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**COMPARISON OF THE FACTORIAL  
MOMENTS IN  $p\bar{p}$ ,  $pp$  AND  $K^-p$   
INTERACTIONS AT 32 GEV/C**

Moscow 1992

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COMPARISON OF THE FACTORIAL MOMENTS IN  $\bar{p}p^-$ ,  
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Intermittency effects have been studied in  $\bar{p}p$ ,  $K^-p$  and  $pp$  interactions at 32 GeV/c. The factorial moments are given for the rapidity and azimuthal angle distributions for all charged particles. A comparison was made with the predictions of the quark-gluon string model and with the data of other experiments.

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Introduction.

It was suggested in [1] to study the factorial moments of multiplicity distributions in the small sells of phase space. Factorial moment of the order  $i$  can be defined in the different forms:

$$F_i = \left\langle M^{i-1} \sum \frac{n_k(n_k-1)\dots(n_k-i+1)}{N(N-1)\dots(N-i+1)} \right\rangle_{ev}, \quad (1a)$$

$$F_i = M^{-1} \sum \left\langle \frac{n_k(n_k-1)\dots(n_k-i+1)}{\langle n_k \rangle_{ev}^i} \right\rangle_{ev}, \quad (1b)$$

where  $M$  is the number of sells initial interval  $\Delta$  of some variable (for example, rapidity) is divided on, the size of this sell is  $\delta = \Delta/M$ ,  $n_k$  is the multiplicity in the sell  $k$  and  $N$  is the number of particles within the interval  $\Delta$  in the event. Symbol  $\langle \dots \rangle_{ev}$  means average over all events. The average in expression (1a) is taken over all bins at first and then over whole events sample ("horizontal"), and vice versa in expression (1b) ("vertical" average). Vertical average reflects the particles multiplicity distinctions in the different sells. One also uses another form of horizontal average [2]:

$$F_i = \frac{\left\langle \frac{1}{N} \sum_k n_k(n_k-1)\dots(n_k-i+1) \right\rangle_{ev}}{\left\langle \frac{N}{N} \right\rangle_{ev}^i} \quad (1c)$$

One has showed in [1] that normalized factorial moments take into account the statistical fluctuations appearing due to finite particle multiplicity in the event and coincide with the moments of the density distribution in the interval  $\delta$ . Normalized factorial moments of the density distribution for the case of independent particle production within the interval with the uniform distribution of any variable are constant and equal to 1. In the opposite case moments will vary with decrease of sell size  $\delta$ . The effect of "intermittency" is the increase of moments by the power

law:

$$F_1 = A M^f = A \delta^{-f} \quad (2)$$

The increase of factorial moments with decrease of the cell size  $\delta$  for rapidity ( $\delta y < 1$ ), azimuthal angle ( $\delta\phi < 1$ ), pseudorapidity was observed in a lot of different experiments [2,3].

This article presents the results of factorial moments investigation in  $\bar{p}p$ ,  $K^-p$  and  $pp$  interactions at the energy 32 GeV. Experimental data were obtained with the bubble chamber "Mirabel" exposed with the beam of RF-separated particles at Serpukhov accelerator. Data processing is described in [4, 5]. 240 thousands  $\bar{p}p$  events, and 35 thousands  $pp$  events and 150 thousands  $K^-p$  were written on the summary data tape. Protons with momentum  $P < 1.2$  GeV/c in the laboratory system were identified by means of track ionization, and other charged particles were considered to be pions. Two different samples of events were used in the analysis. The first one is the sample of events with charged particles multiplicities more or equal 4, for comparison with the data obtained by TASSO- collaboration [6]. 28000  $pp$ , 158000  $\bar{p}p$  and 89000  $K^-p$  remained in this case. The other one is the sample with excluding elastic and single diffractive events. Two-prongs event is considered to be elastic if it has not any associated  $V^0$  and if the square of the mass additional to the positive particle is smaller then 1.5 for  $\bar{p}p$  and 0.7 for  $K^-p$  and also the difference between the flying angle of negative particle and the same angle calculated with the measured momentum of the positive particle (considering this collision to be elastic one) is less then 0.3 [7]. The events with multiplicity less or equal 8 in which there exists positive particle with  $x < 0.88$  or (and) negative one with  $x > 0.88$  are considered to be diffractive and are also excluded from the sample. 97000  $K^-p$  and 175000  $\bar{p}p$  are in this sample.

#### Experimental results.

The dependences of the factorial moments logarithms calculated with the expression (1a) (horizontal average) for

the different divisions of rapidity interval  $-1 < y < 1$  in  $\bar{p}p$  and  $pp$  collisions on the logarithm of the cell size  $\delta y$  and comparison with the quark-gluon string Monte Carlo model (MQGS) [10] are shown on the figure 1(a-d). This model describes multiplicity distributions and inclusive spectra in  $\bar{p}p$  and  $pp$  at our energies rather well. The number of generated events is about the same as in the experiment. The first kind of samples is used in this analysis.

Obtained by means of the expression (1a) (horizontal average) moments values increase with the decrease of  $\delta y$  up to the experimental resolution. The results of the experimental data fit by means of expression (2) in the range  $\delta y < 1$  are presented with the solid lines on the figures; the obtained slopes value  $f_1$  are shown in the table 1. It can be seen that results for  $\bar{p}p$  and  $pp$  collisions conformed rather well and MQGS describes slopes, although the factorial moments values in MQGS are smaller than in the experiment. Table 1 also shows slopes of factorial moments in horizontal average (1a) for  $K^-p$  collisions at 32 GeV/c. One can see that slopes values for  $K^-p$  data differ for corresponding ones for  $\bar{p}p$  and  $pp$  interactions. It should be also noted that at our energies we have the empty-bin effect [11] in the  $F_4(\delta y)$  for the  $\bar{p}p$  and morely for  $K^-p$ . This effect is connected with the limited number of particles in each event and with the limited number of events in the experiment, which cause the underestimated experimental values of the high order factorial moments in the small sells of phase space. In the case of  $pp$ - interactions the number of events in the sample does not allow to calculate the slope value even for  $F_4(\delta y)$ . Slopes values obtained in  $\bar{p}p$  collisions correspond to dependency on the moment order [1]

$$f_1 = \alpha i (i - 1), \quad (3)$$

where  $\alpha = 0.0081 \pm 0.0004$  ( $\chi^2/ND = 0.8$ ).

Factorial moments slopes obtained by means of horizontal average in  $e^+e^-$  annihilation into hadrons at  $\sqrt{s} = 35$  GeV [6] are shown in table 1. It can be seen that in  $e^+e^-$  collisions  $f_1$  correspond to the expression (3) and

their values 1.5 time greater than in our experiment ( $\alpha=0.012\pm 0.003$ ).

We can also approximate the dependance  $f_1$  on the order of moments for  $K^-p$ - data ( $\alpha=0.007\pm 0.001$ ) but the result is not very well ( $\chi^2/ND \approx 4$ ).

Factorial moments  $F_i$  of  $\bar{p}p$  and  $K^-p$  interactions for the non-single diffractive sample in the case of normalisations (1b,c) for the particles from the interval  $-1 < y < 1$  and normalized on the value  $F_1$  ( $\delta y=2$ ) are presented on the figure 2(a-d). One can see that  $F_2$  for  $\bar{p}p$ - and  $K^-p$ - data are not different practically. The third order moment in the vertical average for  $\bar{p}p$  interactions become constant when  $\delta y < 0.2$  but at the same time for  $K^-p$  data the increase of the third order moment continues till experimental resolution. This fact can also be seen from the figure 3 which shows the slope value  $f_1(\delta y_{min})$  (by approximation of the factorial moments with formula (2)) dependance on the range of fit  $\delta y_{min} < \delta y < 1$ . This dependance for the third and forth moments orders in the averages 1b,c differ significantly for  $\bar{p}p$  and  $K^-p$  collisions at 32 GeV/c. For  $K^-p$ - interactions  $f_{3,4}(\delta y_{min})$  are constant and don't depend on the range of fit and for  $\bar{p}p$  data  $f_{3,4}(\delta y_{min})$  decrease with decreasing  $\delta y_{min}$ . On this figure we can also see this dependance for the horizontal average 1a. Slope values in this normalisation  $f_1(\delta y_{min})$  both for  $\bar{p}p$  and  $K^-p$  decrease with increasing the range of fit, however  $f_1$  for  $K^-p$  at all  $\delta y_{min}$  increase faster with increasing the order of moment than for  $\bar{p}p$  data.

The discription of moments by expression (2) for all ranges of fit and all moment orders is rather well.  $\chi_1^2/ND < 1$  in  $K^-p$  for all  $\delta y_{min}$  when one calculates  $F_i$  in the averages 1b,c and  $F_{2,3}$  in the normalisation 1a and  $\chi_1^2/ND=1\div 1.5$  for  $F_4$  in the normalisation 1a when  $\delta y_{min} < 0.1$ . As for  $\bar{p}p$ ,  $\chi_1^2/ND=1\div 2.5$  for all moment orders in the horizontal averages 1a,c for all ranges of fit and  $\chi_1^2/ND < 1$  for the 1b in all cases.

Slope values in the case of vertical normalisation (1b) for the particles from the interval  $-1 < y < 1$  resulted from the approximation of eperimental data of  $\bar{p}p$  and  $K^-p$  interactions for the non single diffractive sample by the formula (2) in the range  $0.2 < \delta y < 1$  for  $\bar{p}p$  and in the range  $0.1 < \delta y < 1$  for  $K^-p$

are shown in table 2. For comparison with our experiment data from  $\pi^+p$  and  $K^+p$  experiment at 250 GeV/c [8] and  $\bar{p}p$  experiment at  $\sqrt{s}=630$  GeV [9] are shown in the table 2. Slope values in  $\bar{p}p$ - collisions at the different energies coincide although one can note that at our energies  $F_1$  become constant when  $\delta y < 0.1 \pm 0.2$  and at SPS energies flattening can not be seen. In meson-nucleon interactions slope values differ from  $\bar{p}p$  ones and increase more rapidly with increase of moments order. This distinction in the behavior of the slope value dependance on the range of fit  $f_1(\delta y_{\min})$  can point out to the difference of the intermittency mechanisms in meson-nucleons and nucleon(anti)nucleon interactions.

Dependence of the logarithm of the second order moment value for particles with  $|y| < 1$  on the logarithm of the azimuthal angle  $\phi$  in  $\bar{p}p$ ,  $K^-p$  and  $pp$  (first sample) interactions is presented on the figure 4. Moments decrease in the range  $\delta\phi > 1$  with decreasing  $\delta\phi$  that can be explained by the conservation of the transverse momentum. There is no any increase of moments in the range  $\delta\phi < 1$ . MQGS predictions agree with experimental data in  $\bar{p}p$  and  $pp$  rather well.

#### Conclusions

In  $\bar{p}p$ ,  $K^-p$  and  $pp$  interactions at 32 GeV/c one observes the increase of factorial moments with decreasing rapidity interval size. There is no any increase of moments with decreasing azimuthal angle interval size in the range  $\delta\phi < 1$ . Model of quark-gluon strings describes dependency on  $\delta\phi$  in  $\bar{p}p$ - and  $pp$  well and lie below the experimental momentum values in the rapidity intervals  $\delta y$ , however, moments increase with decrease of  $\delta y$  is in agreement with MQGS predictions.

Slopes dependance on the range of fit for the third and forth moment orders in the normalisations 1b,c differ significantly for  $\bar{p}p$  and  $K^-p$  collisions at 32 GeV/c. For  $K^-p$  interactions slope values in those averages do not depend on the range of fit, in the case of  $\bar{p}p$  collision slope values decrease with increasing the range of fit. For the second order moment there is not any difference between  $K^-p$  and  $\bar{p}p$  collisions. The moments slope values in the horizontal average 1a decrease with increasing the range of fit.



Moments increase parameters coincide within errors in  $\bar{p}p$  interactions at  $\sqrt{s}=630$  GeV and at our energy. In the meson- nucleon interactions slopes increase faster with increasing moment order than in  $\bar{p}p$  interactions.

In  $e^+e^-$  interactions at  $\sqrt{s}=35$  GeV slopes are 1.5 times greater than in  $\bar{p}p$  collisions. Slope dependence on moment order in these interactions is the same.

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Table 1.

Factorial moment  $\delta y$ -dependence slopes in rapidity interval  $-1 \leq y \leq 1$  (horizontal) for  $\bar{p}p$ ,  $K^-p$  and  $pp$  interactions at 32 GeV/c and for predictions of quark-gluon string model for  $\bar{p}p$  collisions, and in rapidity interval  $-2 \leq y \leq 2$  for  $e^+e^-$  interactions at  $\sqrt{s}=35$  GeV. Approximation has been made in  $\delta y$ -interval  $0.1 \leq \delta y \leq 1$ .

| 1 | $\bar{p}p$<br>32GeV/c | MOGS $\bar{p}p$<br>32GeV/c | $pp$<br>32GeV/c | $K^-p$<br>32GeV/c | $e^+e^-$<br>$\sqrt{s}=35$ GeV |
|---|-----------------------|----------------------------|-----------------|-------------------|-------------------------------|
| 2 | 0.017±<br>0.001       | 0.015±<br>0.001            | 0.015±<br>0.003 | 0.011±<br>0.002   | 0.023±<br>0.003               |
| 3 | 0.046±<br>0.004       | 0.043±<br>0.004            | 0.063±<br>0.014 | 0.057±<br>0.006   | 0.080±<br>0.014               |
| 4 | 0.088±<br>0.012       | 0.060±<br>0.010            |                 | 0.084±<br>0.014   | 0.134±<br>0.052               |

Table 2.

Factorial moment  $\delta y$ -dependence slopes (vertical) in rapidity interval  $-1 \leq y \leq 1$  for  $\bar{p}p$  and  $K^-p$  interactions at 32 GeV/c, and in rapidity interval  $-2 \leq y \leq 2$  for  $\pi^+p$ ,  $K^+p$  at 250 GeV/c,  $\bar{p}p$  with  $\sqrt{s}=630$  GeV. Approximation for  $\bar{p}p$  32 has been made in the interval  $0.2 \leq \delta y \leq 1$  and for other data in the interval  $0.1 \leq \delta y \leq 1$ .

| 1 | $\bar{p}p$ , 32 | $K^-p$ , 32     | $\pi^+p, K^+p$<br>250 | $pp, \sqrt{s}=630$ |
|---|-----------------|-----------------|-----------------------|--------------------|
| 2 | 0.013±<br>0.001 | 0.008±<br>0.002 | 0.008±<br>0.001       | 0.012±<br>0.001    |
| 3 | 0.027±<br>0.002 | 0.034±<br>0.005 | 0.045±<br>0.002       | 0.028±<br>0.002    |
| 4 | 0.045±<br>0.006 | 0.10±<br>0.02   | 0.170±<br>0.010       | 0.049±<br>0.005    |

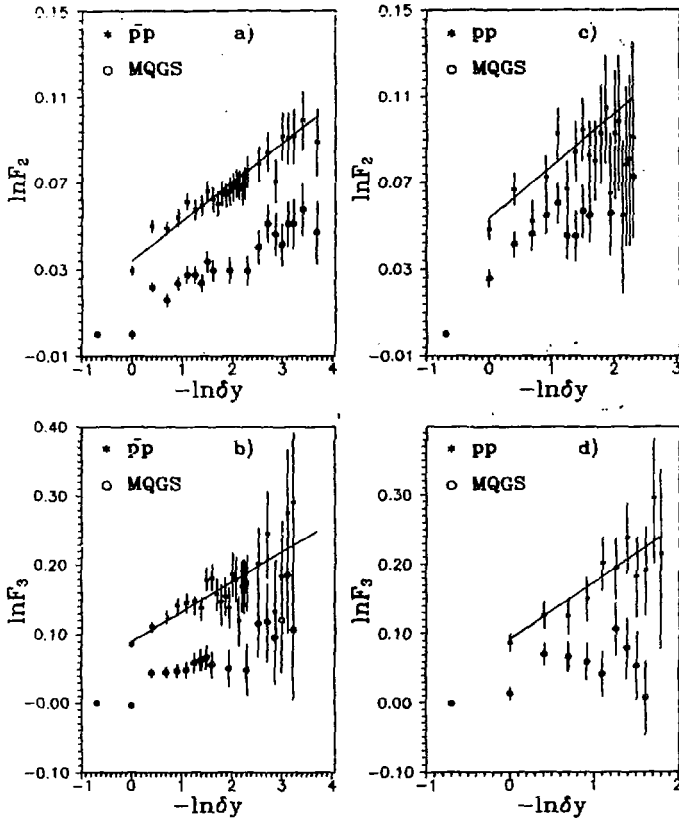


Fig 1. Factorial moments of 2<sup>nd</sup> and 3<sup>rd</sup> orders in  $\bar{p}p$ (a,b) and  $pp$  (c,d) collisions at 32 GeV/c calculated by using formula (1a) for particles from the interval  $-1 \leq y \leq 1$ . Solid line shows the result of approximation of the experimental data by means of formula (2). First kind of samples.

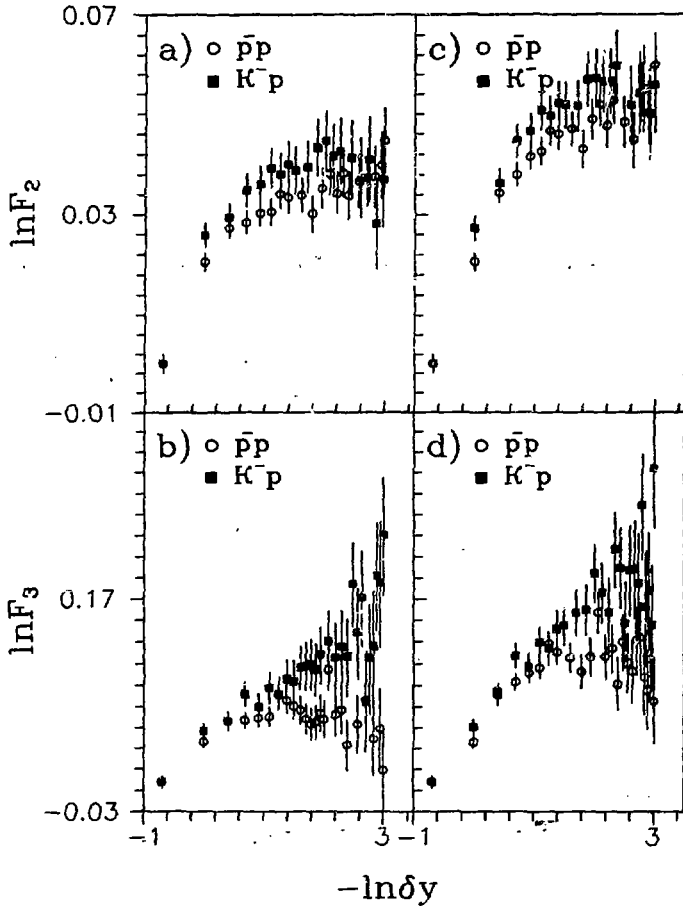


Fig 2. Factorial moments of 2<sup>nd</sup> and 3<sup>rd</sup> orders in non-single diffractive sample of  $\bar{p}p$  and  $K^-p$  collisions at 32 GeV/c calculated by using formula (1b) fig.(a,b) and formula (1c) fig.(c,d) for particles from the interval  $-1 \leq y \leq 1$ . (Moments normalised on the value  $F_1(\delta y=2)$ .)

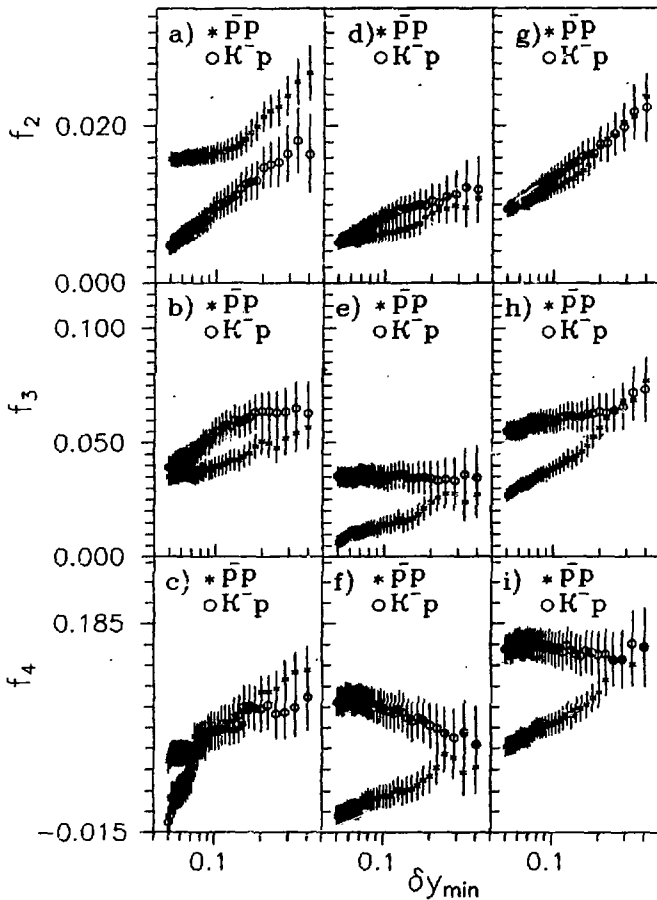


Fig 3. Slopes of 2<sup>nd</sup>, 3<sup>rd</sup> and 4<sup>th</sup> order in non-single diffractive sample of  $\bar{p}p$  and  $K^-p$  collisions at 32 GeV/c for factorial moments calculated by means of formula (1a) fig.a-c, formula (1b) fig.d-f and formula (1c) fig.g-i in interval  $-1 \leq y \leq 1$ .

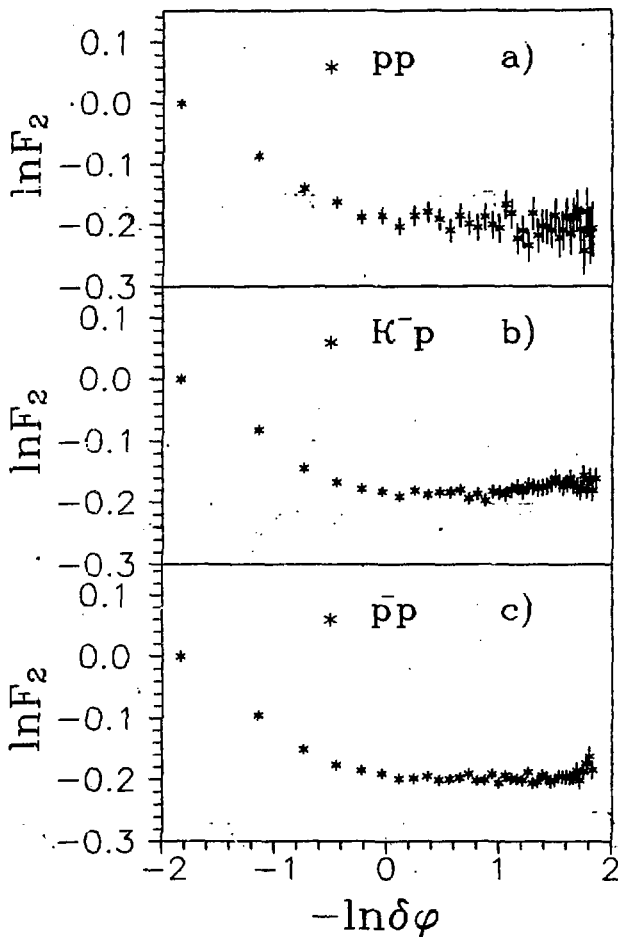


Fig 4. Factorial moments of 2<sup>nd</sup> order (formula 1a) for the particles from interval  $-1 \leq y \leq 1$  when azimuthal angle interval  $0 \leq \phi \leq 2\pi$  is divided on the cells. Sample of events with  $n_{ch} \geq 4$  (the first sample) for  $\bar{p}p, pp$  and  $K^-p$  data is used.

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Сравнение факториальных моментов в  $\bar{p}p$ -,  $pp$ - и  $K^+p$ -...  
взаимодействиях при 32 ГэВ/с.

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