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TO THE PROBLEM OF THE COHERENCE LENGTH OF NEUTRONS

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To the Problem of the Coherence Length of Neutrons

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ABSTRACT

The challenge of the high accuracy of certain optical measurements, the long coherence length of light provokes one to search for possibilities to enlarge the neutron coherence length. A proposal is made to achieve this by using a five or a four plate Bonse-Hart interferometer. A further problem is, whether the neutron beam is composed of wave packets or of overlapping independent monochromatic waves; it is considered that the former is more likely.

Петер Варга: К вопросу о длине когеретности нейтронов
KFKI-1992-30/A.

АННОТАЦИЯ

Высокая точность некоторых оптических измерений, обусловлена болшой длиней когерентности вызывает мысль, нелзя-ли увеличить длину когерентности и нейтронов. К зтой цели предлагается использовать пять- или четырехплатный интерферометер Бонзе-Харта. Рассуждается и проблема из чего состоится нейтронный пучок: из отдельных волновых цугов или из наложения независимых монохроматических волн; анализ поддерживает первый вэгляд.

Varga P.: A neutronok koherenciahosszának kérdéséhez KFKI-1992-30/A

KIVONAT

Egyes optikai kísérletek nagy pontossága, ami a nagy koherenciahosszra vezethető vissza, azt a csábító gondolatot szuggerálja, hogy keressük, milyen módon lehetne a neutronok koherenciahosszát is megnövelni. Javaslatot teszünk, hogy e célra az öt-, illetve a négylemezes Bonse-Hart féle interferométert használják fel. Felvetjük azt a kérdést is, hogy a neutronsugár hullámvonulatok vagy független monochromatikus hullámok szuperpozíciójaként fogható-e fel; az analízis az előbbi javára szól.

I. INTRODUCTION

The interference of light has deliberately been studied for nearly two centuries, whereas the interference of neutrons for only two decades. The triumphal path of light interference was crowned by the new definition of the meter. This can attributable to the excellent work of Michelson even from two reasons: The experiment named after him proved that the speed of light does not depend on the velocity of the inertial frame. Moreover, he began to compare the wavelength of spectral lines with the standard of the meter. This initiative meant, that the wavelength of the atomic time standard could be compared by contemporary means with the ancient standard of the unit of length. The fact that the speed of light could be standardized to an accuracy of nine decimals is due to the large coherence length of the light emitted by atoms.

The path of neutron interferometry is nevertheless triumphal. Experiments proved the power of this method by showing the influence of the gravitational field [1], the rotation of the frame [2], and the motion of the medium [3] on the phase of neutron interference pattern; the 4π rotational invariance of the spinors was demonstrated [4]; loss of the visibility of the interference pattern was observed, when the experimentator wanted to delude the Nature by trying to hit the tail of Schr"odinger's cat [5].

To stress the analogy between the interference of neutrons and light or acoustic waves we refer to [6]. The visibility of neutron interference fringes was measured, it decreased with increasing path difference and vanished after certain delay. By further increasing the delay the visibility recovered . Moreover, the phase of the intereference pattern had a jump of π, when the visibility returned. Fizeau studied the yellow light of sodium in 1863 by interferometer and observed that Newton's rings disappeared at the 500th fringe, but afterwards they turned back and vanished at every 500th fringe. He could follow this up to the 5000th one. Fizeau explained, that this observation was caused by a twin spectral line: it was the discovery of the Na doublet. The dissapearence and the returning of the visibility, observed in [6], was explained in the same way: the output spectrum of the monochromator was analysed, the spectrum showed deviation from Gaussian shape and it could be decomposed with reasonable good fidelity into two shifted Gaussian spectra.

The jump of the phase is the same, which happens when sound is beating: whenever the volume of sound falls to zero, the envelope of the carrier wave changes its sign, which occurs in the phase of the carrier as a jump.

These analogies provoke the idea of finding further interference effects with neutron waves, similar to those of the classical waves.

However, there is a difference between light and neutron interference: in the contrary to that of the light, interference length measured with neutrons is in the order of hundred wavelengths [6, 7] because no line source exists for neutrons, the coherence length is defined by the bandwidth of the device used to produce the spectrum [8].

II. METHODS FOR ENLARGING COHERENCE LENGTH

Spectrum and coherence function are bound by the Wiener-Khintchine theorem, it is obvious that any attempt to increase coherence length will reduce the flow rate of particles because the lengthening is equivalent to cutting off a smaller part of the continuous spectrum offered by the source. Now we analyse some possibilities for increasing the coherence.

1.The simplest way is to enlarge the resolving power of the monochromator by using a better quality crystal.

2.The other way is to select neutrons by the time of flight method. This method enables a relative spectral width of $10^{-3} - 10^{-4}$ to be achieved.

3.In spite of the statement in [9] the dispersion in a medium may help to make the coherence longer. In that article the coherence length of light waves was analysed to see whether it depended on the length of the dispersive medium D. Here, besides the difference between phase and group velocities, the second order dispersion $d^2k/d^2\omega^2$ was taken into account. The authors stated that "...using a simple- minded picture of spreading wave packets, one could easily be misled into thinking that the coherence length will also spread upon propagation in a dispersive medium."

The error was caused by the voluntary change of variables. In eq.(18) of Ref [9] the mutual coherence function is expressed as a function of the length D of the dispersive cell, put into one arm of the interferometer, and of the path difference X due to the phase shifter introduced into the other arm of the interferometer. The change of the variables comes after eq.(18) of [9]: new variables T and $\overline{X}$ are introduced by

$$T = (X + D)/c,$$

$$\overline{X} = D,$$

consequently the length of the dispersion cell disappears. T is called time delay. In Ref.[9] an example is given for a wave packet which has a Gaussian spectrum at the entrance of the interferometer and all the spectralcomponentshave the same initial phase. At this stage the Fourier components of the spectrum have no additive phase, for such a wave packet interference fringes are equidistant, when measured without the dispersion cell. Using the mentioned transformations the authors obtained an expression (25) for the coherence function observable at the output of the interferometer

$$\gamma_{12}(X,T) = \frac{\exp[i\omega_0(X/u_0 - T)]}{[1 - i(\Delta\omega)^2(d^2k/d\omega^2)_{\omega_0}X]^{1/2}} \times \exp\left[-\frac{\frac{1}{2}(\Delta\omega)^2(X/v_0 - T)^2}{[1 - i(\Delta\omega)^2(d^2k/d\omega^2)_{\omega_0}X]}\right],$$

where $\Delta\omega$ stands for the initial line width, u_0 and v_0 are phase and group velocities respectively. Here we have omitted the bar over the X to make the formula identical with the original expression. It seems, it is the path difference X (i.e. the lenghtening of one of the arms of the interferometer) which has an effect on the line width. If we do not use the transformations mentioned, instead of (25) we obtain

$$\gamma_{12}(X,D) =$$
$$\frac{\exp[-i\omega_0[X/c - (1/u_0 - 1/c)D]]}{[1 - i(\Delta\omega)^2(d^2k/d\omega^2)_{\omega_0}D]^{1/2}} \times \exp\left[-\frac{\frac{1}{2}(\Delta\omega)^2[X/c - (1/v_0 - 1/c)D]^2}{1 - i(\Delta\omega)^2(d^2k/d\omega^2)_{\omega_0}D}\right],$$

The shape of the coherence function remains Gaussian but the coherence lenght becomes longer than the initial one. Also the phase of the interference fringes is no longer constant, it shows a slight variation along the interference curve. In reality, the spectrum (a real nonnegative quantity) does not change as a consequence of the dispersion, but each Fourier component gains an additive frequency dependent phase. Nevertheless, the interference can be observed for larger arm-length differences.

4. Now we come to the interferometric method which seems to be possible to implement. The optical analogy of the proposed experiment was perfected in 1961 [10]. The authors demonstrated that a longer coherence than that offered by the light source can be obtained with the aid of an additional interferometer. They put a Fabry-Perot pair into a beam and the output was analysed by a Michelson interferometer (Fig.1). At the input of the Fabry-Perot interferometer the coherence length of the emitted light was less than twice the spacing of the parallel plates.The interfernce

pattern measured with nearly equal arm lengths of the Michelson interferometer vanished when one of the arms was lengthened by an amount which was measured with the same source but without the F-P interferometer. The pattern came back, when the arm-length difference approached the separation of the Fabry-Perot interferometer plates. The effect was repeated for every multiple of the separation. Interference was observed up to the eighth order. The visibility of the interference in the individual orders was reduced in accordance with the exponential law, as the amplitude of the output wave of the F-P interferometer is also reduced expontially after each round-trip. (The same effect can be observed when measuring the longitudinal interference of a laser beam, containing several longitudinal modes.)

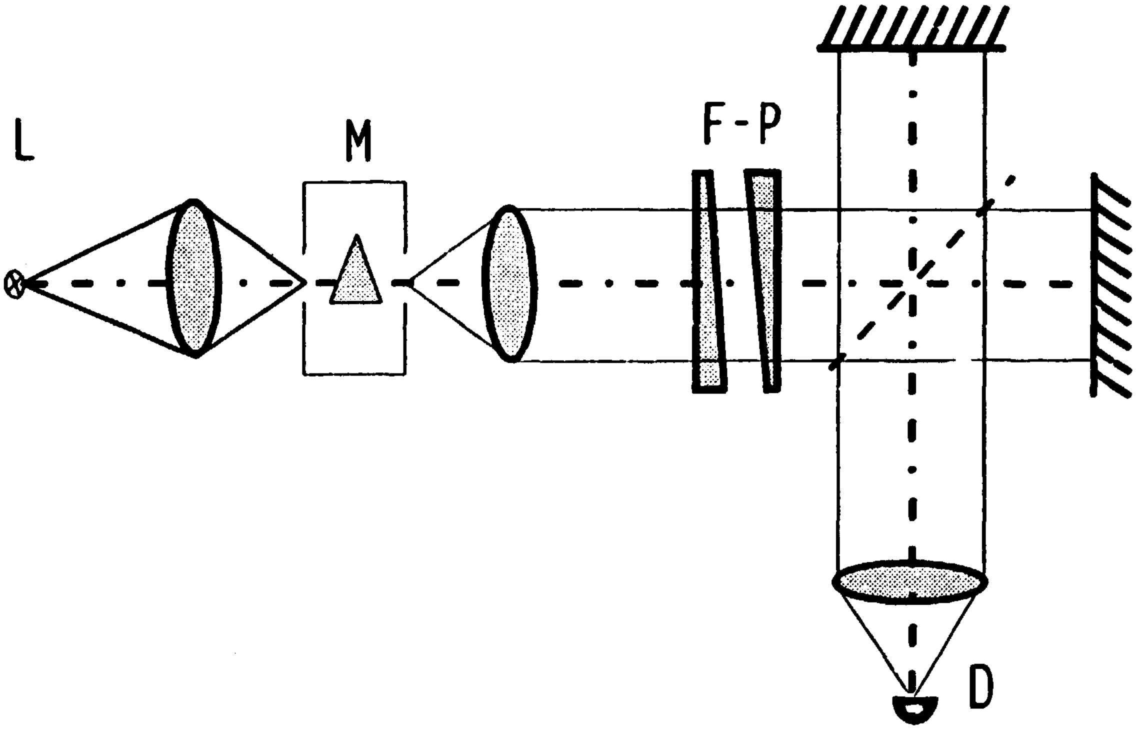

Fig.1. Experimental set up used in [10]. L mercury spectral lamp, M - monochromator, F-P - Fabry-Perot interferometer, D - detector

The effect of the interferometer is principally the same as the effect of a better monochromator: the spectral transfer function of the Fabry-Perot acts like a comb-like filter on the initial spectrum, by cutting off periodic parts of the spectrum.

Though the neutron storage device [11] can in principle be regarded as a Fabry-Perot interferometer, in practice it is virtually impossible to store neutrons without spoiling the phase of the neutron wave. It seems to be realizable to enlarge the coherence length with the aid of a five or a four plate Bonse-Hart device (details of the latter were reported in [12]). Let us take a five plate interferometer (Fig.2) and put two media into the arms of loop *A* between plates *II* and *III*, one with attractive, the other with repulsive pseudopotential. The first speeds up the wave packet, the other slows it down. Consequently, the wave train leaving point X on plate III will consist of two superimposed packets, shifted with respect to each other. In loop D, a third packet with no slowing down/speeding up is added to the twin train, and three superimposed packets reach point Y. If a phase plate is placed between plates IV and V, after mixing at point Y on plate V a pair of trains consisting of three superimposed wave packets interferes, this can be measured at ports 5 and 6. At points Z and Z' one train of three packets is mixed with a single packet, at ports 2, 3, 6 and 7 the interference length is shorter than in the previous case, but longer than with a three plate interferometer.

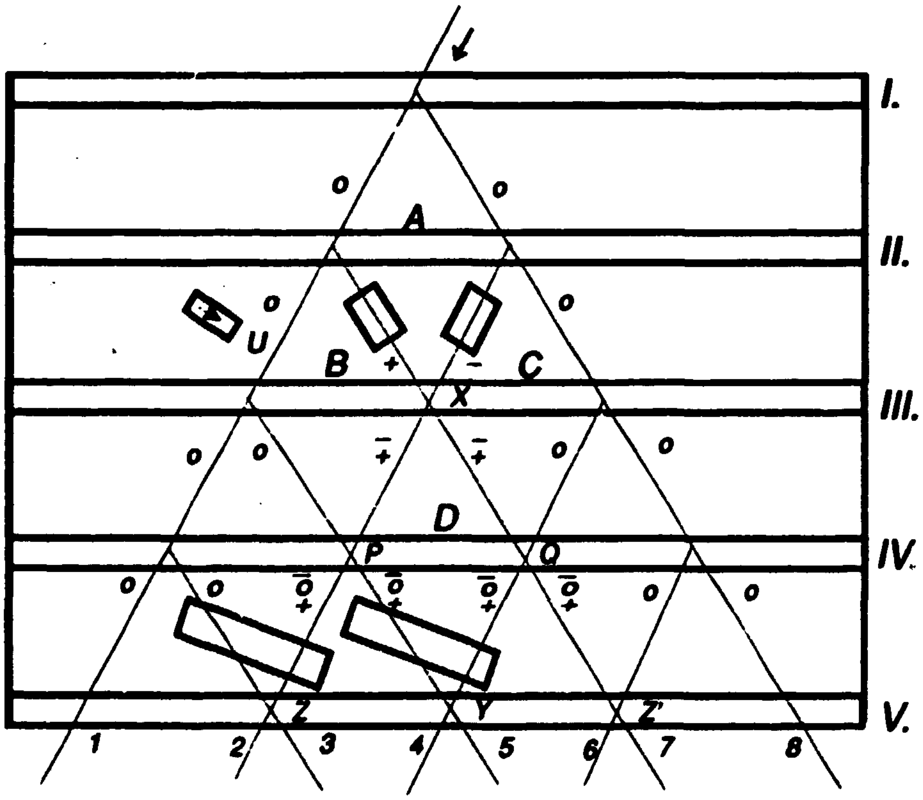

Fig.2. The five-plate neutron interferometer

Let ψ_+, ψ_0 and ψ_- be the wave functions of the neutron with speeding up, with no speeding up/slowing down and with slowing down respectively. The wave function of the train at port 5 is,

$$\psi = \psi_+ + \psi_0 + \psi_- + \overline{\psi}_+ + \overline{\psi}_0 + \overline{\psi}_-,$$

where the bar denotes the phase shift in loop D. Let the speeding up /slowing down introduced in loop A be longer, than the coherence time, but the shift introduced in loop D is less, than the coherence time (i.e. all the packets overlap). The residual density is

$$|\psi|^2 = S + 2\Re(\psi_+\overline{\psi}^*_+ + \psi_0\overline{\psi}^*_0 + \psi_-\overline{\psi}^*_-),$$

where

$$S = |\psi^2_+| + |\psi^2_0| + |\psi^2_-| + |\overline{\psi}^2_+| + |\overline{\psi}^2_0| + |\overline{\psi}^2_-|.$$

If the shift in D is nearly equal the speeding up/slowing down, (only two packets overlap), the density is

$$|\psi|^2 = S + 2\Re[\psi_0\overline{\psi}^*_- + \psi_+\overline{\psi}^*_0].$$

Finally, for the shift of double speeding up/slowing down,

$$|\psi|^2 = S + 2\Re\psi_+\overline{\psi}^*_-.$$

The contrast takes its maximum value only in the first case. In the second order it is not more than 2/3 parth of that of the first; in the third order not more than 1/3. Nevertheless, interference can be observed around five positions of the phase shifter

At ports 2, 3, 4 and 7 a triple train interferes with a single one, the phase shifter has only three positions, around which interference can be observed but the contrast does not differ in various orders - if the absorption of the media is small. If an absorber is put into loop B to cover the path with no slowing down (location U) a triple packet will be combined with a double one at Y on plate V, but the number of orders remains 5 . A four plate interferometer offers similar results: interference between a twin and a single train can be observed behind points P and Q.

III. WAVE PACKETS OR SUPERPOSITION OF INDEPENDENT INFINITE WAVES

In the discussion [8] following the publication of the first coherence experiment [7] was cleared, that last experiment can not distinguish[†] between two models of neutron waves: Does the neutron beam consist of nearly identical wave packets, or it is a simple superposition of infinite sinusoidal waves, where the modulus of the amplitude of the Fourier components is the square root of the spectral density, but the initial phase of each component is independent of the other?

The first case corresponds to a model of electromagnetic wave packets emitted by atoms (the length of the packet is determined by the emission process), the second to the radiation of an incandescent lamp. There is no model for the elementary radiation process of the latter, we have a model for a black body in the equilibrium state only. In this model the spectral components are cavity modes, the energy exchange between modes is perfected by a small soot particle. These modes (Fourier components) can

[†] *Nor can the experiment proposed in the previous section help.*

be regarded as independent. Nor can optical interferometry distinguish between the two ideas, but in optics there is a possibility to measure higher order momenta.

In both cases, however, we have a priori knowledge about the source and about the interactions undergone by the agent under observation. Neutrons, produced somewhere inside the reactor suffer collisions - or at least, they interact with the monochromator crystal that acts as a grating . Every atom of the grating scatters the neutron, the atoms are periodically arranged, the wave reaching the observation point is influenced by this periodicity.

For electromagnetic radiation no dispersion takes place in a vacuum, the periodicity at each point (far enough from the grating or in the focal plane of a lens) is conserved. The finite interference length is defined by the textbook formula for the resolution power of the grating, or in a spectacular way : the coherence length is equal to the path difference of the boundary rays (Fig 3).

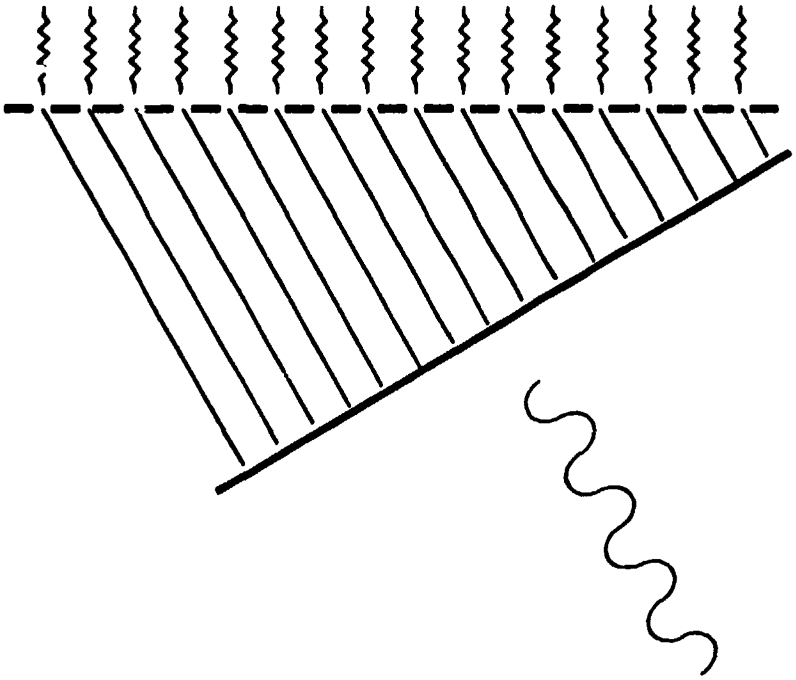

Fig.3. The effect of a periodical structure on the field

If the radiation impinging on the grating has no periodicity, it will be produced by the periodic elements of the grating itself. If there is a periodicity in the primary radiation (spectral line source), intensity is observed only in directions for which the periodicity offered by the grating equals to that which was present in the incident light.

Neutrons, as produced in the fission process and scattered inelastically afterwards, can be taken into analogy with the thermal radiation. The last interaction with the monochromator defines the initial form of the wave function, which obeys on the further path the Schr"odinger equation of free particles. There is no reason to doubt the existence of wave packets, but a real experiment would help to clear up this problem.

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