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INFLUENCE OF CP VIOLATION ON ENERGY SPECTRA
IN $K^+ \rightarrow \pi^\pm \pi^\pm \pi^\mp$ DECAYS

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ABSTRACT

Using the chiral σ model with broken $U(3)_L \otimes U(3)_R$ symmetry we have calculated in the p^4 -approximation a difference between the slope parameters g^+ and g^- describing the energy distributions of "odd" pions in $K^+ \rightarrow \pi^+ \pi^+ \pi^-$ and $K^- \rightarrow \pi^- \pi^- \pi^+$ decays. The result is: $(g^+ - g^-)/(g^+ + g^-) = -(1.9 \pm 0.8)10^{-2} s_2 s_3 \sin \delta$, where s_2 , s_3 and δ are the parameters of the Kobayashi-Maskawa mixing matrix.

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REFERENCE

1. INTRODUCTION

So far a search for the direct CP violation caused by the Kobayashi-Maskawa phase mechanism [1] was concentrated on measurements of the electric dipole moment of the neutron d_n and parameter ϵ' in $K^0 \rightarrow 2\pi$ decays. But the experimental limit $d_n < 1.2 \cdot 10^{-25} \text{ ecm}$ is far from the expected magnitude $d_n^{th} < 10^{-31} \text{ ecm}$ [2]. As for the parameter ϵ' , its magnitude crucially depends on t -quark mass and even zero value of ϵ' would not mean that the direct CP violation is absent, if the t -quark mass is big enough [3-5]. Besides, two measurements of ϵ' giving

$$\text{Re}(\epsilon'/\epsilon) = \begin{cases} (2.3 \pm 0.7)10^{-3} & [6] \\ (0.6 \pm 0.7)10^{-3} & [7] \end{cases}$$

are not in good agreement.

In such a situation, a search for the direct CP violation in $K^+ \rightarrow 3\pi$ decays leading to difference of widths of $K^+ \rightarrow \pi^+ \pi^+ \pi^-$ and $K^- \rightarrow \pi^- \pi^- \pi^+$ decays and difference of the slope parameters in these decays becomes of great interest.

The calculations [8-12] of such effects fulfilled in the lowest, p^2 -approximation in momentum expansion of the amplitude led to conclusion that the relative difference of the widths of $K^+ \pi^+ \pi^+ \pi^-$ and $K^- \rightarrow \pi^- \pi^- \pi^+$ has to be of order of 10^{-6} .

In the lowest p^2 -approximation, the CP-violating effects appear only thanks to interference of the $\Delta I = 1/2$ and $\Delta I = 3/2$ parts of the total amplitude. But in the p^4 -approximation, the parts of the $\Delta I = 1/2$ amplitude corresponding to "penguin" and "non-penguin" diagrams become of different dynamical structure and an additional contribution to CP violating effects appears from $|A_{1/2}|^2$. It was claimed by Bel'kov *et al.* [13-15] that in the p^4 -approximation the result for CP violating effects in $K^\pm \rightarrow \pi^\pm \pi^\pm \pi^\mp$ decays turns out to be larger by two orders in comparison with the value calculated in the p^2 -approximation.

Be it true, the investigation of CP violation in $K^\pm \rightarrow 3\pi$ decays should be a paramount task for the planned ϕ factories.

In view of significance of the question and in connection with some doubts on results of Refs.[13-15] expressed in Refs.[10-16] (however, without the calculation at p^4 order), it is important to investigate the problem again using some independent approach. We do this in the framework of Chiral Pole Model (ChPM) based on linear σ model with broken $U_L(3) \otimes U_R(3)$ symmetry [17]. The application of this model allows to calculate the amplitudes of mesonic processes in any order in p^2 without any considerable difficulties. An employment of this model for a description of $K_{\ell 3}$ and $K_{\ell 4}$ decays [18,19] led to the results practically indistinguishable from the ones obtained in the most general Chiral Perturbation Theory (ChPT) approach [20, 21].

The outline of the paper is as follows. In Sec. 2 we present a calculation of

the $K^\pm \rightarrow 3\pi$ and $K \rightarrow 2\pi$ amplitudes in p^2 -approximation neglecting effect of mixing between the isosinglet scalar $\bar{q}q$ state and gluonic scalar state $G_{\mu\nu}^a G_{\mu\nu}^a$.

The aim of such a calculation is to verify that our approach allows to reproduce the results obtained with algebra of currents and soft-pion technics. And using the data on $K \rightarrow 2\pi$ decays we also fix some constants necessary for $K^\pm \rightarrow 3\pi$ description.

Being convinced of self-consistency of our method, in Sec. 3 we take into account the mixing of the scalar $\bar{q}q$ state, with the gluonic one and calculate the explicit (at the tree-level) expressions of the amplitude under consideration.

Section 4 is devoted to the calculation of a difference between the slope parameters g^+ and g^- for $K^+ \rightarrow \pi^+\pi^+\pi^-$ and $K^- \rightarrow \pi^-\pi^-\pi^+$ transitions. These parameters are defined by the expression

$$|M(K^\pm \rightarrow \pi^\pm - \pi^\pm \pi^\mp)|^2 \sim 1 + g^{(\pm)}Y + h^{(\pm)}Y^2 + k^{(\pm)}X^2 \quad (1)$$

where $Y = (s_3 - s_0)/m_\pi^2$; $X = (s_2 - s_1)/m_\pi$; $s_i = (k - p_i)^2$; $s_0 = m_k^2/3 + m_\pi^2$; k and p_i are 4-momenta of K and π_i mesons.

The difference $g^+ - g^-$ is calculated in the p^4 -approximation. It is done for two reasons. First, a calculation is more simple in this approximation and second, our result can be compared with the results of Refs.[13-15] directly. In our calculation we use an expansion of the $K^\pm \rightarrow 3\pi$ amplitude up to terms p^4/Λ^2 and the leading p^2 -approximation for $\pi - \pi$ scattering amplitudes in calculation of the loop diagrams.

Section 5 contains the conclusions.

All formulae necessary for calculations of the amplitudes described by the diagrams of Figs.1-3 are presented in the Appendix.

2. EFFECTIVE $|\Delta S = 1|$ LAGRANGIAN IN LINEAR $U(3)_L \otimes U(3)_R$ σ MODEL

We start from the effective four-quark lagrangian for $|\Delta S = 1|$ transition in the form [22]

$$L(\Delta S = -1) = -\sqrt{2} G_F \sin \theta_C \cos \theta_C \sum_{i=1}^6 c_i \mathcal{O}_i \quad (2)$$

where

$$\begin{aligned} \mathcal{O}_1 &= \bar{s}_L \gamma_\mu d_L \cdot \bar{u}_L \gamma_\mu u_L - \bar{s}_L \gamma_\mu u_L \cdot \bar{u}_L \gamma_\mu d_L, (\{8_f\}), \Delta I = 1/2) \\ \mathcal{O}_2 &= 2\bar{s}_L \gamma_\mu d_L \cdot \left(\sum_{q=u,d,s} \bar{q}_L \gamma_\mu q \right) - \mathcal{O}_1, (\{8_f\}), \Delta I = 1/2) \end{aligned}$$

$$\mathcal{O}_3 = \mathcal{O}_2 - 5\bar{s}_L \gamma_\mu d_L \cdot \bar{s}_L \gamma_\mu s_L, (\{27\}), \Delta I = 1/2)$$

$$\begin{aligned} \mathcal{O}_4 &= \bar{s}_L \gamma_\mu d_L \cdot \bar{u}_L \gamma_\mu u_L + \bar{s}_L \gamma_\mu u_L \cdot \bar{u}_L \gamma_\mu d_L - \\ &\quad - \bar{s}_L \gamma_\mu d_L \cdot \bar{d}_L \gamma_\mu d_L, (\{27\}), \Delta I = 3/2) \end{aligned}$$

$$\mathcal{O}_5 = \bar{s}_L \gamma_\mu \lambda^a d_L \cdot \left(\sum_{q=u,d,s} \bar{q}_R \gamma_\mu \lambda^a q_R \right), (\{8\}), \Delta I = 1/2)$$

$$\mathcal{O}_6 = \bar{s}_L \gamma_\mu d_L \cdot \left(\sum_{q=u,d,s} \bar{q}_R \gamma_\mu q_R \right), (\{8\}), \Delta I = 1/2) \quad (3)$$

Though this set of operators has to be supplemented by operators corresponding to electroweak penguin and box diagrams, it is sufficient for checking of results of Refs.[13-15] and we shall use this set in our calculations.

To solve the problem of presentation of the 4-quark operators \mathcal{O}_i in terms of products of the currents formed by physical scalar and pseudoscalar mesons the linear σ model with broken $U(3)_L \otimes U(3)_R$ symmetry can be used. The lagrangian is of the form [17,18]

$$\begin{aligned} L &= \frac{1}{2} \text{Tr}(\partial_\mu U \partial_\mu U^\dagger) - c \text{Tr}(UU^\dagger - A^2 t_0^2)^2 - c\xi (\text{Tr}(UU^\dagger - A^2 t_0^2))^2 + \\ &\quad + \frac{F\pi}{2\sqrt{2}} \text{Tr}[(U + U^\dagger)M] + \Delta L_{PS}(U_i) \end{aligned} \quad (4)$$

where

$$U = \hat{\sigma} + i\hat{\pi}$$

and $\hat{\sigma}$ and $\hat{\pi}$ are 3×3 matrices of nonets of scalar and pseudoscalar mesons.

The quark currents forming the operators \mathcal{O}_{1-4} can be written in terms of U using the relations

$$\begin{aligned} \bar{q}_k \gamma_\mu q_\ell &= i[\gamma_\mu U \cdot U^\dagger - U \partial_\mu U^\dagger]_{\ell k}^{\text{Vector}} = \\ &= i(\partial_\mu \hat{\pi} \cdot \hat{\pi} - \hat{\pi} \partial_\mu \hat{\pi} + \partial_\mu \hat{\sigma} \cdot \hat{\sigma} - \hat{\sigma} \partial_\mu \hat{\sigma})_{\ell k}, \\ \bar{q}_k \gamma_\mu \gamma_5 q_\ell &= i[\partial_\mu U \cdot U^\dagger - U \partial_\mu U^\dagger]_{\ell k}^{\text{Axial}} = \\ &= (\partial_\mu \hat{\sigma} \cdot \hat{\pi} - \hat{\sigma} \cdot \partial_\mu \hat{\pi} - \partial_\mu \hat{\pi} \cdot \hat{\sigma} + \hat{\pi} \partial_\mu \hat{\sigma})_{\ell k}. \end{aligned} \quad (5)$$

Using the relations [23]

$$\lambda_8^a \lambda_8^b = 2\delta_{ab}^8 \delta_8^c - \frac{2}{3} \delta_8^a \delta_8^b$$

$$\bar{s}_\mu (1 + \gamma_5) d \cdot \bar{q}_\mu (1 - \gamma_5) q = -2\bar{s}(1 - \gamma_5) q \cdot \bar{q}(1 + \gamma_5) d$$

we can present the operator 0_5 in the form

$$0_5 = -\bar{s}(1 - \gamma_5)q \cdot \bar{q}(1 + \gamma_5)d - \frac{1}{6} \bar{s}\gamma_\mu(1 + \gamma_5)d \cdot \bar{q}\gamma_\mu(1 - \gamma_5)q$$

But

$$\bar{q}_k(1 - \gamma_5)q_\ell = \frac{\sqrt{2}F_\pi m_\pi^2}{m_u + m_d} U_{\ell k}, \quad F_\pi \cong 93 \text{ MeV} \quad [24]$$

Then

$$0_5 = -\frac{2F_\pi^2 m_\pi^4}{(m_u + m_d)^2} (U^\dagger U)_{23} + \dots \quad (6)$$

In this formula we have omitted the part which does not contribute to $K^\pm \rightarrow 3\pi$ transition.

The operator 0_5 induces the non-diagonal transitions described by the effective lagrangian

$$L(0_5) = -\frac{2F_\pi^2 m_\pi^4}{(m_u + m_d)^2} \left[K^+ \pi^- - \frac{1}{\sqrt{2}} K^0 \pi^0 + \sigma_{K^0} \sigma_{\eta'} \left(\frac{2}{\sqrt{3}} \cos \theta_S - \frac{\sin \theta_S}{\sqrt{6}} \right) - \sigma_{K^0} \sigma_\eta \left(\frac{2}{\sqrt{3}} \sin \theta_S + \frac{1}{\sqrt{6}} \cos \theta_S + \dots \right) \right] \quad (7)$$

where σ_{K^0} , $\sigma_{\eta'}$ and σ_η are the scalar partners of K^0 , η' and η mesons and θ_S is a mixing angle between σ_0 and σ_8 :

$$\begin{aligned} \sigma_{\eta'} &= \sigma_0 \cos \theta_S + \sigma_8 \sin \theta_S \\ \sigma_\eta &= -\sigma_0 \sin \theta_S + \sigma_8 \cos \theta_S. \end{aligned} \quad (8)$$

In absence of mixing of isosinglet scalar $\bar{q}q$ state with the gluonic state $G_{\mu\nu}^a G_{\mu\nu}^a$ $\theta_S = \arcsin(1/\sqrt{3})$. This case corresponds explicitly to the situation when usual algebra of currents is working. In our approach, it corresponds to $\xi = 0$ in the lagrangian (4), and to verify our technics, it is useful to evaluate the $K^\pm \rightarrow 3\pi$ amplitude putting $\xi = 0$ to be confident that all known results are well reproduced. And only after that we shall take into account the effects of mixing σ_0 with the gluonic state playing an important role in reality.

The transition $K^+ \rightarrow \pi^+ \pi^+ \pi^-$ induced by operator 0_5 is described in our theory by diagrams shown in Fig.1. In the p^2 -approximation corresponding to soft-pion approximation we get

$$\langle \pi^+(p_1) \pi^+(p_2) \pi^-(p_3) | 0_5 | K^+(k) \rangle_{\xi=0} = \beta \cdot (s_1 + s_2 - 2m_\pi^2) \quad (9)$$

$$\text{where} \quad \beta = 2m_\pi^4 / [(m_u + m_d)^2 \Lambda^2], \quad \Lambda^2 = m_{\sigma_\pi}^2 - m_\pi^2 \quad *) \quad (10)$$

*) Though σ_π exchange does not contribute to the amplitude under consideration, a definition of Λ^2 through $m_{\sigma_\pi}^2$ is convenient in view of the mass relations between different scalars (see Appendix).

The result (9) can be written also in the form (see Ref.[24])

$$\langle \pi^+ \pi^+ \pi^- | 0_5 | K^+ \rangle = \frac{i}{2F_\pi} \langle \pi^+ \pi^- | 0_5 | K_1^0 \rangle (m_K^2 - m_\pi^2)^{-1} (s_1 + s_2 - 2m_\pi^2) \quad (11)$$

To calculate the contributions of the operators 0_{1-4} let us note that

$$\begin{aligned} \bar{u}\gamma_\mu u &\Rightarrow i(\partial_\mu \pi^+ \cdot \pi^- - \partial_\mu \pi^- \cdot \pi^+) + \dots \\ \bar{s}\gamma_\mu d &\Rightarrow i(\partial_\mu \pi^- \cdot K^+ - \partial_\mu K^+ \cdot \pi^-) + i\partial_\mu \sigma_{K^0} \langle \sigma_{33} - \sigma_{22} \rangle_0 + \dots \\ \bar{s}\gamma_\mu d &\Rightarrow \frac{i}{\sqrt{2}}(\partial_\mu K^+ \cdot \pi^0 - \partial_\mu \pi^0 \cdot K^+) + \dots \\ \bar{u}\gamma_\mu d &\Rightarrow i\sqrt{2}(\partial_\mu \pi^- \cdot \pi^0 - \partial_\mu \pi^0 \cdot \pi^-) + \dots \\ \bar{u}\gamma_\mu \gamma_5 u &\Rightarrow -F_\pi \partial_\mu \pi^0 + \dots \\ \bar{d}\gamma_\mu \gamma_5 d &\Rightarrow F_\pi \partial_\mu \pi^0 + \dots \\ \bar{s}\gamma_\mu \gamma_5 d &\Rightarrow \pi^- \partial_\mu \sigma_{K^+} - \partial_\mu \pi^- \cdot \sigma_{K^+} + \frac{1}{2}(\partial_\mu \pi^0 \cdot \sigma_{K^0} - \partial_\mu \sigma_{K^0} \pi^0) + \dots \\ \bar{s}\gamma_\mu \gamma_5 u &\Rightarrow -\sqrt{2} F_\pi R \partial_\mu K^+ + K^+ \partial_\mu \left(\frac{2\sqrt{2}\sigma_0 - \sigma_8}{\sqrt{6}} \right) - \partial_\mu K^+ \cdot \left(\frac{2\sqrt{2}\sigma_0 - \sigma_8}{\sqrt{6}} \right) + \\ &\quad + \frac{1}{\sqrt{2}} (\partial_\mu \sigma_{K^+} \pi^0 - \partial_\mu \pi^0 \sigma_{K^+}) + \dots \\ \bar{u}\gamma_\mu \gamma_5 d &\Rightarrow -\sqrt{2} F_\pi \partial_\mu \pi^- + 2[\pi^- \partial_\mu \left(\frac{\sqrt{2}\sigma_0 + \sigma_8}{\sqrt{6}} \right) - \left(\frac{\sqrt{2}\sigma_0 + \sigma_8}{\sqrt{6}} \right) \partial_\mu \pi^-] + \dots \end{aligned} \quad (12)$$

where

$$R = F_K / F_\pi \quad (13)$$

We have omitted in Eq.(12) the parts of the currents which do not contribute into the amplitude under consideration. The currents $\sum_{q=n,d,s} \bar{q}\gamma_\mu(1 \pm \gamma_5)q$ and $\bar{s}\gamma_\mu(1 \pm \gamma_5)s$ do not contribute to our amplitude.

The currents (12) induce the diagrams shown in Fig.2. In the p^2 -approximation they give:

$$\begin{aligned} M(K^+(k) \rightarrow \pi^+(p_1) \pi^+(p_2) \pi^-(p_3))_{\xi=0} &= \frac{1}{2\sqrt{2}} \sin \theta_C \cos \theta_C \cdot \\ &\cdot [(c_1 - c_2 - c_3 - c_4 + 4_\beta c_5)(s_1 + s_2 - 2m_\pi^2) + 9c_4 \cdot (s_0 - s_3)] \end{aligned} \quad (14)$$

$$\begin{aligned} M(K^+(k) \rightarrow \pi^0(p_1) \pi^0(p_2) \pi^+(p_3))_{\xi=0} &= \frac{1}{2\sqrt{2}} \sin \theta_C \cos \theta_C \cdot \\ &\cdot \left[(c_1 - c_2 - c_3 - c_4) + 4_\beta c_5 \right] (s_3 - m_\pi^2) + \frac{9}{2} c_4 (s_0 - s_3) \end{aligned} \quad (15)$$

Using the same technics we can obtain

$$\begin{aligned}
M(K_1^0 \rightarrow \pi^+ \pi^-) &= \frac{1}{\sqrt{2}} G_F \sin \theta_C \cos \theta_C (m_K^2 - m_\pi^2) [(c_1 - c_2 - c_3 + 4\beta c_5) - c_4] \\
M(K_1^0 \rightarrow \pi^0 \pi^0) &= -\frac{1}{\sqrt{2}} G_F \sin \theta_C \cos \theta_C (m_K^2 - m_\pi^2) [(c_1 - c_2 - c_3 + 4\beta c_5) + 2c_4] \\
M(K^+ \rightarrow \pi^+ \pi^0) &= \frac{1}{\sqrt{2}} G_F \sin \theta_C \cos \theta_C (m_K^2 - m_\pi^2) \frac{3c_4}{2}.
\end{aligned} \tag{16}$$

Analysing the question on consistency of Eqs.(16) with the experimental data on $K \rightarrow 2\pi$ decays one has to take into account that due to $\pi - \pi$ scattering in the final state the combination

$$(c_1 - c_2 - c_3 + 4\beta c_5)$$

acquires the phase factor $e^{i(\delta_0 - \delta_2)}$. Then from comparison with the no data $K \rightarrow 2\pi$ decays we obtain

$$\begin{aligned}
c_4 &= 0.328, \quad \delta_0 - \delta_2 \cong 54^\circ \\
c_1 - c_2 - c_3 + 4\beta c_5 &= -10.13
\end{aligned} \tag{17}$$

Combining the relations (14)-(16) we can represent the matrix elements (14) and (15) in the form

$$\begin{aligned}
M(K^+ \rightarrow \pi^+ \pi^+ \pi^-(p_3)) &= \frac{i}{3F_\pi} M(K_1^0 \rightarrow \pi^+ \pi^-) [1 + y + 6\zeta y] \\
M(K^+ \rightarrow \pi^0 \pi^0 \pi^+(p_3)) &= \frac{i}{6F_\pi} M(K_1^0 \rightarrow \pi^+ \pi^-) [1 - 2y + 6\zeta y]
\end{aligned} \tag{18}$$

where $y = 3E_3/m_K - 1$; and $\zeta = -\frac{M(K^+ \rightarrow \pi^+ \pi^0)}{M(K_S^+ \rightarrow \pi^+ \pi^-)} = \frac{3c_4}{2(c_1 - c_2 - c_3 + 4\beta c_5)}$

The results (18) are the well known results of PCAC and current algebra (see, for example, the review [25]).

CP violation connected with the Kobayashi-Maskawa phase δ leads to appearance of $\text{Im } c_5$. So, in the "tree" approximation the matrix elements (14) and (15) contain a small imaginary part which is of *opposite signs* for $K^+ \rightarrow \pi^+ \pi^+ \pi^-$ and $K^- \rightarrow \pi^- \pi^- \pi^+$ decays.

But to lead to observable effects this imaginary part must interfere with the imaginary part arising from $\pi - \pi$ scattering and having *the same signs* for K^+ and K^- decays. Besides, the appearing phase factors must be connected to different dynamical structures in the amplitude. In the p^2 -approximation, as it follows from Eq.(14) a difference in dynamical structure takes place only between $\Delta I = 1/2$ and $\Delta I = 3/2$ parts of the total amplitude. Consequently, after addition of the imaginary part induced by the loop diagrams shown in Fig.3 to the amplitude (14), the CP-violating effects have to be proportional to interference between $\Delta I = 1/2$ and $\Delta I = 3/2$ parts of the amplitude.

The calculation fulfilled in Ref.[12] for the width difference gave the result

$$\left| \frac{\Gamma(K^+ \rightarrow \pi^+ \pi^+ \pi^-) - \Gamma(K^- \rightarrow \pi^- \pi^- \pi^+)}{\Gamma(K^+ \rightarrow \pi^+ \pi^+ \pi^-) + \Gamma(K^- \rightarrow \pi^- \pi^- \pi^+)} \right| \approx 8.4 \cdot 10^{-4} / \beta c_4 |\text{Im } c_5|$$

which would give the number $(0.11 \pm 0.06)10^{-6}$ if $\text{Im } c_5$ is extracted from ε'/ε by the manner used before the appearance of the papers [3-5]. The close number was obtained in Ref.[8].

So, this result and results (18) show that ChPM approach works good enough reproducing all known results of calculation in the p^2 -approximation. But our approach allows to take into account the next order corrections in p^2 and corrections caused by mixing of σ_0 state with the gluonic one. In fact, we need for such corrections to obtain the correct value of the slope parameter g^\pm and probability.

Using the relations (17) and taking

$$c_1 - c_2 - c_3 - c_4 = -3.2 \tag{22}$$

we should get

$$(g^+)^{th} = -0.168, \quad \Gamma(K^+ \rightarrow \pi^+ \pi^+ \pi^-)^{th} = 2.18 \cdot 10^{-9} \text{ eV} \tag{23}$$

instead of experimental values [27]

$$(g^+)^{\text{exp}} = -0.2154 \pm 0.0035 \quad \text{and} \quad \Gamma(K^+ \rightarrow \pi^+ \pi^+ \pi^-)^{\text{exp}} = (2.97 \pm 0.03) \cdot 10^{-9} \text{ eV} \tag{24}$$

And, of course, we need for such corrections to check the results of Refs.[13-15].

3. THE EFFECTS OF THE HIGHER-ORDER CORRECTIONS IN p^2 AND EFFECTS OF MIXING BETWEEN σ_0 AND GLUONIC STATE

As it was found before [19], the correct description of the observed behaviour of the K_{e4} form factors needs for the value

$$\xi = -0.225 \tag{25}$$

of the parameter ξ introducing at the phenomenological level the effects of mixing of the isosinglet scalar $\bar{q}q$ state with the gluonic one. The result (25) was obtained at the value

$$R = F_K/F_\pi = 1.176 \tag{26}$$

following from the identification of the σ_π meson with the resonance $a_0(980)$ [17]. But this value is in accordance also with the calculation of R using the relations

$$F_K/F_\pi = (0.275 \pm 0.002) \left| \frac{V_{ud}}{V_{us}} \right|$$

and

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

and putting into them the values $|V_{ub}| < 0.007$ and $|V_{ud}| = 0.9735 \pm 0.0015$ used in Ref.[26].

The formulae of the Appendix then lead to the following value of the mixing angle θ_S defined by relations (8):

$$\theta_S = 18.3^\circ \quad (27)$$

At these magnitudes of ξ , R and θ_S the diagrams of Fig.1 give

$$\begin{aligned} < \pi^+(p_1)\pi^+(p_2)\pi^-(p_3)|0_S|K^+(k) > = \\ = \beta \Lambda^2 \sum_{j=1,2} (s_j - m_\pi^2) \left(\frac{0.4307}{m_{\sigma_n}^2 - s_j} + \frac{1.0899}{m_{\sigma_n}^2 - s_j} - \frac{1.5206}{m_{\sigma_K}^2 - s_j} \right) \end{aligned} \quad (28)$$

or

$$= \beta \left[0.6273(s_1 + s_2 - 2m_\pi^2) + 2.1363 \Lambda^{-2} \sum_{j=1,2} (s_j - m_\pi^2)^2 + \dots \right] \quad (29)$$

or

$$\cong \beta \cdot \frac{2m_K^2}{3} \left(0.860 - \frac{3m_\pi^2}{2m_K^2} Y \cdot 1.168 + \dots \right) . \quad (30)$$

The diagrams of Fig.2 give

$$\begin{aligned} < \pi^+(p_1)\pi^+(p_2)\pi^-(p_3)|4\bar{s}_L\gamma_\tau d_L \cdot \bar{u}_L\gamma_\tau u_L|K^+(k) > = \\ = 3(s_0 - s_3) = -3m_\pi^2 Y . \end{aligned} \quad (31)$$

$$\begin{aligned} < \pi^+(p_1)\pi^+(p_2)\pi^-(p_3)|4\bar{s}_L\gamma_\tau u_L \cdot \bar{u}_L\gamma_\tau d_L|K^+(k) > = -2m_K^2 + \\ + \Lambda^2 \sum_{j=1,2} (s_j - m_\pi^2) \left(\frac{0.1803}{m_{\sigma_n}^2 - s_j} - \frac{0.6117}{m_{\sigma_n}^2 - s_j} + \frac{1.4668}{m_{\sigma_K}^2 - s_j} \right) \cong \\ \cong \frac{2m_K^2}{3} \left(-1.8087 - \frac{3m_\pi^2}{2m_K^2} Y \cdot 1.3608 \right) . \end{aligned} \quad (32)$$

It follows from the last two formulae that

$$\begin{aligned} < \pi^+(p_1)\pi^+(p_2)\pi^-(p_3)|40_1|K^+(k) > = \\ = \sum_{j=1,2} \{ 1.96(s_j - m_\pi^2) - 1.45(s_j - m_\pi^2)^2 \Lambda^{-2} + \dots \} = \\ = \frac{2m_K^2}{3} \left(1.80 - 1.63 \frac{3m_\pi^2}{2m_K^2} Y + \dots \right) . \end{aligned} \quad (33)$$

Comparing the expressions (29) and (33) we see that the dynamical structure of the ‘‘penguin’’ and ‘‘non-penguin’’ contributions become different when the terms of order p^4/Λ^2

are taken into account. It means, that some contribution to CP violating effects appears from $|A_{1/2}|^2$. But first of all, the correction from the mixing of σ_0 and $G_{\mu\nu}^a G_{\mu\nu}^a$ and higher p^2 corrections improve an agreement between the theoretical and experimental values of the slope parameter and width of $K^+ \rightarrow \pi^+\pi^+\pi^-$ decay.

Instead of Eq.(14) we have now

$$\begin{aligned} M(K^+ \rightarrow \pi^+\pi^+\pi^-) = \frac{1}{2\sqrt{2}} G_F \sin\theta_C \cos\theta_C \cdot \frac{2m_K^2}{3} [1.80(c_1 - c_2 - \\ - c_3 - c_4) + 0.86 \cdot 4_\beta c_5] \left[1 - \right. \\ \left. - \frac{3m_\pi^2}{2m_K^2} Y \frac{1.63(c_1 - c_2 - c_3 - c_4) + 1.168 \cdot 4_\beta c_5 + 9c_4}{1.80(c_1 - c_2 - c_3 - c_4) + 0.86 \cdot 4_\beta c_5} + \dots \right] \end{aligned} \quad (34)$$

where the terms proportional Y^2 and $X^2 = (s_1 - s_2)^2/m_\pi^4$ are omitted. At the value (17) and (22) of the coefficients c_i we obtain

$$(g^+)^{th} = -0.21 \quad \text{and} \quad \Gamma(K^+ \rightarrow \pi^+\pi^+\pi^-)^{th} = 2.9 \cdot 10^{-9} \text{ eV} \quad (35)$$

in agreement with the experimental value (24).

Now, being convinced of the right influence of the mixing of σ_0 with gluonium and correct influence of the higher-order corrections in p^2 we can pass to the evaluation of the effects of CP violation.

4. THE EFFECTS OF CP VIOLATION IN $K^\pm \rightarrow \pi^\pm\pi^\pm\pi^\mp$ DECAYS. THE p^4 -APPROXIMATION

To calculate these effects we must know the imaginary part of the amplitude caused by $\pi - \pi$ re-scattering, that is, to know the integrals corresponding to the loop diagrams shown in Fig.3.

These integrals are

$$\begin{aligned} F_n = \frac{1}{32\pi^2} \int \left\{ \frac{1}{2} f_n^{+-}(\tilde{s}_1, \tilde{s}_2) A^{++}(r_1, r_2; p_1, p_2) \delta^4(r_1 + r_2 - p_1 - p_2) \frac{d^3r_1}{E_{r_1}} \cdot \frac{d^3r_2}{E_{r_2}} \right. \\ \left. + \left[\left(f_n^{+-}(s_1, \tilde{s}_1) A^{+-}(r_1, r_3; p_2, p_3) + \frac{1}{2} f_n^{00+}(s_1) A^{00}(r_1, r_3; p_2, p_3) \right) \cdot \right. \right. \\ \left. \cdot \delta^4(r_1 + r_3 - p_2 - p_3) + \left(f_n^{+-}(s_2, \tilde{s}_1) A^{+-}(r_1, r_3; p_1, p_3) + \right. \right. \\ \left. \left. + \frac{1}{2} f_n^{00+}(s_2) A^{00}(r_1, r_3; p_1, p_3) \right) \delta^4(r_1 + r_3 - p_1 - p_3) \right] \frac{d^3r_1}{E_{r_1}} \cdot \frac{d^3r_3}{E_{r_3}} \left. \right\} , \end{aligned} \quad (36)$$

where $s_i = (k - p_i)^2$, $\bar{s}_i = (k - r_i)^2$ and A^{++} , A^{+-} and A^{00} are the amplitudes of $\pi^+\pi^+ \rightarrow \pi^+\pi^+$, $\pi^+\pi^- \rightarrow \pi^+\pi^-$ and $\pi^0\pi^0 \rightarrow \pi^+\pi^-$ scattering. We can use for these amplitudes the expressions in the lowest p^2 -approximation. f_n^{++} and f_n^{00} are the matrix elements $\langle \pi^+\pi^+\pi^- | 0_n | K^+ \rangle$ and $\langle \pi^0\pi^0\pi^+ | 0_n | K^+ \rangle$ respectively. In the p^4/Λ^2 -approximation

$$\begin{aligned} f_1^{++}(a, b) &= \sum_{z=a, b} [1.96(z - m_\pi^2) - 1.45(z - m_\pi^2)^2/\Lambda^2] \\ f_5^{++}(a, b) &= \sum_{z=a, b} [0.627(z - m_\pi^2) + 2.136(z - m_\pi^2)^2/\Lambda^2]. \end{aligned} \quad (37)$$

The functions $f_{1,5}^{00}(a)$ differ from $f_{1,5}^{++}$ by abolition of summing over the variable b .

$$f_4^{++}(a, b) = 9 \left[-\frac{2}{3} m_k^2 + \sum_{a, b} (z - m_\pi^2) \right] \quad (38)$$

$$\begin{aligned} f_4^{00}(a) &= \frac{9}{2} \left\{ s_0 - a + \frac{1}{3R(2R-1)\Lambda^2} [(k^2 - 2kr_1)^2 + (k^2 - 2kr_3)^2 \right. \\ &\quad \left. - 2(a - m_\pi^2)^2] \right\}. \end{aligned} \quad (39)$$

With the imaginary part induced by $\pi - \pi$ scattering in the final state, the amplitude under consideration has the form

$$\begin{aligned} M(K^+ \rightarrow \pi^+\pi^+\pi^-) &= \frac{G}{2\sqrt{2}} \sin\theta_C \cos\theta_C \left\{ (c_1 - c_2 - c_3 - c_4)(f_1^{++} + i F_1) + \right. \\ &\quad \left. + c_4(f_4^{++} + i F_4) + 4_\beta c_5(f_5^{++} + i F_5) \right\}, \end{aligned} \quad (40)$$

where c_{1-4} , f_n and F_n are the real values. The constant c_5 has small imaginary parts induced by phase δ . Then

$$\begin{aligned} |M(K^+ \rightarrow \pi^+\pi^+\pi^-)|^2 - |M(K^- \rightarrow \pi^-\pi^-\pi^+)|^2 &= 2G_F^2 \sin^2\theta_C \cos^2\theta_C \beta \operatorname{Im} c_5 \cdot \\ &\cdot \{(c_1 - c_2 - c_3 - c_4)(F_1 f_5^{++} - F_5 f_1^{++}) + c_4(F_4 f_5^{++} - F_5 f_4^{++})\}. \end{aligned} \quad (41)$$

As it follows from the formulae (36)-(39), a non-zero value of $(F_1 f_5 - F_5 f_1)$ appears at the order Λ^{-2} :

$$(F_1 f_5^{++} - F_5 f_1^{++})_{\Lambda^{-2}} = \frac{1}{32\pi\Lambda^2 F_\pi^2} \left(\frac{m_K^2}{3} \right)^4 \left(0.32 - 11.9 \frac{3m_\pi^2}{2m_K^2} Y \right) + 0(\Lambda^{-4}). \quad (42)$$

The value $(F_4 f_5^{++} - F_5 f_4^{++})$ differs from zero in the leading Λ^0 -approximation:

$$(F_4 f_5^{++} - F_5 f_4^{++})_{\Lambda^0} = \frac{1}{32\pi F_\pi^2} \left(\frac{m_K^2}{3} \right)^3 35.4 \frac{3m_\pi^2}{2m_K^2} Y + 0(\Lambda^{-2}). \quad (43)$$

Using Eq.(34) we find at the end that for $K^+ \rightarrow \pi^\pm\pi^\pm\pi^\mp$ decays

$$\begin{aligned} \frac{g^+ - g^-}{g^+ + g^-} &= \frac{\beta \operatorname{Im} c_5 m_K^2}{96\pi F_\pi^2} \left[35.4 c_4 - \frac{11.9m_K^2}{3\Lambda^2} (c_1 - c_2 - c_3 - c_4) \right] \cdot \\ &\cdot \{ [1.80(c_1 - c_2 - c_3 - c_4) + 0.86 \cdot 4_\beta c_5] [1.63(c_1 - c_2 - c_3 - c_4) + 1.17 \cdot 4_\beta c_5 + 9c_4] \}^{-1} \cong \\ &\cong 1.9 \cdot 10^{-2} \frac{\operatorname{Im} c_5}{\operatorname{Re} c_5}. \end{aligned} \quad (44)$$

At the value $|\operatorname{Im} c_5|/\operatorname{Re} c_5 = (1.3 \pm 0.7)10^{-3}$ used by Bel'kov *et al.* in Ref.[14] we should get

$$|(g^+ - g^-)/(g^+ + g^-)| = (2.5 \pm 1.3)10^{-5}$$

instead of the value $1.1 \cdot 10^{-3}$ obtained by authors of Ref.[14] themselves. Our result (45) shows that the contribution to CP violating effects from $|A_{1/2}|^2$ does not exceed 30% of the main contribution from $|A_{1/2} A_{3/2}^* + A_{3/2} A_{1/2}^*|$ and this result contradicts to the statement of Ref.[15]. Our final result on value of $g^+ - g^-$ depends on magnitude $\operatorname{Im} c_5/\operatorname{Re} c_5$. At present it would be careless to use the data on ϵ'/ϵ for extracting this value. We can use, however, the estimate

$$\operatorname{Im} c_5/\operatorname{Re} c_5 = -(1 \pm 0.4)s_2 s_3 \sin\delta \quad (45)$$

based on old calculations of Voloshin [28] together with the fact that $\operatorname{Im} c_5$ has a very weak dependence on m_t in the region $50 \leq m_t \leq 250$ GeV [3-5] ^{*)}. Experimentally

$$s_2 s_3 < 2.6 \cdot 10^{-3}. \quad (46)$$

Therefore we conclude that according to our calculations fulfilled in the framework of Chiral Theory the value $g^+ - g^-$ is limited by the relation

$$\frac{g^+ - g^-}{g^+ + g^-} \lesssim (5 \pm 2)10^{-5} |\sin\delta|. \quad (47)$$

5. CONCLUSIONS

Our calculations pursued an object to verify the statement of Ref.[15] that the contribution to CP violating effects originating from the interference of "penguin" and "non-penguin" pieces of $\Delta I = 1/2$ part of the total $K^\pm \rightarrow 3\pi$ amplitude is considerable (by one-two orders) larger than the one caused by interference between $\Delta I = 1/2$ and $\Delta I = 3/2$ parts of the amplitude. We have found that the relative contribution to $g^+ - g^-$ from $|A_{1/2}|^2$ calculated in the p^4/Λ^2 approximation does not exceed 30% of the contribution from $2\operatorname{Re}(A_{1/2} A_{3/2}^*)$ calculated in the lowest p^2 -approximation.

^{*)} Our coefficient c_5 corresponds to c_6 of Refs.[4,5].

Our result (44) for $g^+ - g^-$ depends on magnitude of $\text{Im } c_5/\text{Re } c_5$. At the same value of this ratio as used in Ref.[14] our result is by 50 times smaller than obtained for $g^+ - g^-$ in Ref.[14]. At reasonable estimate (45) for $\text{Im } c_5/\text{Re } c_5$ we have the result

$$\left| \frac{g^+ - g^-}{g^+ + g^-} \right| \lesssim (5 \pm 2)10^{-5} |\sin \delta|$$

which is by twenty times smaller than the value obtained in Ref.[15].

There is no problem in our approach to calculate the $K^+ \rightarrow 3\pi$ amplitude in next orders in p^2 , or take into account the contributions of so-called electroweak penguin and box operators. Such calculations are more cumbersome only. Together with calculation of the width difference $\Gamma(K^+ \rightarrow \pi^+\pi^+\pi^-) - \Gamma(K^- \rightarrow \pi^-\pi^-\pi^+)$ it will be done elsewhere.

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Here we present the formulae for the coupling constants and masses of the scalar mesons necessary for calculation of the amplitudes described by the diagrams shown in Figs.1-3.

$$\begin{aligned} g_{\sigma_\eta, \pi^+\pi^-} &= g_{\sigma_\eta, \pi^0\pi^0} = -\frac{\Lambda^2}{\sqrt{3} F_\pi} \left\{ \sqrt{2}[1 + \xi(2R + 1)] \cos \theta_S + [1 - 4\xi(R - 1)] \sin \theta_S \right\}, \\ g_{\sigma_\eta, \pi^+\pi^-} &= g_{\sigma_\eta, \pi^0\pi^0} = -\frac{\Lambda^2}{\sqrt{3} F_\pi} \left\{ -\sqrt{2}[1 + \xi(2R + 1)] \sin \theta_S + [1 - 4\xi(R - 1)] \cos \theta_S \right\}, \\ g_{\sigma_\eta, K^+K^-} &= g_{\sigma_\eta, K^0\bar{K}^0} = -\frac{\Lambda^2}{\sqrt{3} F_\pi} \left\{ \sqrt{2}[R + \xi(2R + 1)] \cos \theta_S - \right. \\ &\quad \left. - \frac{1}{2}[10R - 9 + 8\xi(R - 1)] \sin \theta_S \right\}, \\ g_{\sigma_\eta, K^+K^-} &= g_{\sigma_\eta, K^0\bar{K}^0} = -\frac{\Lambda^2}{\sqrt{3} F_\pi} \left\{ -\sqrt{2}[R + \xi(2R + 1)] \sin \theta_S \right. \\ &\quad \left. - \frac{1}{2}[10R - 9 + 8\xi(R - 1)] \cos \theta_S \right\}, \\ g_{\sigma_{K^0}, K^+\pi^-} &= g_{\sigma_{K^0}, K^0\pi^-} = \sqrt{2} g_{\sigma_{K^0}, K^+\pi^0} = -\sqrt{2} g_{\sigma_{K^0}, K^0\pi^0} = -\frac{\Lambda^2(2R - 1)}{\sqrt{2} F_\pi}, \end{aligned}$$

where $R = F_K/F_\pi$.

$$\begin{aligned} m_{\sigma_\pi}^2 - m_\pi^2 &\equiv \Lambda^2 = (m_K^2 - m_\pi^2)(R - 1)^{-1}(2R - 1)^{-1}, \\ m_{\sigma_K}^2 - m_\pi^2 &= \Lambda^2(2R - 1)R, \\ m_{\sigma_{\eta'}}^2 - m_\pi^2 &= \Lambda^2 \left\{ 1 + 2R(R - 1)(\cos \theta_S - \sqrt{2} \sin \theta_S)^2 + \right. \\ &\quad \left. + \frac{1}{3} \xi [(2R + 1) \cos \theta_S - 2\sqrt{2}(R - 1) \sin \theta_S]^2 \right\}, \\ m_{\sigma_\eta}^2 - m_\pi^2 &= \Lambda^2 \left\{ 1 + 2R(R - 1)(\sin \theta_S + \sqrt{2} \cos \theta_S)^2 + \right. \\ &\quad \left. + \frac{1}{3} \xi [(2R + 1) \sin \theta_S + 2\sqrt{2}(R - 1) \cos \theta_S]^2 \right\}. \end{aligned}$$

The mixing angle θ_S depends on ξ and R as follows:

$$\theta_S = \frac{1}{2} \arctan \left\{ 2\sqrt{2} \frac{1 + \xi(2R + 1)(3R)^{-1}}{1 - \xi(2R + 1)^2[1 - 8(R - 1)^2(2R + 1)^{-2}][6R(R - 1)]^{-1}} \right\}$$

At $\xi = 0$ $\sin \theta_S = 1/\sqrt{3}$:

At $\xi = -0.225$ and $R = 1.176$ the angle $\theta_S = 18.3^\circ$.

It should be noted that the following equations take place:

$$\begin{aligned} \frac{g_{\sigma_{n'}}^2 \pi \pi}{m_{\sigma_{n'}}^2 - m_\pi^2} + \frac{g_{\sigma_n}^2 \pi \pi}{m_{\sigma_n}^2 - m_\pi^2} &= \frac{\Lambda^2}{F_\pi^2} (1 + 2\xi), \\ \frac{g_{\sigma_{n'}}^2 \pi \pi}{(m_{\sigma_{n'}}^2 - m_\pi^2)^2} + \frac{g_{\sigma_n}^2 \pi \pi}{(m_{\sigma_n}^2 - m_\pi^2)^2} &= \frac{1}{F_\pi^2}, \\ \frac{g_{\sigma_{n'}} K K g_{\sigma_{n'}} \pi \pi}{m_{\sigma_{n'}}^2 - m_\pi^2} + \frac{g_{\sigma_n} K K g_{\sigma_n} \pi \pi}{m_{\sigma_n}^2 - m_\pi^2} + \frac{m_{\sigma_K}^2 - m_K^2}{2F_\pi^2} &= \\ &= -\frac{\Lambda^2}{F_\pi^2} (1 + 2\xi). \end{aligned}$$

Due to these relations an expansion of the $\pi - \pi$ and $K - \pi$ scattering amplitudes begins from the p^2 (or $m_{K,\pi}^2$) terms:

$$\begin{aligned} A(\pi^+(k) \rightarrow \pi^+(p_1)\pi^+(p_2)\pi^-(p_3)) &= \\ &= -\frac{2\Lambda^2}{F_\pi^2} (1 + 2\xi) + \sum_{\substack{\sigma=\sigma_n, \sigma_{n'} \\ i=1,2}} \frac{g_{\sigma}^2 \pi \pi}{m_\sigma^2 - s_i} = \frac{1}{F_\pi^2} (s_1 + s_2 - 2m_\pi^2) + \dots \\ A(\pi^+(k) \rightarrow \pi^0(p_1)\pi^0(p_2)\pi^+(p_3)) &= \\ &= -\frac{\Lambda^2}{F_\pi^2} (1 + 2\xi) + \sum_{\sigma=\sigma_n, \sigma_{n'}} \frac{g_{\sigma}^2 \pi \pi}{m_\sigma^2 - s_3} = \frac{1}{F_\pi^2} (s_3 - m_\pi^2) + \dots \end{aligned}$$

where $s_i = (k - p_i)^2$.

For the $(2\pi 2K)$ amplitude the mixing between σ_0 and gluonium leads to some change of lowest-order expression. Namely,

$$\begin{aligned} A(K^+(k) \rightarrow K^+(p_1)\pi^+(p_2)\pi^-(p_3)) &= -\frac{\Lambda^2}{F_\pi^2} (1 + 2\xi) + \frac{(m_{\sigma_K}^2 - m_K^2)}{2F_\pi^2(m_{\sigma_K}^2 - s_2)} + \\ &+ \frac{g_{\sigma_{n'}} K K g_{\sigma_{n'}} \pi \pi}{m_{\sigma_{n'}}^2 - s_1} + \frac{g_{\sigma_n} K K g_{\sigma_n} \pi \pi}{m_{\sigma_n}^2 - s_1} = \frac{1}{2F_\pi^2} [s_2 - m_\pi^2 + (s_1 - m_\pi^2)\gamma] + \dots \end{aligned}$$

where

$$\gamma = 1 - 2F_\pi^2 \left[g_{\sigma_{n'}} \pi \pi g_{\sigma_{n'}} K K / (m_{\sigma_{n'}}^2 - m_\pi^2)^2 - g_{\sigma_n} \pi \pi g_{\sigma_n} K K / (m_{\sigma_n}^2 - m_\pi^2)^2 \right].$$

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FIGURE CAPTIONS

- Fig.1 Diagrams induced by operators O_5 . Open circles denote the vertices caused by strong interaction.
- Fig.2 Diagrams induced by operators O_{1-4} .
- Fig.3 Loop diagrams taking into account the re-scattering of pions. Shadowed circles denote $K^+ \rightarrow \pi^+(\pi^0)\pi^-(\pi^0)\pi^+$ transition described by the diagram shown in Fig.1 and Fig.2.

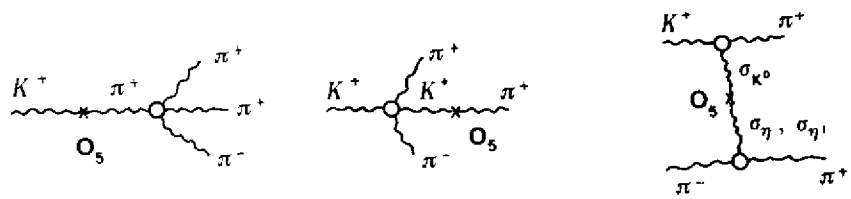


Fig. 1

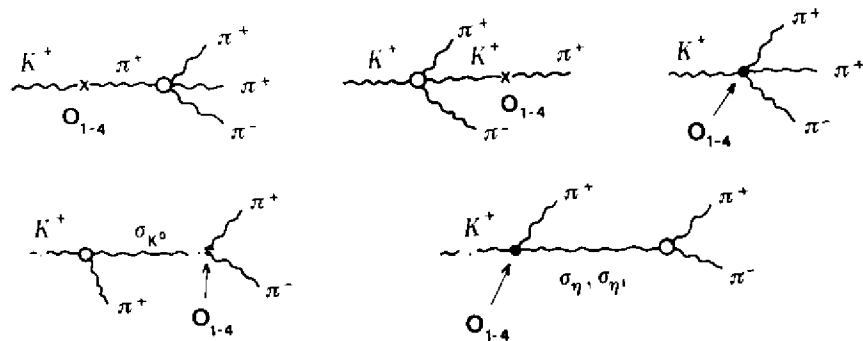


Fig. 2



Fig. 3