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CP Violation in Higgs Decays

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We study CP violation in fermion pair decays of Higgs boson. We identify some CP odd observables related to the tree level decay amplitude. We find that a few thousand Higgs boson decay events can already provide important information about CP violation. If the Higgs boson is produced, such an analysis could be carried out at the SSC, LHC and NLC.

CP violation was found in the neutral K meson system and can be explained within the Minimal Standard Model, where the source of CP violation is the Kobayashi–Maskawa phase[1]. If this is the only source of CP violation the observed baryon asymmetry in the universe may not be accommodated [2], so additional sources may be needed. The multi-Higgs doublet models, where CP violation can exist in the Higgs sector[3], may provide the additional sources for CP violation necessary to explain the baryon asymmetry of the universe[2]. The possible effects of CP violation from such models have been studied in different processes, e.g. the effects in the production of the $t\bar{t}$ system were studied in [4], the effect in the top quark decay was studied in [5,6], and the effects in the neutral Higgs decay were investigated in [7,8]. In all the above mentioned works the effects of CP violation come from the one loop level.

The motivation for our work is to note that in multi-Higgs doublet models CP violating effects in fermion pair decays of a neutral Higgs boson exist already at the *tree-level* and hence can be very large. In this letter we will study these effects in two cases: a) the Higgs boson is heavy enough to decay into top quark and anti-top quark, b) the Higgs is light and the fermion pair is a τ lepton pair. Before going into the detailed decay channels, we discuss at first some general features in $H \to f\bar{f}$. The most general decay amplitude for this decay is

$$T_{fi} = \bar{f}(a_f + i\gamma_5 b_f)f \, , \tag{1}$$

where a_f and b_f are in general complex numbers. If both a_f and b_f are nonzero, CP is violated. To probe CP violation the polarization of the fermion pair *must* be measured. Since the fermions we will consider here are top quark and τ lepton, the polarization information can be obtained through their decays. One can define a density matrix R for the process $H \to f\bar{f}$, where the $f(\bar{f})$ is polarized and the polarization is described by a unit polarization vector $\mathbf{n}_{f(\bar{f})}$ in the $f(\bar{f})$-rest frame. With the amplitude in eq.(1) the CP violating part of the density matrix is given by,

$$R_{CP} = N_f \beta_f \{\mathrm{Im}(a_f b_f^*) \hat{\mathbf{p}}_{\mathbf{f}} \cdot (\mathbf{n}_{\bar{\mathbf{f}}} - \mathbf{n}_{\mathbf{f}}) - \mathrm{Re}(a_f b_f^*) \hat{\mathbf{p}}_{\mathbf{f}} \cdot (\mathbf{n}_{\mathbf{f}} \times \mathbf{n}_{\bar{\mathbf{f}}})\} \ , \tag{2}$$

where N_f is a normalization constant, and $\hat{\mathbf{p}}_{\mathbf{f}}$ is the three momentum direction of the fermion and $\beta_f = \sqrt{1 - 4m_f^2/M_H^2}$. At the tree-level a_f and b_f are real and $\mathrm{Im}(a_f b_f^*)$ is zero. R_{CP} contains all information about experimental observables. The expectation values of CP odd and CPT odd observables are proportional to $Im(ab^*)$. To study these observables one needs to know the absorptive amplitudes Ima_f and Imb_f which can only be generated at loop levels. Observables of this type has been studied in [7]. To detect possible CP violation at the tree-level one should use CP odd and CPT even observables which are related to $\mathrm{Re}(a_f b_f^*)$. In these observables one naturally expects bigger CP violating effects than in CP odd and CPT odd observables. We now study some of these observables in the two cases mentioned before. We will neglect the imaginary amplitudes of a_f and b_f.

Case a). $H \to t\bar{t}$. As is well known, the top quark is heavy and will then decay quickly before forming hadronic states. This makes it possible to analyze the spin of the top quark[9]. We consider the decays,

$$\begin{aligned} t &\to W^+ b \to \ell^+ \nu b \ , \\ \bar{t} &\to W^- \bar{b} \to \ell^- \bar{\nu} \bar{b} \ . \end{aligned} \tag{3}$$

We use the lepton momenta defined in the W rest frames to construct the CP odd and CPT even observable O_t and the corresponding asymmetry

$$\begin{aligned} O_t &= \hat{\mathbf{p}}_t \cdot (\hat{\mathbf{q}}_+ \times \hat{\mathbf{q}}_-) \ , \\ A_t &= \frac{N(O_t > 0) - N(O_t < 0)}{N(O_t > 0) + N(O_t < 0)} \ , \end{aligned} \tag{4}$$

where $\hat{\mathrm{q}}_+(\hat{\mathrm{q}}_-)$ is the momentum direction of $\ell^+(\ell^-)$ in the $W^{+(-)}$ rest frame. These momenta are related to the corresponding momenta measured in the Higgs rest frame through two Lorenz boosts: one tranforms the top quark into its rest frame, and the other transforms the W boson into its rest frame. It should be pointed out that the top quark and the W rest frames can be reconstructed in experiment even though the neutrinos in eq.(3) escape and the $b(\bar{b})$ quark jet may be not distinguished from the $\bar{b}(b)$ quark in experiment[10]. Using the tree level results from the MSM for the decay matrices of the decays in eq.(3) we obtain,

$$\begin{aligned} &< O_t >= -\frac{4}{9}\beta_t \frac{\alpha_t^2(a_t b_t)}{\beta_t^2 a_t^2 + b_t^2}\,,\quad < O_t^2 >= \frac{2}{9}\,, \\ &A_t = -\frac{9\pi}{16} < O_t >\,, \end{aligned} \tag{5}$$

where $\alpha_t = M_W(2m_t + M_W)/(m_t^2 + 2M_W^2)$ is the polarization parameter for t. We also give in eq.(5) the variance $< O_t^2 >$ of the observable O_t. The statistical error can then be determined by $\delta < O_t > = \sqrt{< O_t^2 > / N_{event}}$, here N_{event} is the number of the available events used to measure the observable (note, $\delta A_t = N_{event}^{-\frac{1}{2}}$). The CP violating effects can be very large, several tens of events can give useful information about $x = a_t/b_t$. For example, if $|x|$ lies from 0.58 to 3.8 for M_H = 400GeV and m_t = 150GeV, then with one hundred available events the absolute value of A_t is already larger than $2 \cdot \delta A_t$.

We have also worked out one observable which is constructed by the lepton momenta q'_+ and q'_- measured in the Higgs rest frame:

$$\begin{aligned} &< O'_t >=< \hat{\mathrm{p}}_t \cdot (\mathrm{q}'_+ \times \mathrm{q}'_-) >= \frac{4\beta_t a_t b_t}{\beta_t^2 a_t^2 + b_t^2}\left(\frac{1}{36z} \cdot \frac{z^2 + 2z + 3}{z + 2}\right)^2\,, \\ &< (O'_t)^2 >= 6M_W^4\left(\frac{z^3 + 2z^2 + 3z + 4}{120z(z + 2)}\right)^2\,. \end{aligned} \tag{6}$$

Here $z = m_t^2/M_W^2$. Comparing the quantities in eq.(4) $< O'_t >$ is less sensitive to x, but, it may be easier to measure than those in eq.(4).

Case b). $H \to \tau^-\tau^+$. In this case we use the following decay mode to analyze the polarization of the τ leptons,

$$\tau^- \to \pi^- \nu_\tau, \quad \tau^+ \to \pi^+ \bar{\nu}_\tau \ , \tag{7}$$

Denoting $\hat{\mathbf{p}}_+(\hat{\mathbf{p}}_-)$ as the moving direction of the $\pi^+(\pi^-)$ in the $\tau^+(\tau^-)$ rest frames, as in case a), we have,

$$\begin{aligned} &< O_\tau >=< \hat{\mathbf{p}}_\tau \cdot (\hat{\mathbf{p}}_+ \times \hat{\mathbf{p}}_-) >= -\frac{4}{9}\beta_\tau \alpha_\tau^2 \frac{(a_\tau b_\tau)}{\beta_\tau^2 a_\tau^2 + b_\tau^2} \ , \ < O_\tau^2 >= \frac{2}{9} \ , \\ &A_\tau = \frac{N(O_\tau > 0) - N(O_\tau < 0)}{N(O_\tau > 0) + N(O_\tau < 0)} = -\frac{9\pi}{16} < O_\tau > \ . \end{aligned} \tag{8}$$

Here $\alpha_\tau = 1$. One can also use the leptonic decay mode of the τ leptons instead of using the decays in eq.(8). In this case $\alpha_\tau = 1/3$.

The momenta of the τ leptons are difficult to measure, so it is also difficult to reconstruct the τ rest frame. It may be possible to overcome this difficulty by constructing CP odd correlations between the momenta measured in the laboratory system and any beam direction if the Higgs boson is produced at some e^+e^- or $p\bar{p}$ colliders.

The above results can also be applied to $H \to \mu^-\mu^+$. In this case the polarization of muon is analysed by its leptonic decay with $\alpha_\mu = 1/3$.

It is clear from the formula for A_f that the asymmetry can be of order one if a_f and b_f are about the same strength. There is experimental constraint for the parameter $a_f b_f$ from the neutron electric dipole moment measurement. However, at the present the constraint is not very strong [11]. The maximal value for $A_{max} = \alpha_i^2 \pi/8$ is not ruled out. The parameters a_f and b_f can receive nonzero contributions from some models at the tree level. Let us consider two Higgs-doublet models. In these models the gauge group is the same as the MSM, i.e., $SU(3)_C \times SU(2)_L \times U(1)_Y$ and has two Higgs representations transforming under $SU(2)_L$ as doublet. It is possible to have CP violation in the Higgs sector in these models, in which there are three physical neutral (H_j mass eigenstates) and one charged physical Higgs bosons. In order to prevent large flavour changing neutral currents at the tree level, some discrete symmetries are imposed to the Yukawa sector and the choice of the discrete symmetry is not unique. Different discrete symmetries result in different Yukawa interaction. A possible Yukawa nteraction Lagrangian is:

$$\begin{aligned}L = (\sqrt{2}G_F)^{1/2}[&\bar{U}_i U_i m_{U_i}(\mathrm{d}_{1j} - \cot\beta \mathrm{d}_{2j}) + i\bar{U}_i\gamma_5 U_i m_{U_i}\cot\beta \mathrm{d}_{3j} \\ &+\bar{D}_i D_i m_{D_i}(\mathrm{d}_{1j} + \tan\beta \mathrm{d}_{2j}) + i\bar{D}_i\gamma_5 D_i m_{D_i}\tan\beta \mathrm{d}_{3j} \\ &+\bar{L}_i L_i(\mathrm{d}_{1j} + \tan\beta \mathrm{d}_{2j}) + i\bar{L}_i\gamma_5 L_i m_{L_i}\tan\beta \mathrm{d}_{3j}]H_j \ ,\end{aligned} \tag{9}$$

where U_i, D_i and L_i are the up-, down-quarks and charged leptons respectively, the sub-indices i runs for different generations and j runs for the three different neutral Higgs particles, d_{ij} are the mixing angles of the Higgs mass matrix, and $\tan\beta$ is the ratio of the vacuum expectation values of the two Higgs doublets. If $\mathrm{d}_{1j}\mathrm{d}_{3j}$ and $\mathrm{d}_{2j}\mathrm{d}_{3j}$ are non-vanishing, CP is violated. We can easily read off the parameters a_f and b_f from the above Lagrangian.

Without loss of generality, let us assume that H_1 is the Higgs boson to be produced and its decays to be analyzed. For $H \to t\bar{t}$ we have ,

$$\begin{aligned}a_t &= -(\sqrt{2}G_F)^{1/2} m_t(\mathrm{d}_{11} - \cot\beta \mathrm{d}_{21}) \ , \\ b_t &= -(\sqrt{2}G_F)^{1/2} m_t \cot\beta \mathrm{d}_{31} \ .\end{aligned} \tag{10}$$

In the region where $M_H > 2m_t$, if one assumes that the MSM prediction is roughly correct for the branching ratios for the Higgs decay, then the main decay modes are $H_1 \to W^+W^-$, ZZ. Nevertheless, $H_1 \to t\bar{t}$ also has a substantial branching ratio, i.e., for $m_t = 150GeV$ and $M_H = 400GeV$, $B_t(H_1 \to t\bar{t})$ is about 0.17. Taking the semi-leptonic branching ratios of the top quark decays into account, CP violation may be observed at 90% confidence level (here we only considered statistic error) with 6,000 Higgs bosons. However, in the model we consider, d_{11} may be very small, and the decay rate for $H_1 \to W^+W^-$, ZZ is proportinal to d_{11}^2. In this case, the branching ratio for $H_1 \to t\bar{t}$ may be large. It is possible to observe CP violation with less than one thousand Higgs bosons.

In the range where $M_H < 2M_W$, the main decay modes is $H \to b\bar{b}$. Because the b quark forms hadrons before it can decay through weak interactions the information about the polarization of the b quark is washed out, thus only the leptonic decay channels can be used for CP test. From eq.(9) we find for the τ leptons:

$$a_\tau = -(\sqrt{2}G_F)^{1/2} m_\tau (d_{11} + \tan\beta d_{21}) ,$$

$$b_\tau = -(\sqrt{2}G_F)^{1/2} m_\tau \tan\beta d_{31} . \tag{11}$$

In this case the branching ratio for Higgs to τ pair is

$$B_\tau(H_1 \to \tau\bar{\tau}) \approx \frac{m_\tau^2}{m_\tau^2 + 3m_b^2} \approx 0.04 . \tag{12}$$

and $B(\tau \to \pi^- \nu_\tau) \approx 11\%$. About 6×10^4 Higgs bosons are required to see the maximal CP violation. However, as pointed out earlier, the Yukawa interaction can be different from those we are discussing, the branching ratio may be large.

In the range $M_H > 2M_Z$, if the MSM prediction for Higgs decay branching ratio is roughly correct, then the branching ratio for $H_1 \to \tau^+\tau^-$ is very small. However, the situation here is similar as discussed for $H_1 \to t\bar{t}$, it is possible in two Higgs doublet models to have larger branching ratio for $H_1 \to \tau^+\tau^-$. This process may still be a good place to test CP invariance in this region.

The above analysis can be carried out for $H_1 \to \mu\bar{\mu}$. Of course the branching ratio for this decay is much smaller, $B(H_1 \to \mu\bar{\mu}) \approx 10^{-4}$. We need about 10^6 Higgs bosons to see CP violation. This is still achievable at SSC.

The above discussion can be easily genaralized to multi-Higgs models.

In conclusion, we have studied CP violation in $H \rightarrow f\bar{f}$. We have proposed the study of CP odd observables to which CP violation at the tree level can contribute. The predictions for the observables are worked out, and two Higgs doublet models are discussed. Less than one thousand Higgs bosons may provide important information about CP violation. The analysis discussed in the above can be easily carried out at the SSC, LHC, and NLC.

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