small $\mathbf{k}$ of spectrum density of energy $< E_{ss}(\mathbf{k}) >= G_{ss}(\mathbf{k})$. Indeed, from (24) and (15) we have

$$< E_{ss}(\mathbf{k}) > \sim k^{C_2 - 11/3} . \tag{30}$$

Thus, the same constant C_2 determines both the time dependence laws (26) - (29) and the small $\mathbf{k}$ asymptotic behaviour of energy density. Two different assumptions have been done earlier for exponent value in (30). It is natural to suppose that $< E_{ss}(\mathbf{k}) >$ has a finite non-zero limit at $\mathbf{k} = 0$. In this case C_2 must be $C_2 = 11/3$ in (30). By taking this value of C_2 we obtain from (27)-(29) the well-known laws

$$\begin{aligned} r_c(t) &\sim (at+b)^{2/5} , \\ e(t) &\sim (at+b)^{-6/5} , \\ W(t) &\sim (at+b)^{-11/5} . \end{aligned} \tag{31}$$

The earlier hypothesis of Loitsianskii [3] starts with the assumption that not only $< E_{jj}(\mathbf{k}) >$ but also $< E_{sj}(\mathbf{k}) >$ has a well determined limit at $\mathbf{k} = 0$. Since $< E_{sj}(\mathbf{k}) >$ includes the transversal projector $\mathcal{P}_{sj} = \delta_{sj} - k_s k_j / k^2$ we must compensate

the additional multiplier k^{-2} and, thus, C_2 will be $C_2 = 17/3$. It gives the following values for the exponents in (27)-(29) (2/7, −10/7, −17/7, respectively) which differ from those in (31). There exists, however, strong objection against the Loitsianskii hypothesis [3]. Therefore, further on the first value of $C_2 = 11/3$ is used. Note also that in principle the value of C_2 can be found analysing the asymptotic behaviour of the solution of equation (1) with some initial condition.

It has to be verified if the used assumption about finite limit of $G^{vv}_{jj}(\mathbf{k} = 0)$ is in accordance with relation (13). If the matrix G^{vv}_{sj} is non-degenerate we conclude that the inverse matrix $(G^{vv}(\mathbf{k}))^{-1}_{jj}$ has a finite limit at $\mathbf{k} = 0$, too. However, all elements of Σ in (13) are proportional to k^2 at small $\mathbf{k}$. To eliminate the contradiction between these facts we must admit that the function $\tilde{\lambda}(\mathbf{k})$ is proportional to $1/k$ for small $\mathbf{k}$. Then the trace of $(\tilde{\lambda}^2\Sigma)_{jj}(\mathbf{k})$ in (13) is finite and the other terms vanish. Note that such behaviour of $\tilde{\lambda}(\mathbf{k})$ is not in contradiction with vanishing right side of (12) for $\mathbf{k} = 0$ because $\Sigma^{vv} \sim k^2$ and $\tilde{\lambda}\Sigma^{vv} \sim k \to 0$.

The obtained laws of correlation length and turbulence intensity behaviour allow us to verify our assumptions formulated in Sec.2. Let us consider for clarity the case of turbulence behind the grid. For this purpose we replace t to z/V everywhere (V is the mean velocity of the flow). Rewriting (31) into usual notation [4, 5, 6] we have

$$\begin{aligned} r_c(z) &= CL\frac{v_0}{V}(\frac{z-z_0}{L})^{2/5} , \\ e(z) &= \frac{v_0^2}{2}(\frac{z-z_0}{L})^{-6/5} , \\ W(z) &= \frac{3v_0^2 V}{5L}(\frac{z-z_0}{L})^{-11/5} , \end{aligned} \tag{32}$$

where v_0 is rms velocity and z_0 defines the co-ordinate of 'effective source'. Parameter v_0/V is usually small [4, 5, 6]. The discussed assumption is correct if the variations of any of the functions $\Phi(z)$ considered in (32) are relatively small at the distances of order r_c, i.e. $r_c\partial_z ln\Phi(z) \ll 1$. For the power law functions (32) far from source ($z \gg z_0$) it is true if

$$C\frac{v_0}{V}(\frac{L}{z})^{3/5} \ll 1 . \tag{33}$$

However, one has to keep in mind that the developed turbulence condition $k_d r_c \gg 1$ may be violated for too large z. Indeed, although the correlation length $r_c(z)$ increases as $z^{2/5}$, the microscale $r_d \equiv k_d^{-1} = (\nu/W(z))^{1/4}$ increases more rapidly (what can be seen from (32) that $r_d \sim z^{11/20}$). Thus, the necessary condition of the developed turbulence

$$\frac{r_c}{r_d} \sim (\frac{VL}{\nu})^{3/4}(\frac{v_0}{V})^{3/2}(\frac{z}{L})^{-3/20} \gg 1 \tag{34}$$

is valid if the Reynolds number $Re \equiv VL/\nu$ is sufficiently large, $Re \gg 1$.

The inequalities (33), (34) have been fulfilled in experimental conditions [4, 5, 6]. The obtained experimental results for functions $r_c(z)$, $e(z)$, $W(z)$ are in good agreement with the expressions (32).

4 Equations for scaling functions

The relations (21), (23) allow us to exclude the time-dependent terms from equation (18). By setting $2\gamma = C_2 = 11/3$ we obtain

$$C_1(1-3/11y\partial_y)g_{sj}(y) = y^{2/3}\{g(y)[\Sigma^{vu}(y)+f(y)\Sigma^{uu}(y)]\}_{sj}\,. \quad (35)$$

The relations (17) and (35) become the full set of equations for the scaling functions $g_{sj}(y)$, $f_{sj}(y)$ if the Σ-diagrams (14) are calculated. To do it we write explicitly the tensor structure of g_{sj}, f_{sj}. As we said above (Sec.2) these functions have two parts:

$$\begin{aligned} g_{sj}(y) &= \mathcal{P}_{sj}g_P(y) + i\mathcal{Q}_{sj}g_Q(y)\,, & (36)\\ f_{sj}(y) &= \mathcal{P}_{sj}f_P(y) + i\mathcal{Q}_{sj}f_Q(y)\,, & (37)\end{aligned}$$

and the inverse matrix $(g^{-1})_{sj}$ is

$$g^{-1}_{sj}(y) = [g_P^2(y)-g_Q^2(y)]^{-1}[\mathcal{P}_{sj}g_P(y) - i\mathcal{Q}_{sj}g_Q(y)]\,. \quad (38)$$

In accordance with (11), (15), (16) the propagators G^{vu}_{sj}, G^{uu}_{sj} have the corresponding forms:

$$\begin{aligned} G^{vu}_{sj}(\mathbf{k},t) &= R^{-1/3}(t)k^{-2/3}[\mathcal{P}_{sj}(g_Pf_P+g_Qf_Q)_y\\ &+ i\mathcal{Q}_{sj}(g_Pf_Q+f_Qg_P)_y]\,, & (39)\\ G^{uu}_{sj}(\mathbf{k},t) &= R^{-4/3}(t)k^{7/3}[\mathcal{P}_{sj}(g_Pf_P^2+2g_Qf_Pf_Q+g_Pf_Q^2)_y\\ &+ i\mathcal{Q}_{sj}(g_Qf_P^2+2g_Pf_Pf_Q+g_Qf_Q^2)_y]\,, & (40)\end{aligned}$$

where the subscript $(\ldots)_y$ denotes that all g and f in brackets depend on $y = kr_c(t)$. We remind that $\Sigma(y)$ in equations (17), (35) must be calculated with $|\,\mathbf{k}\,| = 1$, $R = 1$.

The elements (14) of Σ matrix have the analogous form as (36), (37):

$$\Sigma_{sj}(y) = \mathcal{P}_{sj}\Sigma_P(y) + i\mathcal{Q}_{sj}\Sigma_Q(y)\,. \quad (41)$$

We can write the Σ matrix in the following form:

$$\Sigma(y) = (2\pi)^{-3}\int d\mathbf{q}[1-(\mathbf{qn})^2/q^2]A(\mathbf{q},\mathbf{p},y)\,, \quad (42)$$

where $\mathbf{n} \equiv \mathbf{k}/k$, $\mathbf{k}$ is an external momentum, $\mathbf{p} \equiv \mathbf{n}-\mathbf{q}$ and $\mathbf{q}$ is an integration undimensional variable. By substituting propagators (15), (36), (39), (40) to (14) for scalar functions $\Sigma_P(y)$ and $\Sigma_Q(y)$ we obtain

$$\begin{aligned} A^{uu}_P &= \frac{1}{2}q^{-11/3}p^{-11/3}\{g_P(qy)g_P(py)p^{-2}[1-3(\mathbf{nq})+q^2+2(\mathbf{nq})^2]\\ &- g_Q(qy)g_Q(py)qp^{-1}\}\,, & (43)\\ A^{uu}_Q &= q^{-11/3}p^{-14/3}g_P(qy)g_Q(py)[1-2(\mathbf{nq})]\,, & (44)\\ A^{vv}_P &= q^{-11/3}p^{-8/3}\{g_P(qy)(g_Pf_P+g_Qf_Q)_{py}[2(\mathbf{nq})-1-q^2(\mathbf{nq})]\\ &+ g_Q(qy)(g_Qf_P+f_Qg_P)_{py}qp[1-(\mathbf{nq})]\}\,, & (45)\\ A^{vv}_Q &= q^{-11/3}p^{-8/3}[1-(\mathbf{nq})][g_Q(qy)(g_Pf_P+g_Qf_Q)_{py}q\\ &- g_P(qy)(g_Qf_P+f_Qg_P)_{py}p]\,, & (46)\\ A^{vv}_P &= A^{vv}_{1P}+A^{vv}_{2P}\,, \qquad A^{vv}_Q = A^{vv}_{1Q}+A^{vv}_{2Q}\,,\\ A^{vv}_{1P} &= q^{-2/3}p^{-8/3}\{(g_Pf_P+g_Qf_Q)_{qy}(g_Pf_P+g_Qf_Q)_{py}[2q^2(\mathbf{nq})-(\mathbf{nq})-q^4]\\ &+ (g_Qf_P+f_Qg_P)_{qy}(g_Qf_P+f_Qg_P)_{py}qp[(\mathbf{nq})-q^2]\}\,, & (47)\\ A^{vv}_{1Q} &= 2q^{-2/3}p^{-5/3}(g_Pf_P+g_Qf_Q)_{qy}(g_Qf_P+g_Pf_Q)_{py}[q^2-(\mathbf{nq})]\,, & (48)\\ A^{vv}_{2P} &= q^{-11/3}p^{1/3}\{g_P(qy)(g_Pf_P^2+2g_Qf_Pf_Q+g_Pf_Q^2)_{py}[1-q^2(\mathbf{nq})\\ &- (\mathbf{nq})+q^4]+g_Q(qy)(g_Qf_P^2+2g_Pf_Pf_Q+g_Pf_Q^2)_{py}qp(q^2-1)\}\,, & (49)\\ A^{vv}_{2Q} &= q^{-11/3}p^{4/3}\{-g_Q(qy)(g_Pf_P^2+2g_Qf_Pf_Q+g_Pf_Q^2)_{py}qp\\ &+ g_P(qy)(g_Qf_P^2+2g_Pf_Pf_Q+g_Qf_P^2)_{py}(1-q^2)\}\,. & (50)\end{aligned}$$

The equations (17), (35) have been considered in our paper [1] in the limit of $k \to \infty$. It has been shown there that the solution of these equations exists if all functions g_P, g_Q, f_P, f_Q are constant. Note that $g_p(\infty)$ may be equated to $g_P(\infty) = 1$ because it is the definition of the scale $R(t)$, in fact. The obtained values [1] of the other parameters are: $g_Q \simeq \pm 1.0$, $f_Q(\infty)/f_P(\infty) \simeq \mp 1.7$., but $f_P(\infty) \sim [1-g_Q^2(\infty)]^{-1/2}$ was not well determined by reason of large relative error in calculation of small quantity $[1-g_Q^2(\infty)]$.

Unlike [1] the momentum representation used here is more useful for nontrivial k-dependence scaling functions. However, this leads to necessity to take into account the counterterms in (7) which are intended to cancel the UV divergences in Σ_1^{vv}, Σ_2^{vv} diagrams [1]. Their main divergences are proportional to $\Lambda^{11/3}$ (Λ - the cut-off parameter) but these are cancelled in the sum of diagrams $\Sigma^{vv} = \Sigma_1^{vv} + \Sigma_2^{vv}$ [1]. The odd divergences which are proportional to $k^2\Lambda^{5/3}$ for Σ_P^{vv} and $k\Lambda^{2/3}$ for Σ_Q^{vv} may be eliminated by the corresponding choice of the constants φ_0, ψ_0 in (7). We do not write these subtractions in (47)-(50) owing to their complexity.

5 Conclusion

In the present paper the generalization of statistical approach for decaying turbulence based on the principle of maximal randomness of velocity field has been obtain. Our consideration was based essentially on the hypothesis that the statistical quantities of velocity field vary relatively slowly in time (for homogeneous turbulence) or along any axis (for the flow behind the grid). It allowed us to obtain the set of self-consistent equations describing the evolution of equal-time correlation functions. We have searched for a scaling solution of these equations in which correlation functions depend on t implicitly through the correlation length and dissipation rate. A natural supposition has been done about the asymptotics of pair correlation function in the ranges of large and small wave vectors $\mathbf{k}$, namely, the power-law decay in inertial interval with some exponent and a finite limit at $\mathbf{k} = 0$. As a result, the variables t and $\mathbf{k}$ in all equations have been separated. We have solved the time-dependent part of these equations and have obtained the well-known power laws of correlation length and turbulence intensity which are in good agreement with the experimental data. At the same time the found laws justified our main assumption.

Much more complicated equations have been obtained for the scaling functions depending on a dimensionless variable - the product of the wave vector and the correlation length. For the asymptotically large values of this variable (inertial interval) these equations have solution of the expected power form with Kolmogorov exponent. The other parameters (in particular, gyrotropy coefficient) of the solution have been obtained in our previous paper [1]. The absence of both UV and IR divergences in the equations were substantial for the existence of this solution. We expect to find the numerical solution in all range of the wave vectors. This solution must coincide with the one found in inertial interval (the solution selection). It will allow to determine the quantity of the spontaneous gyrotropy not only in the inertial interval, but also in the energy-containing range and to compare the total helicity with experimental data [7, 8].

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