

Formation and Decay of Toroidal and Bubble Nuclei and the  
Nuclear Equation of State

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ABSTRACT

Multifragmentation, following the formation of toroidal and bubble nuclei, is observed with an improved Boltzmann-Uehling-Uhlenbeck (BUU) model for central  $^{92}\text{Mo} + ^{92}\text{Mo}$  collisions. Guided by our BUU model, we propose two signatures: (1) because of the geometries of bubbles and toroids and because of the *cold* breakup at low temperatures, we predict enhanced cross sections for fragments with *nearly-equal* masses at *small* center-of-mass energies. (2) the coplanarity of these nearly-equal fragments could carry important information concerning the geometry of the sources. This, in turn, could provide information about the stiffness of the equation of state.

1. Introduction

Based on the Boltzmann-Nordheim-Vlasov calculations, Moretto et al. recently observed multifragmentation following the formation of nuclear "disks" for Mo+Mo collisions.<sup>1</sup> They argued that such multifragmentation was due to surface instabilities of the Rayleigh-Taylor kind. On the other hand, subsequent calculations<sup>2,3,4,5</sup> based

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on similar models predicted bubble and/or toroidal geometries, analogous to those studied by C.Y. Wong<sup>4</sup> some time ago.

To investigate the dependence on the equation of state (EOS) and to look for experimental observables, we have performed improved BUU calculations<sup>5</sup> for  $^{92}\text{Mo}+^{92}\text{Mo}$  collisions. In our calculations, we have included Coulomb interactions and have used a Lattice Hamiltonian method<sup>7</sup> to propagate test particles. This method provides a reasonable nuclear surface and accurate energy conservation. For details of this model, see Refs.<sup>5,8</sup>.

## 2. Bubbles and Toroids and the Dependence on the EOS

In the left four columns of Fig. 1, we display, respectively, the top and front views of the BUU calculations for  $^{92}\text{Mo}+^{92}\text{Mo}$  collisions at  $b=0$  and  $E/A = 85$  MeV calculated with the stiff EOS (nuclear compressibility  $K = 375$  MeV). Clearly, due to the early compression and subsequent expansion, a metastable torus is gradually formed with its normal axis parallel to the beam direction. This torus eventually breaks up simultaneously, though slowly, into fragments of nearly-equal sizes with their radii approximately equal to the minor radius<sup>6</sup> of the torus at breakup. In our study,<sup>5</sup> we find that as the incident energy is increased, a torus with a larger aspect ratio<sup>6</sup> is formed which subsequently breaks up into a larger number of fragments. This result is consistent with calculations within the liquid-drop model.<sup>9</sup>

In contrast, in the calculations with the soft EOS (see the right four columns of Fig. 1), a nuclear bubble starts to emerge when the system expands to its maximum at  $t \approx 60$  fm/c. Similar to the formation of a torus, the inner surface of the bubble starts from zero and continues to increase while the outer surface remains relatively unchanged. This bubble stays for  $t \approx 60$  fm/c and then breaks up simultaneously into several fragments (fragments are emitted isotropically).

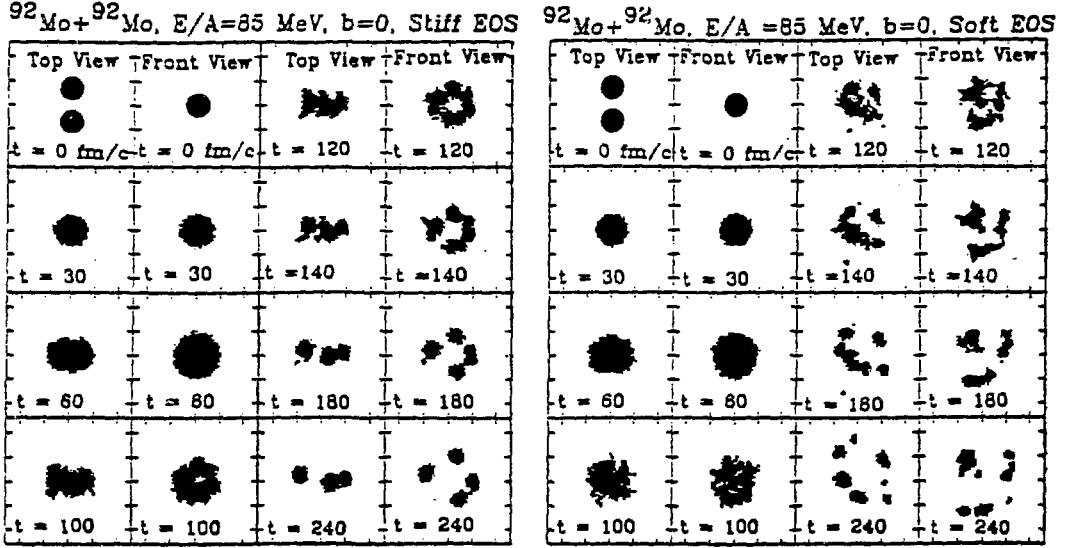


Fig. 1 BUU calculations with the stiff EOS (left four columns) and the soft EOS (right four columns) for  $^{92}\text{Mo}+^{92}\text{Mo}$  collisions at  $E/A=85$  MeV,  $b=0$ . Only areas with densities  $\rho \geq 0.1\rho_0$  are shown. The scales between neighboring ticks are 10 fm.

### 3. Predicted Observables

To guide experimental efforts for searching for the formation of toroidal nuclei, we estimate the kinematics of the final fragments *prior* to the decay of the toroidal nuclei, at  $t = 120$  fm/c, from our numerical BUU simulations,<sup>5</sup> and the results are listed in Table 1. We further estimate various components of excitation energies using techniques outlined in Refs.<sup>5,8,10</sup> and find that the toroidal nuclei formed are quite cold, with thermal excitation energies per nucleon,  $E_{\text{thermal}}/A \approx 1\text{-}2$  MeV, at breakup ( $t \approx 120$  fm/c). Thus the breakup process is a cold breakup process at low temperatures similar to the cold scission of an initially hot system well understood in fission processes. Because of the geometry of the toroids and because of cold breakup at low temperatures, the decay fragments will have approximately similar masses, thus enhancing the cross sections for fragments with nearly-equal sizes at kinematic

Table 1 Multifragment decay from a metastable toroidal nucleus in  $^{92}\text{Mo}+^{92}\text{Mo}$  collisions predicted by our improved BUU model

$E/A$ (MeV)	IMF	multiplicity	$E_k/A$ (MeV)	$E_k$ (MeV)	$\theta_{max}$
75	$^{20}\text{Ne}$	3	1.8	36	$16.7^\circ$
85	$^{12}\text{C}$	4	2.4	29	$19.1^\circ$
100	$^7\text{Li}$	5	2.9	20	$20.9^\circ$

regions discussed below.

Guided by the BUU calculations, we predict a typical case of fragmentation into three  $^{20}\text{Ne}$ -like fragments in a coplanar final state for  $^{92}\text{Mo}+^{92}\text{Mo}$  collisions at  $E/A=75$  MeV, each with kinetic energy per nucleon in the C.M. frame,  $E_k/A \approx 1.8$  MeV, or total energy 36 MeV per  $^{20}\text{Ne}$  fragment. These energies are very small and the fragments will therefore be focused to laboratory angles  $\theta_{lab} \lesssim \theta_{max} \approx 16.7^\circ$ . At higher energies, we predict typical cases of four  $^{12}\text{C}$ -like and five  $^7\text{Li}$ -like fragments, respectively at  $E/A=85$  and 100 MeV. On the average, as the incident energy is increased, a larger number of IMF's with smaller mass per fragment is emitted to larger critical lab angles  $\theta_{max}$ . We note here that the specific values of the multiplicities listed in Table 1 are used for the convenience of estimating the kinematics. In reality, large fluctuations in IMF multiplicities are expected, which is beyond what the BUU model can predict. However, although the kinetic energy may depend on the mass and multiplicity, each fragment should have approximately the same C.M. kinetic energy *per nucleon*, thus these nearly-equal fragments are focused to angles less than  $\theta_{lab} \lesssim 20^\circ$ . We emphasize the specific kinematic regions because, in the systems considered, more than half of the mass and energy is emitted prior to the decay of the bubble and toroidal nuclei.

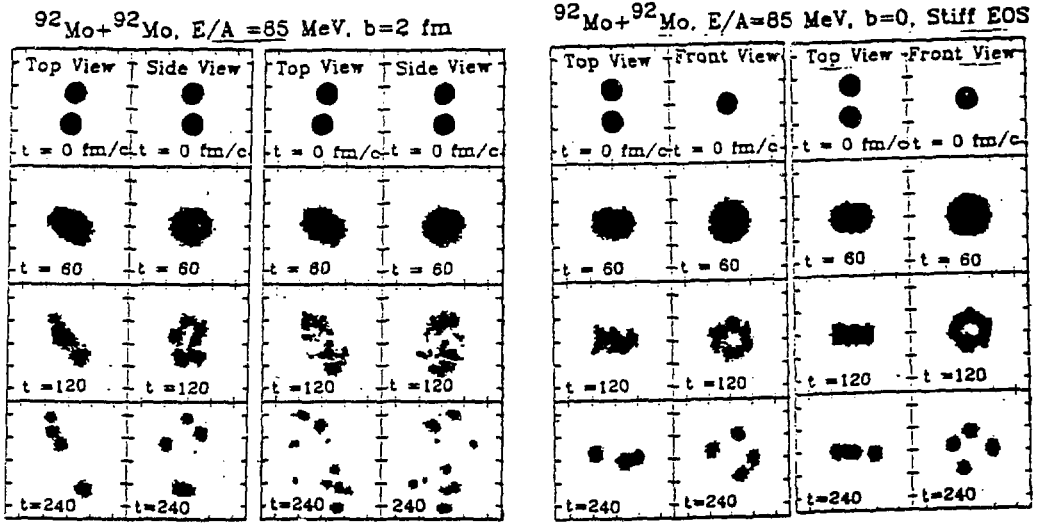


Fig. 2 Dependence on the impact parameters (left 4 columns) and on the number of test particles (right 4 columns). The details of the figure are discussed in the text.

These earlier emissions could make the experimental observations very complicated. Thus the best chance to see the enhancement of nearly-equal fragments is to focus the analysis to the specific kinematical regions where they are produced.

#### 4. Dependence on Impact Parameters

The left four columns of Fig.2 show BUU calculations for the stiff (the two leftmost columns with two views indicated in the figure) and the soft EOS (Columns 3 and 4 from the left), respectively, at impact parameter  $b=2$  fm and  $E/A=85$  MeV. Instead of an emission plane perpendicular to the beam direction, the emission plane (oblate shape when calculated with the stiff EOS) is now rotated by an angle whose value depends on the impact parameter at impact parameters  $b < 3$  fm.<sup>5</sup> For the calculation with the soft EOS, an emission pattern corresponding to a prolate shape is found. Because of the toroidal and bubble geometries and because of low temperatures at freezeout, the fragments emitted in central collisions ( $b < 3$  fm) will be nearly-equal in

size. At large impact parameters,  $b \gtrsim 3$  fm, whether the toroidal or bubble geometry is formed becomes questionable and one observes remnants of the projectile-like and target-like residues. Clearly, these peripheral collisions differ significantly from the central collisions, where the bubble or toroidal geometry is formed and one observes several fragments of nearly-equal sizes with very small C.M. energies.

## 5. Dependence on the Number of Test Particles

The right four columns of Fig. 2 show calculations for two different number of test particles,  $N_{test}=80$  (columns 5 and 6, two views) and  $N_{test}=200$  (columns 7 and 8) for calculations with the stiff EOS at  $E/A=85$  MeV. Similar emission patterns are observed. Further calculations with the soft EOS confirm this result. Thus the event shapes depend little on the number of test particles.

## 6. Conclusions

In summary, with our improved BUU model, we predict multifragmentation following the formation of metastable toroidal and bubble nuclei in  $^{92}\text{Mo}+^{92}\text{Mo}$  collisions. Based on our numerical simulations, we propose the following signatures for detecting the new phenomena: 1) because of the geometries of bubbles and toroids and because of the *cold* breakup at low temperatures, we predict enhanced cross sections for fragments with *nearly-equal* masses at *small* center-of-mass energies. Because of the small C.M. kinetic energies, these nearly-equal fragments are therefore focused to angles  $\theta_{lab} \lesssim 20^\circ$  (Table 1); 2) the coplanarity of the emission pattern for the intermediate mass fragments with nearly-equal masses and C.M. energies at forward kinematics ( $\theta_{lab} \lesssim 20^\circ$ ) may carry important information concerning the geometry of the sources. This, in turn, could provide information about the stiffness of the equation of state. Indeed, after we completed this work, <sup>5</sup> we became aware of a paper

by Bruno *et al.*<sup>11</sup> where they reported a significant enhancement for fragments with nearly-equal masses at kinematical regions similar to our predictions. This could be the first experimental evidence for the formation of bubble or toroidal geometry in heavy-ion collisions.

## 7. Acknowledgement

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