INSTITUTE OF PLASMA PHYSICS
CZECHOSLOVAK ACADEMY OF SCIENCES

RADIAL DIFFUSION OF TOROIDALLY TRAPPED PARTICLES INDUCED BY LOWER HYBRID AND FAST WAVES

L. Krlín

IPPCZ - 327

October 1992

RESEARCH REPORT

POD VODÁRENSKOU VĚŽÍ 4, 180 69 PRAGUE 8
CZECHOSLOVAKIA

RADIAL DIFFUSION OF TOROIDALLY TRAPPED PARTICLES INDUCED BY LOWER HYBRID AND FAST WAVES

L. KRLÍN
Institute of Plasma Physics, Czech. Acad. Sci., P.O. Box 17, Prague 8, Czechoslovakia

Abstract

The interaction of RF field with toroidally trapped particles (bananas) can cause their intrinsic stochasticity diffusion both in the configuration and velocity space [1,2,3]. In RF heating and/or current drive regimes, RF field can interact with plasma particles and with thermonuclear alpha particles. The aim of this contribution is to give some analytical estimates of induced radial diffusion of alphas and of ions.

1 Canonical apparatus

For the study of RF field-banana interaction, the Hamiltonian approach is very suitable. We use the system of canonically conjugated coordinates Q_i and momenta P_i, derived and discussed in [4,5,2]. Here, Q_1 is the angle of the cyclotron rotation, and Q_2 and Q_3 are the poloidal and toroidal angles, respectively. Further, $P_1 = \frac{Mv_\perp^2}{2\omega_c}$, $P_2 = \frac{r^2 eB_T}{2}$, P_3 is determined as $1/2Mv_\parallel^2 = \frac{(P_3 - eRA_\varphi)^2}{2MR^2}$, $R = R_0 + r\cos Q_2$. R_0, a are the major and minor radii, respectively, r is the radial position of the guiding centre of a particle, A_φ is the toroidal component of the vector potential A, ω_c is the cyclotron frequency and B_T is the toroidal magnetic field. M is the mass and e is the charge of a particle, respectively.The corresponding Hamiltonian H_0 of a particle in the toroidal system is

$$H_0 = \omega_c P_1 + \frac{(P_3 - eRA_\varphi)^2}{2MR^2}. \tag{1}$$

The corresponding Hamiltonian equations are integrable and represent - for banana parameters - the banana motion.

The effect of LHW (in its electrostatic wave representation) or of FW (as a circular left-handed polarized wave) can be described by the total Hamiltonian $H = H_0 + H_1$, where $H_{1(LHW,FW)}$ is

$$H_{1(LHW)} = e\Phi_0 \sum_n J_n(k_\perp \rho_c) \exp(in(Q_1 + Q_2) + ik_\parallel R_0 Q_3 - i\omega t) \tag{2}$$

$$H_{1(FW)} = \frac{\omega_c e E_\perp}{k_\perp \omega} \sum_n n J_n(k_\perp \rho_c) \exp(in(Q_1 + Q_2) + ik_\parallel R_0 Q_3 - i\omega t) \tag{3}$$

where Φ_0 is the amplitude of the potential of LHW, $E_\perp$ is the electric component of FW, and where ω, $k_\parallel$, $k_\perp$ are the frequency and the parallel and perpendicular components of the wave vector k. The banana trajectory is given by the values r_0, $v_{\perp 0}$, $v_{\parallel 0}$, where r_0 is the largest value of the radius of the trajectory in the poloidal plane, and $v_{\perp 0}$ and $v_{\parallel 0}$ are the corresponding velocities of the particle in the same position in the equatorial plane. To obtain (2), we used for H_1 the simple expression $H_1 = e\Phi$, where Φ is the potential of the electrostatic wave in the form

$$\Phi = \Phi_0 \exp^{i(k_\perp r + k_\| R_0 Q_3 - \omega t)} = \Phi_0 \exp^{i\psi} .$$

Expressing $r = r(Q_1, Q_2, P_1, P_2)$, using the periodicity of r on Q_1, Q_2 and expanding the expression for Φ into the Fourier series, we obtain (2). The expression (3) was obtained by similar way. The total Hamiltonian H, including the effect of RF field, depends in the toroidal coordinates and momenta r, θ, ϕ, p_r, p_θ, p_ϕ on the components of A, A_θ, A_ϕ. Expressing $A_\theta = \bar{A}_\theta + \tilde{A}_\theta$, where $\tilde{A}_\theta$ represents the circular wave (again with the argument ψ), transforming into Q_i, P_i coordinates, expanding the Hamiltonian H into the form $H = H_0 + H_1$ and expanding $\exp^{i\psi}$ again into the Fourier series, we obtain (3).

2 Interaction of toroidally trapped alpha particles with LHW

For this case of this interaction, the resultant effect depends on the value of $k_\perp \rho_c$ (where ρ_c is the Larmor radius). A nonnegligible interaction can exist only for $k_\perp \rho_c > n$ (see e.g.[6]). For typical ITER parameters and for sufficiently high frequency f (in [6], $f = 5$GHz), the interaction is negligible. Nevertheless, there are still some possibilities of a stronger interaction. In [7], the proposal of the current drive, generated by intensive LHW pulses, penetrating into the plasma core, appeared. It is possible that in this case, a part of LHW spectrum can possesss a large $k_\perp$ component. Further, using the multijunction grill technique, the existence of the subsidiary part of the spectrum with e.g. three times larger $k_\|$ (and, therefore, also larger $k_\perp$) cannot be avoided [8]. Consequently, it is still reasonable to discuss a possible effect of LHW on alphas.

The LHW-banana interaction possesses two properties, making exact analytical discussion impossible. During one banana period, a particle can cross several resonances of the type

$$\omega - k_\| v_\| - n\omega_c = 0 \tag{4}$$

the number of which (together with their position) depends on the banana geometry. Further, LHW appears in the tokamak geometry in complicated geometrical structures (cones). To obtain some relevant analytical results, a suitable approximative approach is necessary.

The strongest LHW-banana interaction occurs in the region of the banana tip. The banana geometry (given by r_0, $v_\perp$, $v_\|$) in the given magnetic field and for given frequency gives the possibility to find along the banana the number of resonances ,Δn. Replacing therefore the actual resonant effect by means of Δn resonances in the tip, we obtain for the discussed banana the upper limit of the resonance effect. The changes of the relevant coordinates by crossing of one resonance are (see also [9])

$$\Delta P_{i(i=1,2)} = -e\Phi_0 J_n(k_\perp \rho_c) n I \tag{5}$$

$$\Delta P_3 = \Delta P_1 k_\| R_0 \tag{6}$$

$$I = 2^{1/3}[n v_{\|0} \omega_B \omega_{c0} R_0^{-2} q^{-1} r_0 (1 + r_0/R_0)^{-2}]^{-1/3} 3^{-2/3} 1/\Gamma(2/3). \tag{7}$$

Here, ω_B is the banana frequency for well trapped particles, ω_{c0} is the cyclotron frequency in the centre (for $r = 0$), q is the averaged safety factor in the banana region and Γ is the gamma function. Further, $|k_\perp \Phi_0| = E_\perp$ (the perpendicular component of the electric field). Using the inverse transformation to the original coordinates we can determine the radial shift of the banana tip, Δr, and estimate the radial diffusion coefficient,D_r as

$$D_r = < (\Delta r)^2 > \omega_B / 2\pi \tag{8}$$

supposing a fully stochastic regime. The symbol <> means the averaging over the resonant phases. For the global estimation, we shall use the averaged value of D_r

$$D_{r,av} = [\int n_{\alpha 0} dr_0]^{-1} \int\int\int dr_0 dv_{\perp 0} dv_{\|0} \Delta n n_{\alpha 0} \frac{D_r}{\Delta v_{\|0} \Delta v_{\perp 0} \Delta r_0} \tag{9}$$

Here, $n_\alpha(r_0)$ is the density of trapped particles, the regions $\Delta v_{\|0}$, $\Delta v_{\perp 0}$ are determined by the banana geometry and Δr_0 is given by the considered region. The limits of the integration are as follows:

$$r_{max} = 1,\ r_{min} = 0,\ v_{\|max} = v_{\perp 0}(\frac{2r_0}{R_0 - r_0})^{\frac{1}{2}},\ v_{\|0} = 0,\ v_{\perp 0max} = v_\alpha,\ v_{\perp 0min} = v_\alpha(\frac{R_0 - r_0}{R_0 + r_0})^{\frac{1}{2}}$$

where v_α is the total velocity of alpha particle (with the energy 3.52 MeV). Δn is again the total number of resonances, appearing during one banana period. We have evaluated approximately the value of $D_{r,av}$ for ITER parameters, for n_α as

$$n_\alpha = n_{\alpha 0}[1 - (r/a)^2]^2 \tag{10}$$

for the region $r_0(0.75a, a)$ and for $E_\perp = 10^5 Vm^{-1}$. The last value corresponds approximately to ITER parameters and to 100 MW of power. We obtain

$$D_{r,av} \approx 1.3 \times 10^{-3} m^2 s^{-1}. \tag{11}$$

This rather slow diffusion declares that for LHCD (and far less for the mentioned case of the subsidiary LHW spectrum) the diffusion of alphas cannot influence the energy balance. The LHCD with the intensive pulses requires further study.

3 Interaction of toroidally trapped particles with FW

The heating of plasmas by means of FW is usually considered in the minority ions or second harmonics schemes. FW will also interact with banana ions. Here, we shall approximately estimate the effect of FW on banana particles from the minority part. This part is here supposed to be represented by protons (in the deuterium plasma). Also in this case , the resonant interaction is given by the resonant condition (4) and can be treated analogously to LHW, using for the Hamiltonian H_1 its FW representation. Contrary to the LHW case, the resonance for FW case appears approximately on one resonance surface (going e.g. through r_0). A banana trajectory can penetrate through this surface maximally four times per one banana period.

Passing the resonance region, the change Δr_0 (and, analogously, $\Delta v_{\perp 0}$, $\Delta v_{\|0}$) of the banana can be determined , using the expression for banana trajectory [3], $r = r(v_{\perp 0}, v_{\|0}, r_0, Q_2)$ and using the Hamiltonian $H_{1,FW}$,as [10]

$$\Delta r_0 = \Delta r_0(\Delta r, \Delta v_\perp, \Delta v_\|, r_0, v_{\perp 0}, v_{\|0}, Q_{2,res}), \tag{12}$$

and, analogously, for further parameters. Here, $Q_{2,res}$ is the poloidal angle Q_2 for the resonance region. Further, Δr, $\Delta v_\perp$, $\Delta v_\|$ are the resonant changes of instantaneous values of r, $v_\perp$, $v_\|$ by crossing of the banana trajectory through the resonance (4). They are determined by the system (5-7) by use of the canonical perturbation method.

Analogously, only the case with the resonance in the banana tip can be treated without difficulties. Choosing further the resonance region for $r = 0$, considering $E_\perp = 10^2 Vcm^{-1}$, $k_\perp = 10^2 m^{-1}$, the shifts in r_0 and in Q_{2max} are related as $\Delta r_0 \approx -r_0 \Delta Q_{2max}$. Consequently, there is a strong coupling between the change of the poloidal angle od the tip, and the radial diffusion.

Let us estimate (for a typical banana) the characteristic time, $\Delta\tau$, in which this banana changes its poloidal angle of the tip, Q_{2max}, from $Q_{2max} = \pi$ to $Q_{2max} = \pi/2$, considering changes only in the perpendicular velocity and neglecting all other changes. Choosing $r_0 = a$, $Q_{2max} = \pi$, the perpendicular energy $W_{\perp 0} = 1.6keV$, and correspondingly, $W_{\| 0} = 1.8keV$, the mentioned change in Q_{2max} requires the increase $\Delta W_\perp \approx 15.4keV$. Considering further the energy diffusion coefficient, D_W as

$$D_W =< (\omega_c \Delta P_1)^2 > \omega_B/2\pi \quad (13)$$

(where <> means the averaging over the phases of resonant banana-RF interaction) we obtain the maximal value of D_W as $D_W \approx 2.7 \times 10^{-26} J^2 s^{-1}$. The lowest value of D_W corresponds to trapped particles with $Q_{2max} = \pi$, the largest value to particles with $Q_{2max} = \frac{\pi}{2}$, both for the same r_0.

The characteristic time, $\Delta\tau$, is then given as $D_W \Delta\tau = (\Delta W_\perp)^2$, from which $\Delta\tau \approx 8\times 10^{-4}$. Considering the lowest possible value of D_W, we obtain the following limit

$$8 \times 10^{-4} s < \Delta\tau < 2.5 \times 10^{-2} s. \quad (14)$$

The diffusion length, given by the neoclassical diffusion coefficient, can be for this time defined as $\Delta l = (D_{neo} \Delta \tau)^{\frac{1}{2}}$ and is for the weakest RF diffusion $\Delta l = 0.1m < a$. Consequently, for larger diffusion coefficients from the region (14), the diffusion length can be expected considerably larger. Since a considerable part of the trapped minority ions will change its position to the lower magnetic part during this time, it can be assumed that trapped minority ions will predominately occupy the low magnetic field part. This effect can have interesting physical consequences, as has been mentioned by Hsu [10] and Chen [11]. (For heating on the second harmonics, the exodus of trapped plasma ions from the higher magnetic field part can be also expected. This surplus of ions can create a poloidal electric field, as mentioned in [10,11]).

4 Quasilinear lines of diffusion. Estimate of the stochasticity threshold

Having the total Hamiltonian H in the action-angle form $= H(J_i, w_i)$, and considering all frequencies ω_i independent on J_i, the energy integral, given by the integration of the canonical equations, will have the form of Kennel-Engelmann constraint (see Becoulet [9])

$$v_{\perp 0}^2(1 - \omega/\omega_{c0}) + v_{\| 0}^2 = const. \quad (15)$$

(the subscripts label values on the equatorial plane). Admitting a weak nonlinearity in banana oscillations, this constraint will be modified as

$$v_{\perp 0}^2(1 - \omega/\omega_{c0} + \frac{H_2 N_2 \omega_B}{8a\omega_{c0}}) + v_{\| 0}^2 = const. \quad (16)$$

Here H_2 represents the energy of poloidal oscillations, α is given as $\alpha = 2Mr_0^2\omega_B^2$ and is substantially larger then H_2 and N_2 is the resonant harmonics of the banana frequency. The modification can therefore bring some changes only for large N_2.

Concerning the stochasticity threshold, the more exact estimate requires - due to 3D character of banana motion - the numerical simulation. For rough estimate,the threshold can be determined, using the fact that the main changes of the phase, which appears in H_1, are given by ω_c. Then the stochasticity threshold requires (see also [12])

$$\triangle \psi = \frac{\partial \omega_c}{\partial r} \triangle r\tau_B \geq \pi \quad (17)$$

We have obtained for LHW and our parameter the value $\triangle\psi = 0.1$, whereas for FW the value 1.4 . We can therefore expected for LHW lower degree of the stochastization then for FW.

Acknowledgement — The author is indebted to Drs. R. Klíma, Z. Gášek, P. Pavlo and V. Petržílka for valuable discussions.

References

[1] Whang K.W.and Morales G.J.: Nucl.Fusion 23 (1983) 481.

[2] Krlín L. Pavlo P., Tlučhoř Z. and Gášek Z.: Plasma Phys.Contr.Fusion 29 (1987) 1653.

[3] Gášek Z., Krlín L. and Tluchoř Z.: Czech.J. Phys.B 37 (1987) 571.

[4] Gášek Z., Krlín L. and Tlučhoř Z.: Phys. Lett.A 135 (1989) 284.

[5] Krlín L.: Czech.J.Phys.B 17 (1967) 112.

[6] Pavlo P., Krlín L., Tlučhoř Z. (1991) Nucl.Fusion 31, 711.

[7] Cohen B.I.et al. in Proc. of Int. School of Plasma Physics, Chebrex 1988, 597.

[8] Huřták O. and Preinhaelter J., accepted in IEEE Transaction on Plasma Science.

[9] Becoulet A., Gambier D.J. and Samain A.: Phys.Fluids B3 (1991) 137.

[10] Hsu J.Y. et al.: Phys.Rew.Lett.53, (1984) 564.

[11] Chen L., Vaclavik J. and Hammet G.W.: Nucl.Fusion 28 (1988) 389.

[12] Helander P., Lisak M. and Anderson D.: in Proc. of 13th Int. Conf. on Plasma Physics and Contr. Fusion Research, Washington, D.C., 1990, IAEA IV-101.

Appendix.

In the foregoing, the system of equations (12), determining the change of banana parameters under the influence of RF fields, has been mentioned. Here, we give an example of this system in a simplified form, namely, for the case when the resonance zone goes through the plasma centre and for $Q_{2max} \approx \frac{\pi}{2}$. Then the changes of basic banana parameters, Δr_0, $\Delta v_{\perp 0}$, $\Delta v_{\| 0}$, ΔQ_{2max} are (for the case of $v_{\| 0}$ in the direction of the plasma current)

$$\triangle r = \triangle r_0 + [-\frac{\Delta r_0}{2r_0} - \frac{\Delta v}{v} + 2(v_{\perp 0} \triangle v_{\perp 0} + 2v_{\| 0} \triangle v_{\| 0})\frac{1}{v_{\perp 0}^2 + 2v_{\| 0}^2}]\delta + \frac{1}{2}\delta \triangle Q_{2max} \cos\frac{\pi}{4} \quad (18)$$

$$\triangle Q_{2max} = -2\frac{\Delta v_\perp}{v}\frac{R_0}{R_0 + 2r_0}[(\frac{r_0}{2R_0})^{\frac{1}{2}}\frac{\omega_{c0} v}{q_0 \omega_{cpol}} + 2 - \frac{R_0}{R_0 + r_0}] \quad (19)$$

$$\triangle v_{\perp 0} = \frac{\omega_{c0} v_\perp \triangle v_\perp}{v_{\perp 0}\omega_c} \quad (20)$$

$$\triangle v_{\| 0} = \frac{1}{v_{\| 0}}[v \triangle v - \frac{\omega_{c0}}{\omega_c} v_\perp \triangle v_\perp]. \quad (21)$$

Here, ω_{cpol} is the cyclotron frequency in the poloidal field, δ is the thickness of the banana.

The second remark is connected with the problem of the quasilinear lines of diffusion. Let us consider that the poloidal oscillations are described by the Hamiltonian $H_2 = H_2(J_2)$ (which forms the part of the total separable Hamiltonian H). In the foregoing, we discussed the effect of the nonlinearity of banana oscillations. Nevertheless, the incorporation of the poloidal motion into the total Hamiltonian obviously extends the quasilinear constraint also into this degree of freedom. The action J_2 is determined by P_2 as

$$J_2 = \frac{1}{2\pi} \oint P_2 dQ_2. \tag{22}$$

Since P_2 is in our canonical representation also the measure of the radial displacement, the constraint in the quasilinear diffusion will now include also the radial displacement of bananas.

Approximately, the coupling between the energy absorption and the radial shift can be determined, using our expression (18 - 21) from the banana dynamics under the effect of RF field. For small changes, we obtain

$$\frac{\Delta v_{\parallel 0}}{\Delta v_{\perp 0}} = \frac{v_{\perp 0}}{v_{\parallel 0}} [\frac{v \Delta v \omega_c}{v_\perp \Delta v_\perp \omega_{c0}} - 1][1 + \frac{\omega_c}{2\omega_{c0}^2} \frac{\partial \omega_c}{\partial r} \Delta r_0 \frac{v_{\perp 0}^2}{v_\perp \Delta v_\perp}]^{-1}. \tag{23}$$

This equation therefore couples the shifts $\Delta v_{\parallel 0}$, $\Delta v_{\perp 0}$ and Δr_0. Here, ω_c satisfies the resonant condition

$$\omega - \omega_c - k_\parallel v_\parallel = 0 \tag{24}$$

and Δv, $\Delta v_\perp$ (changes of v, $v_\perp$ in the resonant point) must be determined - for given banana and the resonant position - from the canonical equations.