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S.M.Sergeev

$Z_N^{\otimes n-1}$ broken model

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Abstract

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Using the ψ-vectors method for the Belavin elliptic R matrix we obtain the A_{n-1} generalization of the *Kashiwara-Miwa model.*

Аннотация

С.М.Сергеев. Двумерная статистическая модель с нарушенной $Z_N^{\otimes n-1}$ инвариантностью: *Препринт ИФВЭ 92-7. — Протвино, 1992. — 4 с., библиогр.: 6.*

Используя метод ψ-векторов для эллиптической R-матрицы Белавина мы получаем A_{n-1} обобщение модели Кашивары-Мивы.

This work is inspired by the $Z_N^{\otimes n-1}$ generalization of the chiral Potts model (the BKMS model) [1]. In the limit $K_{\alpha\beta} = 1$ this model becomes the A_{n-1} generalization of the Fateev– Zamolodchikov model [2]. The latest one admits the elliptic deformation [3]. It is natural to look for an elliptic extension of the limit of BKMS model, using the method, developed in [1], for the elliptic ψ-vectors [4] of the Belavin R-matrix [5].

Hereafter we shall borrow the notations for algebra structures from [4]. Let ε_μ be a basis of R^n. Denote

$$\hat{\mu} = \varepsilon_\mu - \varepsilon, \quad \varepsilon = \sum_{\mu=0}^{n-1} \varepsilon_\mu$$

Let

$$a = \sum_{\mu=0}^{n-1} a^\mu \varepsilon_\mu, \quad a^\mu \in C$$

$$a = a_0 + \zeta, \quad \sum_\mu a_0^\mu = 0, \quad a_0^\mu \in Z/n$$

So a_0 is a weight of A_{n-1}, ζ is arbitrary and fixes the class of a.

For the Belavin R-matrix [5]

$$R(z) = \sum_{\alpha+\beta=\gamma+\delta} R_{(\gamma-\alpha)}^{(\delta-\alpha)} E_{\gamma\alpha} \otimes E_{\delta\beta} \tag{1}$$

where

$$R_{(y)}^{(x)} = \frac{h(z)\theta^{(x-y)}(z+\lambda,\tau)}{\theta^{(x)}(z,\tau)\theta^{(-y)}(\lambda,\tau)}$$

$$h(z) = \frac{\prod_{j=0}^{n-1}\theta^{(j)}(z,\tau)}{\prod_{j=1}^{n-1}(0,\tau)}$$

$$\theta^{(j)}(z,\tau) = \theta\begin{bmatrix} \frac{1}{2} - \frac{j}{n} \\ \frac{1}{2} \end{bmatrix}(z,\tau)$$

there exist the ψ - vectors [4]

$$\psi_a^{(\mu)}(z)_\alpha = \psi_a^{(\mu)}\theta^{(\alpha)}(z - n\lambda a^\mu, \tau) \tag{2}$$

such that the following equation holds [4]:

$$\sum_{\gamma\delta} R_{\alpha\beta}^{\gamma\delta}(z_1 - z_2)\psi_{a-\hat{\mu}}^{(\mu)}(z_1)_\gamma \psi_a^{(\nu)}(z_2)_\delta =$$

$$= \sum_k W\begin{bmatrix} a-\hat{\mu} & a-\hat{\nu}+\hat{k} \\ a & a+\hat{\nu} \end{bmatrix}(z_1 - z_2)\psi_{a-\hat{\mu}+\hat{k}}^{(\mu+\nu-k)}(z_1)_\alpha \psi_{a-\hat{\mu}}^{(k)}(z_2)_\beta \tag{3}$$

Define the inverse vectors $\bar{\psi}_c^{(\mu)}(z)^\alpha$ by the relation:

$$\sum_{\alpha=0}^{n-1} \bar{\psi}_a^{(\mu)}(z)^\alpha \psi_a^{(\nu)}(z)_\alpha = \delta_{\mu\nu} \tag{4}$$

Now one has to distinguish the *classes of face spins a. We shall write (a,ζ)* instead of a.

Following [1] introduce the ϕ - vectors through the relations:

$$\phi_{a,\zeta}^{(\mu)}(z)_\alpha = \psi_{a,\sigma(\zeta)}^{(\mu)}(-n\lambda - z)_\alpha \tag{5}$$

$$\sum_a \bar{\phi}_{a-\hat{\mu},\zeta}^{(\mu)}(z)^\alpha \phi_{a-\hat{\nu},\zeta}^{(\nu)}(z)_\alpha = \delta_{\mu\nu}$$

where constant numbers $\psi_a^{(\mu)}$ have to be chosen so that

$$W\begin{bmatrix} a-\hat{\mu} & a-\hat{\nu}+\hat{k} \\ a & a+\hat{\nu} \end{bmatrix}(z) = W\begin{bmatrix} a-\hat{\mu} & a \\ a-\hat{\nu}+\hat{k} & a+\hat{\nu} \end{bmatrix}(z) \tag{6}$$

Note, that the condition for

$$\mathcal{L}_{\alpha\beta} = \bar{\psi}_{a,\zeta}^{(\mu)}(z)^\alpha \psi_{a,\zeta'}^{(\mu)}(z)_\beta \tag{7}$$

to be the $\mathcal{L}$ – operator for (1) is $\zeta'^\mu = \zeta^\mu + u$.

In the spirit of [1] we define W by the relation

$$W_{\zeta\zeta'}(a,b)\sum_\alpha \psi_{a,\zeta}^{(\mu)}(z)_\alpha \bar{\psi}_{b,\zeta'}^{(\nu)}(z)^\alpha = W_{\zeta\zeta'}(a+\hat{\mu}, b+\hat{\nu})\sum_\alpha \bar{\phi}_{a,\zeta'}^{(\mu)}(z)^\alpha \phi_{b,\zeta}^{(\nu)}(z)_\alpha \tag{8}$$

The other equations for W follow from (8).

The conditions for W to be independent of z read

$$\zeta' = \zeta + u, \quad \sigma(\zeta) = \zeta, \quad \sigma(\zeta') = \zeta - u$$

$$2\lambda \sum_a \zeta^a = t\frac{\tau}{n} + s + \lambda(n-1) \tag{9}$$

We shall choose the simply solution to (9):

$$\zeta^a = \frac{\alpha}{n}, \quad t = s = 0$$

In order to write down the solution to (8) denote

$$f_\lambda(z) = f(\lambda z) \text{ for any } f(z)$$

$$h_\lambda(z) = \theta_\lambda^{(0)}(z, \frac{\tau}{n}) \tag{10}$$

$$D(a) = \prod_{\alpha<\beta} \frac{h_\lambda(a^\alpha - a^\beta)}{h_\lambda(\zeta^\alpha - \zeta^\beta)}$$

The *solution to* (8) *reads*

$$\frac{W(a+\hat{\mu}-\hat{\nu}, b)}{W(a,b)} = \sqrt{\frac{D(a+\hat{\mu}-\hat{\nu})}{D(a)}} \prod_\alpha \frac{h_\lambda(a^\mu - b^\alpha - u)}{h_\lambda(a^\nu - b^\alpha - 1 - u)} \tag{11.a}$$

$$\frac{W(a, b+\hat{\nu}-\hat{\mu})}{W(a,b)} = \sqrt{\frac{D(b+\hat{\nu}-\hat{\mu})}{D(b)}} \prod_\alpha \frac{h_\lambda(b^\mu - a^\alpha + u)}{h_\lambda(b^\nu - a^\alpha + 1 + u)} \tag{11.b}$$

up to the gauge provides the cyclicity in the case

$$\lambda = \frac{p - p'\frac{\tau}{n}}{N}, \quad \text{p-a prime number} \tag{12}$$

The deformation of $K=1$ BKMS model can be obtained when $\lambda = 1/N - \tau/n$. *Note, that cases of complex* λ *can be obtained by modular transformation of real* λ cases, so we choose $\lambda = 1/N$ without loss of generality.

The W-weights (11) obey the unitarity relation

$$\sum_{(l)} \frac{W_u(m,l)W_{-u}(l,k)}{W_u(0,0)W_{-u}(0,0)} = \delta_{mk}\Phi(u)$$

where

$$\Phi(u) = \frac{\theta_\lambda^{(0)}(nNu, N\tau')}{\theta_\lambda^{(0)}(nu, \tau')} \frac{\prod_{k=0}^{n-1} \theta_\lambda^{(0)}(u + \frac{k}{n} - 1, \tau')^k \theta_\lambda^{(0)}(\frac{k}{n} + u, \tau')^{n-k}}{\theta_\lambda^{(0)}(nNu, nN\tau')^n} \tag{13}$$

up to a constant factor, providing $\Phi(0) = 1$, $r' = \frac{r}{n}$. The function Φ is obtained for the subset of integer spins a: $a_0^\mu \in Z$.

As it is shown by R.Kashaev (private communication, see also [6]), the Yang-Baxter equation for factorized S matrix, intertwinnig the $\mathcal{L}$ operators (7), is equivalent to the Star-Star Relation

$$\frac{W_{q-q'}(m_3,m_2)}{W_{q-q'}(m_4,m_1)}\sum_g \frac{W_{q-p}(g,m_1)W_{p-q'}(m_4,g)W_{q'-p'}(g,m_3)}{W_{q-p}(g,m_2)} =$$

$$\frac{W_{p-p'}(m_4,m_3)}{W_{p-p'}(m_1,m_2)}\sum_g \frac{W_{q-p}(m_3,g)W_{p-q'}(g,m_2)W_{q'-p'}(m_1,g)}{W_{q-p}(m_4,g)} \qquad (14)$$

that we have checked numerically together with the Yang-Baxter equation itself and found them true.

The most interesting feature of BKMS model is the equivalence to the generalization of $3-d$ Zamolodzhikov model [7]. Unfortunately the elliptic deformation in sense of $3-d$ model includes interactions among all the spines along $1-d$ chain and seems to be nonlocal.

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С.М.Сергеев

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Редактор А.А.Антипова. Технический редактор Л.П.Тимкина.

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Институт физики высоких энергий, 142284, Протвино
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6 коп.

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