LEPTODERMOUS EXPANSION STUDIES IN FINITE NUCLEI

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I. Introduction :-

The term "leptodermous" means having a thin skin. For any system this term is applicable if it is essentially homogenous almost all over the bulk except at the surface region. In the light of this definition one can ask the question: whether a nucleus is leptodermous? In fact this term for the first time had been used by W.J. Swiatecki and his coworkers (1,2) in developing the Liquid Drop (LD) Model and later the Droplet Model (DM) for the masses of nuclei. In addition to this concept one also uses the fact that the total energy of a nucleus can be written as an integral of energy density which again is a function of the nucleon densities. Then it is possible to express the total energy as the sum of a bulk term and a surface layer term. This type of behaviour is the result of saturating and short range nature of the nuclear force. This is the basic concept behind the leptodermous idea in nuclei.

In the present talk we shall briefly discuss these models, their shortcomings and their eventual improvements. In the 2nd part we shall highlight how this leptodermous concept can be used to obtain finite nucleus incompressibility (K_A) as a series whose zeroth order term is the infinite nuclear matter incompressibility (K_∞). Finally we shall discuss how this quantity can be calculated theoretically and whether this expansion is convergent. Also the present status of determination of this quantity will be presented.

II. Mass Formula Based on LDE :-

We have already mentioned earlier that a nucleus can be considered as a leptodermous system. Then the surface region of a finite nucleus deviates from uniform distribution occuring in the bulk region. The width of this diffused region is usually small compared to the characteristic spatial dimension of the nucleus. In the nuclei the ratio of the interparticle spacing to the nuclear radius is of the order of $A^{-1/3}$. In addition to this parameter the asymmetry parameter $I = (N-Z)/A$ should be considered to obtain a LDE of the nuclear mass. Then the various terms in the expansion can be written as follows.

Order in $A^{-1/3} \rightarrow$

$A \quad A^{2/3} \quad A^{1/3} \quad \ldots$

Order in $I^2 \downarrow$ $\quad I^2A \quad I^2A^{2/3} \quad \ldots$

$\quad I^4A \quad \ldots$

The first order terms in this expansion $A, A^{2/3}$ I^2A alongwith the Coulomb term constitute the first mass formula known as Bethe-Weizsacher mass formula. It is well-known that this had a qualitative success only. Myers and Swiatecki later used these terms assuming a nucleus as a incompressible liquid drop for the macroscopic energy term. This is the basis of the famous LD Model and is quite successful to give quantitative agreements. However the predicted value for the nuclear radius parameter ' r_o ' differ with that of from electron scattering analysis by 6-10%.

A further refined version in the Leptodermous expansion (LDE) is the Droplet Model of nuclei, in which higher order terms upto $A^{1/3}$ and the correspondent cross terms in I^2 i.e. $I^2 A^{2/3}$, I^4A are used. In this model nuclei are considered as compressible liquid drops to take care of the compression effects. This results in having two different fluids namely neutron and proten fluids characterized by parameters,

$$\epsilon = -(\rho_c - \rho_\infty)/(3\rho_\infty) \text{ and } \delta = (\rho_{nc} - \rho_{pc})/\rho,$$

where ρ_c corresponds to the central density of the finite nucleus. This model provides a comprehensive description of wide range of nuclear static properties like nuclear shape, size, density etc. Since these are well known, we shall only highlight in the following its shortcomings.

One of the severe criticisms of the DM model is the 'Nuclear Squeezing Effect (3)'. It has been observed that the central density of a finite nucleus in LDE is always squeezed in contrast to the opposite behaviour in HF. This follows from a comparative study of the nuclear radie in both DM and Hartree-Fock (HF) Model with Gogny (4) force. It has been further shown (3,5) that even higher order terms in the LDE cannot explain this nuclear squeezing effect and hence the very convergence of LDE in nuclear mass is questionable.

Further analysis by Treiner, Myers and Swiatecki (5) shows that in addition to LDE terms, one needs, at some stage a term of the type $\exp(-C A^{-1/3})$ to explain this squeezing effect. On the otherhand this exponential term can not generate the LDE terms already present and hence corresponds to the non-leptodermous effect. This non-leptodermous term as expected is only effective in the low mass region. This is the basis of the recent Finite Range Droplet Model (FRDM) (6) for the masses of nuclei.

In concluding the first part of LDE studies on nuclei, we see that models based on LDE is quite successful for most of the static properties of nuclei. However for very small nuclei, these models are likely to show descrepances in the low mass region. Hence non-leptodermous terms should be considered, which in fact is the basis of the recent FRDM (6) model for the masses of nuclei.

III. LDE OF FINITE NUCLEUS INCOMPRESSIBILITY(K_A)

Before going to obtain an expression for K_A, we must discuss its importance. Out of all the nuclear maller constants, K_∞ remains undetermined even till now. Its value is of crucial importance for the very occurance of supernova explosions. Hence it is desirable to get some information on this quantity from laboratory nuclear physics. The most reliable source has been the giant isoscalar monopole resonances, the so-called breathing modes in finite nuclei. These breathing mode energies can be used to extract K_∞ in two ways. In the first approach, RPA calculations of the breathing modes are made in several finite nuclei with various effective forces, each characterized by different values of K_∞. The force that has the best agreement with experiment indirectly determines K_∞. This approach gives values lying in the range 215 to 230 MeV for this quantity K_∞. The second approach (8) is more direct in the sense that the breathing mode energies are expressed directly in terms of K_A as

$$K_A = (m/\hbar^2) \langle r^2 \rangle E^2_{br} \qquad (1)$$

and then adopting the scaling model for the breathing mode energies E_{br}, a LDE of K_A can be obtained, which is given by

$$K_A = K_V + K_{sf} A^{-1/3} + K_{vs} I^2 + K_{coul} Z^2 A^{-4/3} + \ldots \qquad (2)$$

Eqn. (2) when fitted with the breathing mode data, permits in principle to obtain not only K_V (identified as K_∞ in the scaling model) but also other coefficients K_{sf}, K_{vs}.......etc.

Here in the present talk we shall discuss this LDE of K_A in detail and the subsequent calculation of various co-efficients in Semiinfinite Nuclear Matter (SINM) (9) using various density dependent Skyrme forces. This will help us to examine the convergence of the LDE of K_A. Finally we shall conclude regarding the present status of the experimental finding of K_{∞}.

LDE OF K_A :-

The total energy of a nucleus (assumed spherical) can be written as

$$E = e^{\infty}(\rho_c, \delta_c)A + 4\pi R^2 b_0(\rho_c, \delta_c) + 8\pi R\, b_1(\rho_c, \delta_c) + \ldots \quad \ldots \qquad (3)$$

Where $\delta_c = (\rho_{cn} - \rho_{cp})/\rho_c$, e^{∞} is the energy per nucleon in infinite nuclear matter while b_0 and b_1 relate to surface properties as described in SINM(9). The mass number 'A' being given by $\int \rho(\vec{r})\, d^3r$, can be inverted to obtain

$$R = (\rho_0/\rho_c)^{1/3}(r_0 A^{1/3} - c_0 a_0) + O(A^{-1/3}) \qquad (4)$$

Where ρ_0 is the equilibrium density of symmetric infinite nuclear matter (INM). a_0 is the value of $a_p = a_n$, in the generalized Fermi distribution for the densities given by

$$\rho_q(\vec{r}) = \frac{\rho_{cq}}{[1+\exp\{(\vec{r}-\vec{c}_q)/a_q\}]^{\nu_q}} \qquad (5)$$

$$c_0 = \int_{-\infty}^{\infty} dy\, [\{1+\exp(y)\}^{-\nu} + \{1+\exp(-y)\}^{-\nu} - 1] \qquad (6)$$

The volume and surface quantities can be expanded in a series in

$$\epsilon = (\rho_c - \rho_0)/\rho_0 \quad \text{and} \quad \delta_c \text{ as}$$

$$e^{\infty}(\rho_c, \delta_c) = a_v + 1/18\, K_v \epsilon^2 - 1/162\, K' \epsilon^3 + \delta_c^2 (J + 1/3\, L\epsilon + \ ) + \delta_c^4 (M + 1/3\, U\epsilon + \ ) + \ldots$$

$$b_0(\rho_c, \rho_c) = \sigma_0 + 1/3\, C\epsilon + 1/18\, D\epsilon^2 + + \rho_c^2 \{\tau_0 + 1/3\, L\epsilon \quad + \} + \cdots$$

$$b_1(\rho_c, \rho_c) = \mu_0 + 1/3\, X\epsilon + 1/18\, Y\epsilon^2 + \qquad (7)$$

Where $\sigma_0 = \frac{1}{4\pi r_0^2}\, a_{sf}$ and $\tau_0 = \frac{9}{16\pi r_0^2}\, J\rho Q$ are respectively surface energy and surface stiffness coefficients, and μ_0 is the curature energy coefficient given by

$$\mu_0 = \frac{1}{8\pi r_0}\, a_{cv} + c_0 a_0 \sigma_0$$

These three quantities are defined with respect to the equilibrium configuration of SINM and all other quantities are derivatives of these three at limiting density ρ_c.

Finally the LDE in K_A is obtained by using the scaling model of nuclear compressions given by

$$\rho_q(\vec{r}) = \lambda^3 \rho_q(\lambda\vec{r}),$$

where λ is the scaling factor. Then

$$K_A = \left[\frac{d^2 e(\lambda)}{d\lambda^2}\right]_{\lambda=1}$$

$$= 9\bar{\rho}^2\left[\left(\frac{d^2 e}{d\rho_c^2}\right)_{\rho_0} + (\bar{\rho}-\rho_0)\left(\frac{d^3 e}{d\rho_c^3}\right)_{\rho_0} + \frac{1}{2}(\bar{\rho}-\rho_0)^2\left(\frac{d^4 e}{d\rho_c^4}\right)_{\rho_0} + \cdots\right],$$

where $\bar{\rho} = \rho_c^{eq}(A, I)$ is obtained by minimizing eqn.(3) with respect to ϵ and ρ_c for a given value of I and A. Then a tedious algebra leads to the LDE expression (eqn.2) of K_A (details can be found in Ref. 10).

Calculation of Coefficients :-

As mentioned earlier all the incompressibility coefficients of LDE consists of quantities related to INM and SINM properties of nuclei. For this calculation we have used various (9) Skyrme (11) forces. For SINM related quantities we use ETF-Skyrme formalism (Details in Ref.9). This calculation yields all the incompressibility coefficients which are presented in Table 1 for the four Skyrme forces that we have considered. One can easily see from this table that the 2nd order symmetric term $K_4 I^4$ can be totally neglected.

Direct calculation of K_A

Scaling model also helps to directly (12) calculate K_A from the scaling behaviour of the energy density functional in Skyrme-ETF approach, from the definition

$$K_A^S = \frac{1}{A} \frac{\partial^2 (E/A)}{\partial \lambda^2}\bigg|_{\lambda=1} ,$$

where λ is the scaling parameter. Then K_A^S can be easily calculated in this approach. Two different sets of finite-nucleus incompressibility calculations are performed.

i) N=Z nuclei, Coulmb force switched off :-

For such nuclei LDE of K_A reduces to

$$K_A = K_v + K_{sf} A^{-1/3} + K_{cv} A^{-2/3}$$

which enables us to make a test of the convergence in powers of $A^{-1/3}$. The results so obtained for 23 nuclei with $10 \leqslant A \leqslant 6000$ for various Skyrme forces are plotted in Fig.1. The y-axis intercept gives K_{sf} while the slope gives K_{cv}. The agreement of these values with those of SINM calculations is quite satisfactory, which indicates that LDE of K_A is converging rapidly.

Table 1. Coefficients of leptodermous expansion(MeV)

	SkM	RATP	SkA	S_3
K_v	216.6	239.5	263.1	355.4
K_{sf}	-230.9	-260.6	-284.6	-375.4
K_{vs}	-349.0	-338.3	-441.1	-456.0
K_{coul}	-4.70	-4.94	-5.14	-6.07
K_4	38.3	-5.7	105.9	-19.3
K_{ss}	496.8	312.6	874.6	383.4
K_{cv}	-129.0	-140.7	-142.3	-149.2

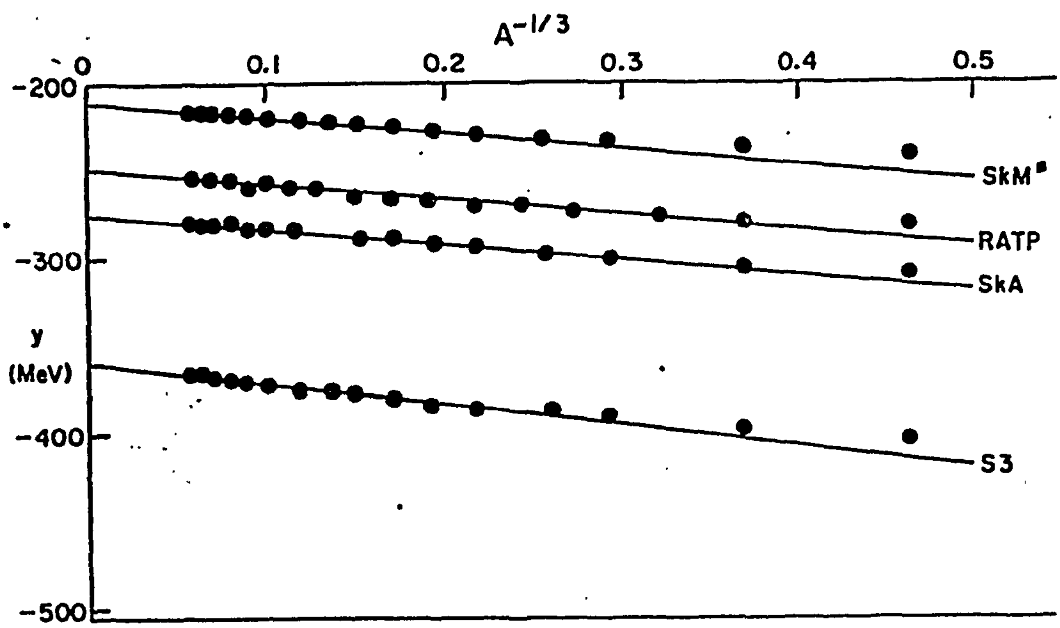


Fig.1

ii) Real Nuclei :-

In order to study the role of surface symmetry term $K_{ss} I^2 A^{-2/3}$ we have also calculated K_A of real spherical nuclei like ^{16}O, ^{40}Ca, ^{48}Ca, ^{56}Ni, ^{90}Zr, ^{112}Sn, ^{132}Sn, ^{140}Ce and ^{208}Pb,

all with the Coulomb force switched on. Then we plot the quantity

$$Z = K_A - K_V - K_{sf} A^{-1/3} - K_{cv} A^{-2/3} - K_{coul} Z^2 A^{-4/3}$$

$$- K_{sym} I^2$$ versus $I^2 A^{-1/3}$ in the Fig. 2.

From the graph we can easily see that surface symmetry term is significant for real nuclei far from stability such as 1 ^{132}Sn. Also we have plotted in Fig. 2 (straight line) $K_{ss} I^2 A^{-1/3}$ with the SINM value of K_{ss}. The agreement is remarkably good.

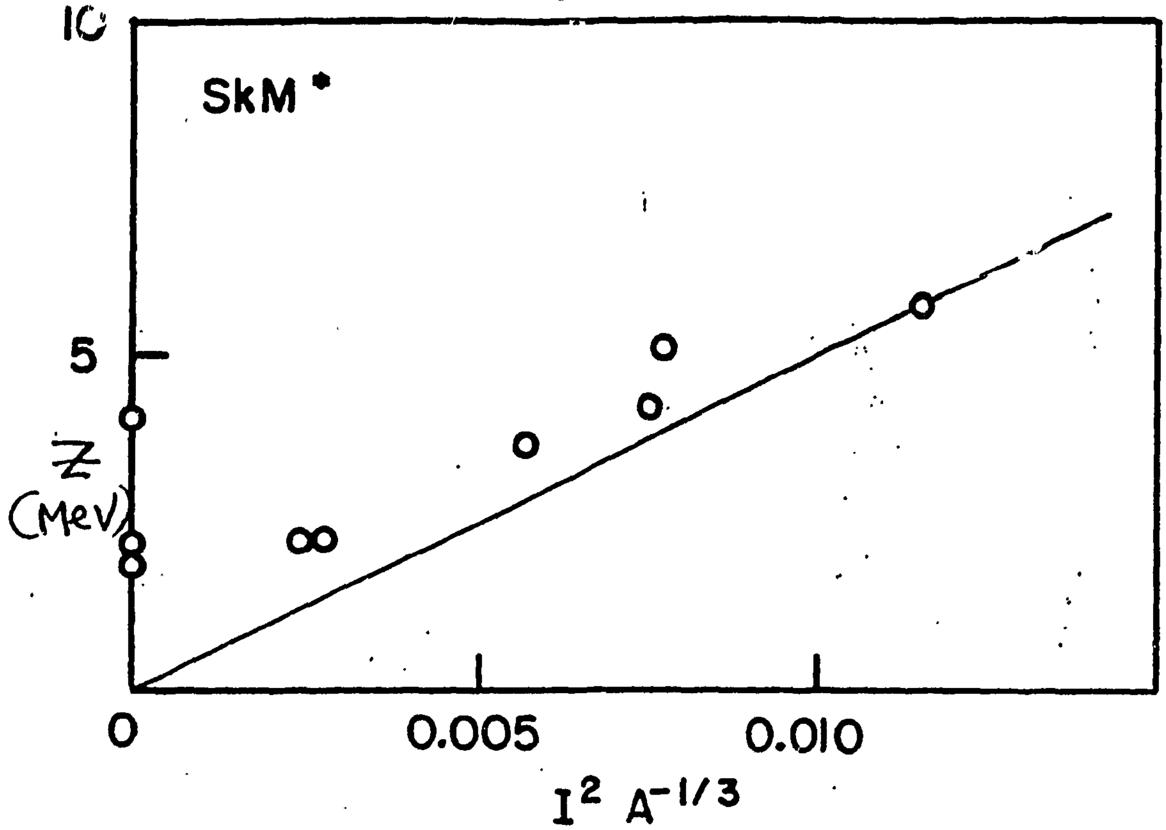
Thus we see that our incompressibility calculations both in the direct scaling model approach and in SINM model shows that LDE of incompressibility is convincingly convergent if the curvature and surface-symmetric terms are taken into account.

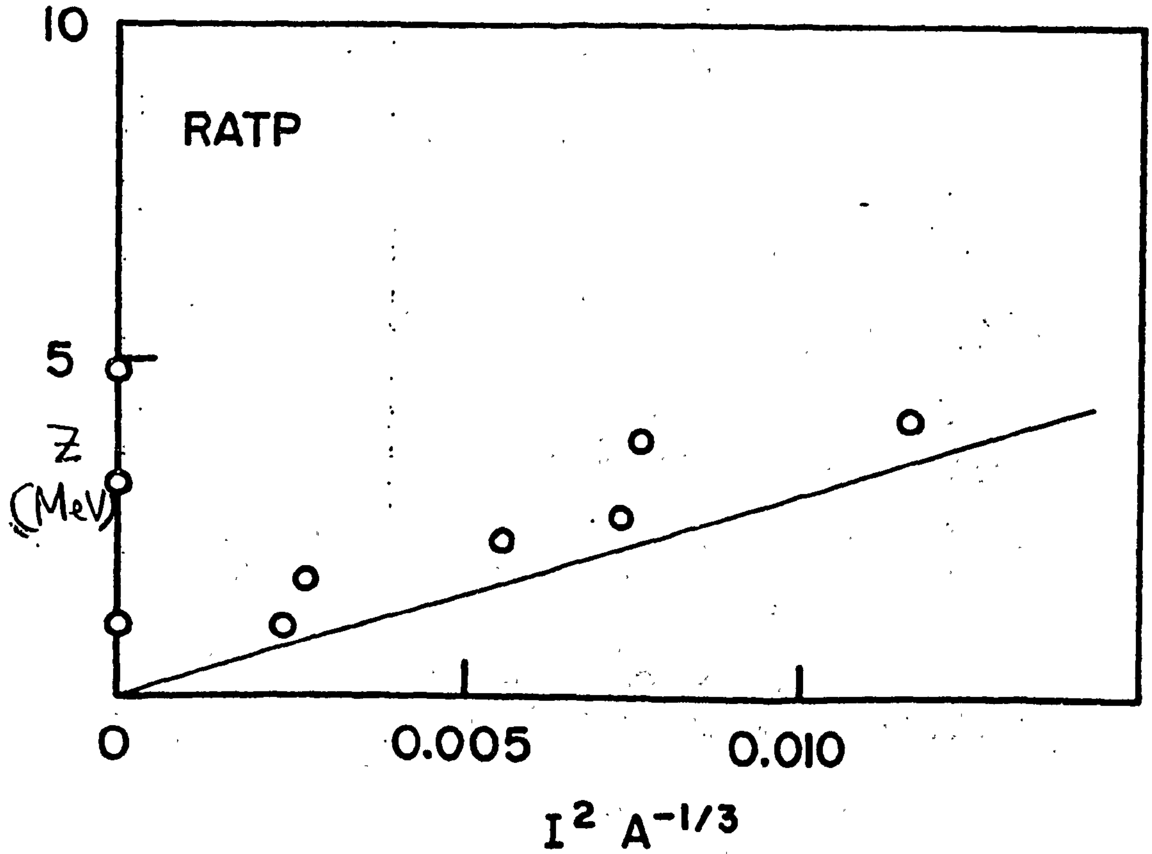
Computation of K_{∞} from Breathing Mode Data

Recently Sharma et al (13) used the LDE of incompressibility in an emperical fitting of the Groningen data (14) to obtain all the coefficients alongwith K_{∞}. They find this value to be 300 ± 25 MeV. However for the fitting procedure to succeed, more data in this regard is desirable.

III. General concluding Remarks :-

In general, we see that nuclei can be farely considered as leptodermous. It is true both


10
SkM*
5
Z (MeV)
0
0.005
0.010
I² A⁻¹/³


10
RATP
5
Z (MeV)
0
0.005
0.010
I² A⁻¹/³

for the static properties of nuclei like mass, density, radius etc. and for dynamic property like incompressibility inspite that nuclei are complex many-body systems. We have already mentioned that the leptodermous feature is a menifestation of the short-range nature of the nuclear force and its saturating property. Of course the models based on this feature have some shortcomings which are predominant only in light-mass nuclei. As far as nuclear mass is concerned, these shortcomings can be more or less rectified with the inclusion of a non-leptodermous exponential term. Regarding incompressibility, its LDE within the framework of scaling model is farely convergent and can be used to obtain the yet-undetermined nuclear matter parameter K_{∞}. Recent fitting procedure of this quantity results in assigning a value equal to 300 ± 25 MeV.

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