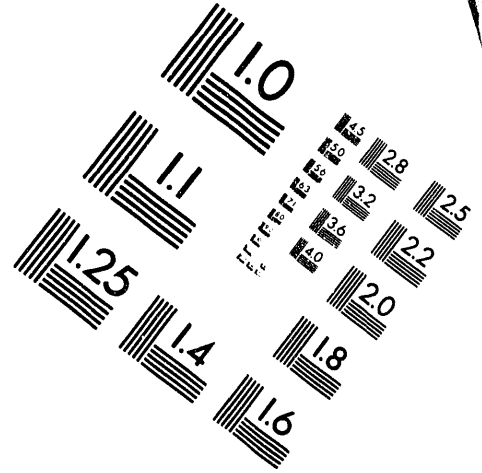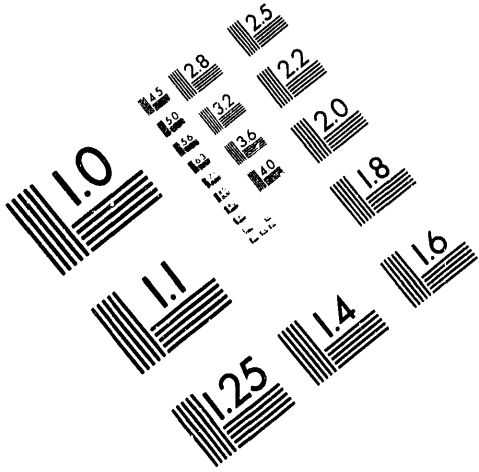# AIIM

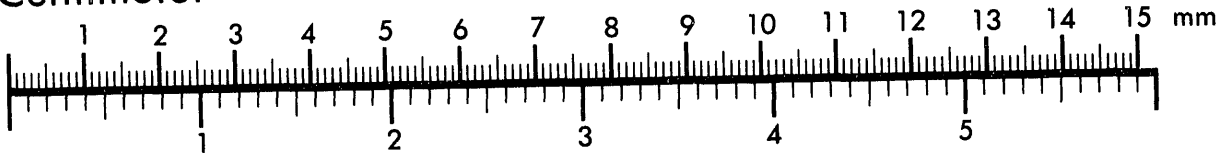**Association for Information and Image Management**
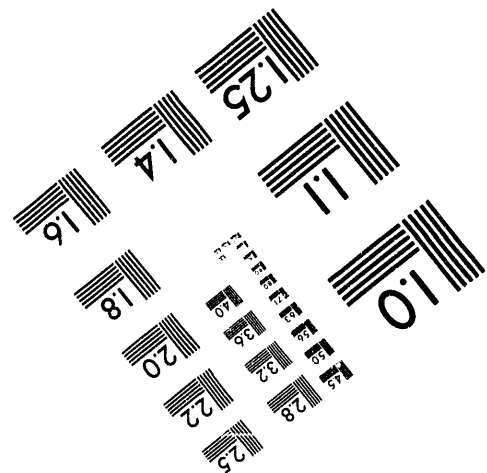
1100 Wayne Avenue, Suite 1100
Silver Spring, Maryland 20910

301/587-8202

Centimeter

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 mm

Inches

1.0

1.1

1.25 1.4 1.6

4.5 2.8 2.5
5.0 3.2 2.2
5.6 3.6
6.3 4.0 2.0
1.8
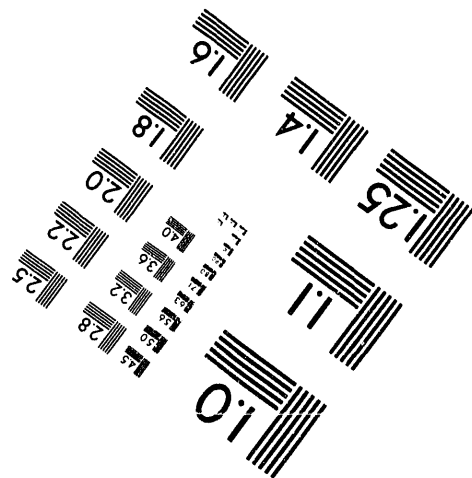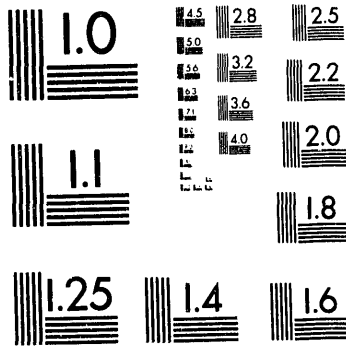
MANUFACTURED TO AIIM STANDARDS
BY APPLIED IMAGE, INC.

1 of 1

RECEIVED
JUN 1 1 1993
OSTI

One-loop fluctuation-dissipation formula
for bubble-wall velocity

PETER ARNOLD

Department of Physics, FM-15, University of Washington, Seattle, WA 98195

The limiting bubble wall velocity during a first-order electroweak phase transition is of interest in scenarios for electroweak baryogenesis. Khlebnikov has recently proposed an interesting method for computing this velocity based on the fluctuation-dissipation theorem. I demonstrate that at one-loop order this method is identical to simple, earlier techniques for computing the wall velocity based on computing the friction from particles reflecting off or transmitting through the wall in the ideal gas limit.

PREPARED FOR THE U.S. DEPARTMENT OF ENERGY

MASTER

February 1993

There has recently been considerable interest in the limiting bubble wall velocity during a first-order electroweak phase transition.[1-4] The dynamics of the bubble wall bears on scenarios for electroweak baryogenesis mediated by the anomalous violation of baryon number in the Standard Model.[5-10] During the phase transition, bubbles of the symmetry-broken phase are nucleated inside the symmetric phase, and these bubbles then expand to convert the entire Universe into the symmetry-broken phase. At zero temperature, in vacuum, the velocity of the bubble walls would asymptotically approach the speed of light; at finite temperature, however, collisions of particles off of the bubble wall provide a source of friction and force a smaller limiting velocity. A simple model for this friction is to compute the momentum transfer to the wall from an ideal gas of thermally-distributed particles colliding with, and either reflecting from or transmitting through, the bubble wall.* This model is valid provided the thermal mean free path of those particles is large compared to the thickness of the bubble wall. (This is not a particularly good assumption for the real case of the electroweak phase transition, but is often used to establish a lower limit on the bubble wall velocity.) Khlebnikov[3] has recently proposed an interesting alternative method for computing the bubble wall velocity u, based on the fluctuation-dissipation theorem, for cases where $u \ll 1$. When he puts his general formalism into practice with a sample one-loop calculation, he claims to find slightly different results than expected from other methods. In this paper, however, I shall show that the one-loop result obtained from the fluctuation-dissipation theorem is identical to the simple model of reflection and transmission in the limit of small wall velocities and large mean free path. (Khlebnikov's general formalism is still of interest because it applies to higher-loop calculations as well, always provided $u \ll 1$. For example, the effects of rescattering, which go beyond the approximations considered in this paper, originate in higher-order loops.)

---

* This has been discussed, in various limits, in refs. [1,2,11,12].
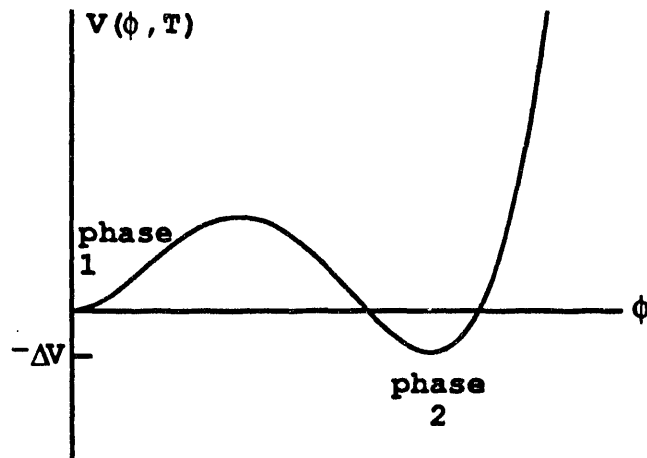
Fig. 1. Qualitative form of the effective, finite-temperature Higgs potential during a first-order electroweak phase transition.
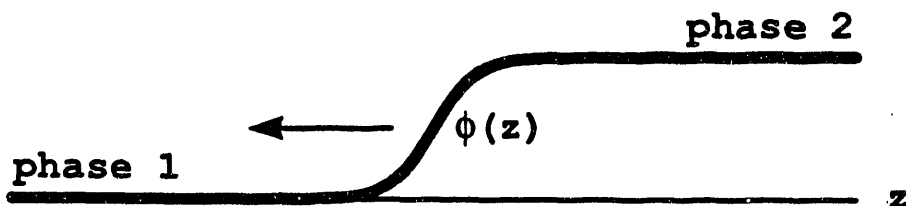
fig. 2. Qualitative profile of the bubble wall.

I shall also reexamine a toy model calculation considered by Khlebnikov, where he found a logarithmic suppression of the bubble wall velocity. It will be possible to see the physical origin of this logarithm (and of another logarithm that he accidentally omitted) by using the picture of reflection and transmission at the bubble wall.

1. Review of simple reflection/transmission model

Figure 1 shows the qualitative form of the effective, finite-temperature Higgs potential when the phase transition begins: the symmetric $\phi = 0$ phase (phase 1) is unstable to decay into the asymmetric vacuum (phase 2). Figure 2 shows qualitatively the profile of a bubble wall interpolating between the phases and which, as drawn, will move from right to left. Since particle masses depend on the value of ϕ, masses will be different in the two phases.

2

If the mean free path is large compared to the wall thickness, then we may separately treat the collision of each particle in the plasma with the bubble wall. In the wall rest frame, the wall will be struck by a Lorentz-boosted thermal distribution of particles. I shall focus on the case of bosons. The usual Bose distribution $n_p = 1/[\exp(\beta E_p) - 1]$ is the form, in the thermal rest frame, of the Lorentz scalar $1/[\exp(\beta p \cdot v) - 1]$ where v is the four-velocity of the thermal bath. In the rest frame of the bubble wall, moving with wall velocity u relative to the thermal bath, this distribution is

$$f_p = \frac{1}{\exp[\beta\gamma(E_p + \vec{p} \cdot \vec{u})] - 1}. \tag{1.1}$$

Take $\vec{u}$ to be in the z direction, and consider a particle of energy E and momentum $\vec{p} = (\vec{p}_\perp, p_z)$ incident from the symmetric phase. Both E and $\vec{p}_\perp$ will be conserved. The momentum transferred to the wall will be $2p_{1z}$ if the particle is reflected and $p_{1z} - p_{2z}$ if it is transmitted, where

$$p_{1z} = \sqrt{E^2 - p_\perp^2 - m_1^2}, \qquad p_{2z} = \sqrt{E^2 - p_\perp^2 - m_2^2}, \tag{1.2}$$

and m_1 and m_2 are the masses of the particle in the two phases. The pressure on the wall from particles incident from the symmetric phase is then

$$P_1 = \int \frac{d^3 p_1}{(2\pi)^3} v_{1z} \theta(v_{1z}) f_{p1} \left[2p_{1z}\mathcal{R}(p_1) + (p_{1z} - p_{2z})\mathcal{T}(p_1)\right], \tag{1.3}$$

where $\mathcal{R}(p_1)$ and $\mathcal{T}(p_1)$ are the reflection and transmission coefficients for particles encountering the wall and $\vec{v}_1$ is the incident particle velocity $\vec{p}_1/E$. The pressure P_2 from particles incident from the asymmetric phase is similar.* Also, the difference of the Higgs potential between the two phases exerts pressure on the wall. The net pressure is

$$P_{\text{net}} = P_1 - P_2 - \Delta V_{\text{cl}}, \tag{1.4}$$

---

* I shall assume throughout that there the temperature does not vary appreciably across the wall. This is the case for the electroweak phase transition; see refs. [1] and [12].

3

where V_{cl} is the classical, zero-temperature Higgs potential. (The finite-temperature effective potential will appear momentarily.)

Consider this result in the limit of zero wall velocity u, and change integration variables in (1.3) from p_z to E:

$$P_{\text{net}}(u=0) = \int \frac{\mathrm{d}^2 p_\perp \, \mathrm{d}E}{(2\pi)^3} \frac{1}{e^{\beta E} - 1} \left\{ 2p_{1z}\theta(-p_{2z}^2) \right.$$
$$\left. + 2(p_{1z} - p_{2z})[\mathcal{R}(p_\perp, E) + \mathcal{T}(p_\perp, E)]\theta(p_{2z}^2) \right\} - \Delta V_{cl}, \tag{1.5}$$

where I have used the fact from one-dimensional quantum mechanics that reflection and transmission coefficients do not depend from which side the particle approaches. $\theta(\pm p_{2z}^2)$ above is a short-hand notation for $\theta[\pm(E^2 - p_\perp^2 - m_2^2)]$, which specifies whether or not the particle is classically allowed in phase 2. The first term in braces above corresponds to particles from phase 1 which are not energetic enough to enter phase 2 and so must reflect from the bubble wall; the second term corresponds to more energetic particles, which may come from phase 1 or phase 2 and may reflect or transmit. Now use $\mathcal{R} + \mathcal{T} = 1$ and integrate by parts to get

$$P_{\text{net}}(u=0) = T \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \left[\ln\left(1 - e^{-\beta E_1}\right) - \ln\left(1 - e^{-\beta E_2}\right) \right] - \Delta V_{cl}, \tag{1.6}$$

which is simply the difference $-\Delta V_{\text{eff}}$ in the one-loop finite-temperature effective potential between the two phases.[12]

At *non*-zero wall velocity, the net pressure (1.3) may then be written as

$$P_{\text{net}} = \int \frac{\mathrm{d}^3 p_1}{(2\pi)^3} v_{1z}\theta(v_{1z})\Delta f_{p1} \left[2p_{1z}\mathcal{R}(p_1) + (p_{1z} - p_{2z})\mathcal{T}(p_1) \right]$$
$$- \int \frac{\mathrm{d}^3 p_2}{(2\pi)^3} v_{2z}\theta(-v_{2z})\Delta f_{p2} \left[2p_{2z}\mathcal{R}(p_1) + (p_{2z} - p_{1z})\mathcal{T}(p_1) \right] - \Delta V_{\text{eff}}, \tag{1.7}$$

where $\Delta f_p = f_p - n_p$. In the limit of small wall velocity u, $f_p - n_p \approx u\beta p_z n_p(n_p + 1)$, and (1.7) simplifies to

$$P_{\text{net}} = -\Delta V_{\text{eff}} + \beta u \int \frac{\mathrm{d}^2 p_\perp \mathrm{d}E}{(2\pi)^3} n(E)[n(E) + 1]\left\{ 2p_{1z}^2\theta(-p_{2z}^2) \right.$$
$$\left. + \left[2(p_{1z}^2 + p_{2z}^2)\mathcal{R}(p_\perp, E) + (p_{1z} - p_{2z})^2\mathcal{T}(p_\perp, E) \right]\theta(p_{2z}^2) \right\} + O(u^2), \tag{1.8}$$
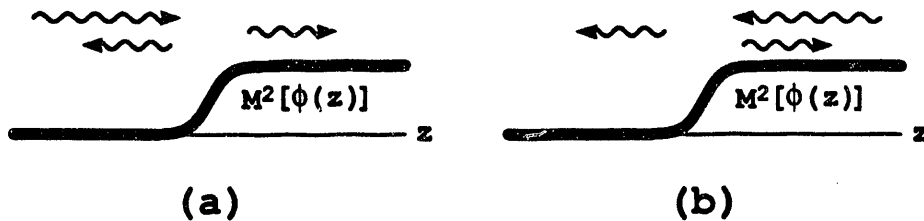
4

(a)   **(b)**

Fig. 3. Schematic depiction of two solutions to the Schrödinger equation describing a particle propagating in the background of the wall. A wave may be incident from either (a) the left or (b) the right, and it reflects and transmits at the wall. The heavy solid line depicts the potential.

where p_{1z} and p_{2z} will henceforth always be understood to be positive, according to (1.2). The limiting velocity of the wall is determined by setting $P_{\mathrm{net}} = 0$.

Equation (1.8) is the formula that I shall show is equivalent to the one-loop fluctuation-dissipation result of Khlebnikov.[3] For the sake of simplicity and concreteness, I shall work with a generalization of the toy scalar model considered by Khlebnikov. The Lagrangian is

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V(\phi) + \frac{1}{2}(\partial\Phi)^2 - \frac{1}{2}M^2(\phi)\Phi^2, \tag{1.9}$$

where ϕ is the usual Higgs field, and Φ is another particle in the theory that, analogous to W and Z bosons, has a mass that depends on the VEV of ϕ. [Khlebnikov's specific model was $M^2(\phi) = M_0^2 + 2g\phi$.] In the presence of a background field $\phi_{\mathrm{cl}}(z)$ describing the bubble wall in its rest frame, the Φ field has the effective Lagrangian

$$\mathcal{L}_{\mathrm{eff}} = \frac{1}{2}(\partial\Phi)^2 - \frac{1}{2}M^2(\phi_{\mathrm{cl}}(z))\Phi^2. \tag{1.10}$$

This is just the problem of a particle propagating in a classical potential. The S matrix may be found by solving the classical equation of motion, taking a solution with a single incoming wave, and extracting the amplitudes of outgoing waves. Fourier transforming $(t, \vec{x}_\perp)$ to $(E, \vec{p}_\perp)$ and noting that the latter are conserved, the equation of motion is just

$$\left[-\partial_z^2 + M^2(\phi_{\mathrm{cl}}(z))\right]\Phi = (E^2 - p_\perp^2)\Phi, \tag{1.11}$$
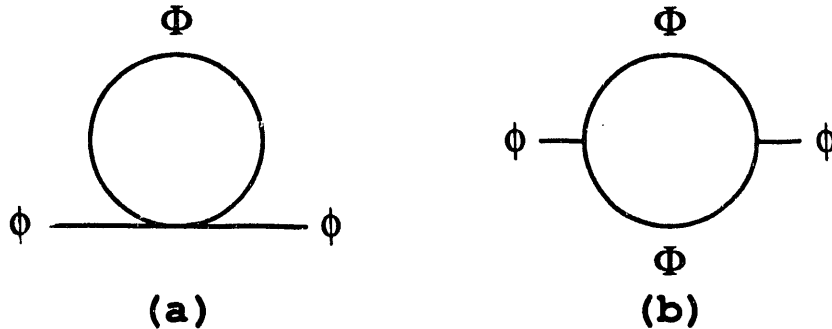
5

fig. 4. One-loop contributions of Φ to the Higgs self-energy.

which is nothing more than the Schrödinger equation for a unit mass particle in the potential $2M^2(\phi_{cl}(z))$ with energy $2(E^2 - p_\perp^2)$. For incident waves from the right or left, it has solutions shown qualitatively in fig. 3, and the reflection and transmission coefficients extracted from these solutions are the $\mathcal{R}$ and $\mathcal{T}$ which appear in (1.8).

2. Fluctuation-dissipation theorem at one loop.

I shall now show that one also obtains the result (1.8) from the fluctuation-dissipation theorem at one-loop. Khlebnikov treats the difference in vacuum energies in fig. 1 as a small perturbation. Imagine turning off that perturbation, so that the vacua are degenerate. A bubble wall in this situation would then remain stationary. Now turn the perturbation back on. The non-equilibrium response of the system to this perturbation can then be related, via the fluctuation-dissipation theorem, to the *equilibrium* calculation of a retarded correlator. Specifically, assuming the bubble wall velocity is small, he finds

$$u = -\Delta V_{\text{eff}} \lim_{\omega \to 0} \left[\frac{1}{\omega} \int dz dz' \, \partial_z \phi_{cl}(z) \operatorname{Im}\Pi_R(\omega; \vec{k}_\perp = 0; z, z') \, \partial_{z'} \phi_{cl}(z') \right]^{-1}, \qquad (2.1)$$

where Π_R is the retarded self-energy of the Higgs field ϕ. Π_R is projected above onto the translational zero-mode $\partial_z \phi_{cl}(z)$ of the bubble wall. In position space, $\Pi_R = \langle [\phi(r), \phi(r')] \rangle$ is a function of two space-time coordinates. I have Fourier transformed the t, x, and y components above to ω and $\vec{k}_\perp$ but have left the z coordinates in position space. The trick is to now compute the desired limit of Π_R in the *background* of the bubble wall $\phi_{cl}(z)$. I shall examine this problem at one-loop order.

6

For simplicity, return to the scalar model (1.9) and focus on the one-loop contributions to Π_R from Φ, which are shown in fig. 4. Only fig. 4b has an imaginary part. To get its contribution to $\text{Im}\Pi_R$, first consider evaluating the self-energy in Euclidean space for Euclidean frequencies $i\nu$. Figure 4b gives

$$\Pi(i\nu; 0; z, z') = \frac{1}{2}\mathcal{M}'(z)\mathcal{M}'(z')T\sum_{p_0}\int\frac{d^2p_\perp}{(2\pi)^2}$$
$$\times\left(\frac{1}{p_0^2 - p_\perp^2 - \Delta^2}\right)_{zz'}\left(\frac{1}{(p_0 + i\nu)^2 - p_\perp^2 - \Delta^2}\right)_{z'z}, \quad (2.2)$$

where the sum is over discrete $p_0 = i2\pi nT$, Δ^2 is the operator

$$\Delta^2 = -\partial_z^2 + M^2(\phi_{\text{cl}}(z)), \quad (2.3)$$

and

$$\mathcal{M}'(z) = \partial_\phi M^2(\phi_{\text{cl}}(z)). \quad (2.4)$$

Now let $f_\kappa^\pm(z)$ be the eigenfunctions of Δ^2 defined by

$$\Delta^2 f_\kappa^\pm = \kappa^2 f_\kappa^\pm \quad (2.5)$$

and normalized to

$$\int dz\, f_\kappa^{a*}(z)f_{\kappa'}^b(z) = \delta^{ab}\delta(\kappa^2 - \kappa'^2). \quad (2.6)$$

Later on, we shall see that a useful basis for the $f^\pm$ will be the solutions depicted in fig. 3, representing scattering of particles incident from a definite side of the wall. By inserting a complete set of states, (2.2) may be rewritten as

$$\Pi(i\nu; 0; z, z') = \frac{1}{2}\mathcal{M}'(z)\mathcal{M}'(z')\int d\kappa^2 d\kappa'^2 T\sum_{p_0}\int\frac{d^2p_\perp}{(2\pi)^2}$$
$$\times f_\kappa^{a*}(z)\frac{1}{p_0^2 - p_\perp^2 - \kappa^2}f_\kappa^a(z')f_{\kappa'}^{b\,*}(z')\frac{1}{(p_0 + i\nu)^2 - p_\perp^2 - \kappa'^2}f_{\kappa'}^b(z), \quad (2.7)$$

where sums over $a, b = \pm$ are implicit. At this point, the steps to obtain $\text{Im}\Pi_R$ are exactly the same as they would be in the case of *free* scalar propagators and may be found, for

example, in refs. [13,3]. First the frequency sum over p_0 is performed by the usual contour trick to obtain

$$
\Pi(i\nu; 0; z, z') = \frac{1}{2}\mathcal{M}'(z)\mathcal{M}'(z') \int d\kappa^2 d\kappa'^2 f_\kappa^{a*}(z) f_{\kappa'}^b(z) f_{\kappa'}^{b\,*}(z') f_\kappa^a(z') \int \frac{d^2 p_\perp}{(2\pi)^2}
$$
$$
\times \left\{ \frac{n(E)}{2E} \left[\frac{1}{(E+i\nu)^2 - (p_\perp^2 + \kappa'^2)} + \frac{1}{(E-i\nu)^2 - (p_\perp^2 + \kappa'^2)} \right] \right.
$$
$$
\left. + \frac{n(E')}{2E'} \left[\frac{1}{(E'+i\nu)^2 - (p_\perp^2 + \kappa^2)} + \frac{1}{(E'-i\nu)^2 - (p_\perp^2 + \kappa^2)} \right] \right\}
$$
$$
+ \text{(T-independent terms)},
$$
(2.8)

where

$$
E = \sqrt{p_\perp^2 + \kappa^2}, \qquad E' = \sqrt{p_\perp^2 + \kappa'^2}. \tag{2.9}
$$

The retarded self-energy is then obtained by replacing $i\nu$ by $\omega + i\epsilon$. It is now straightforward to take the imaginary part,

$$
\text{Im}\Pi_R(\omega; 0; z, z') = -\frac{1}{2}\mathcal{M}'(z)\mathcal{M}'(z') \int d\kappa^2 d\kappa'^2 f_\kappa^{a*}(z) f_{\kappa'}^b(z) f_{\kappa'}^{b\,*}(z') f_\kappa^a(z') \int \frac{d^2 p_\perp}{(2\pi)^2}
$$
$$
\times \frac{\pi}{4EE'} [n(E) - n(E')][\delta(\omega + E - E') - \delta(\omega - E + E')],
$$
(2.10)

and to take the $\omega \to 0$ limit. Switching integration variables from κ to E,

$$
\text{Im}\Pi_R(\omega; 0; z, z') \to -\pi\beta\omega\mathcal{M}'(z)\mathcal{M}'(z') \int \frac{d^2 p_\perp}{(2\pi)^2} \int dE
$$
$$
\times n(E)[n(E)+1] f_\kappa^{a*}(z) f_\kappa^b(z) f_\kappa^{b\,*}(z') f_\kappa^a(z').
$$
(2.11)

Projecting this onto the bubble wall zero mode then transforms Khlebnikov's result (2.1) into:

$$
\frac{\Delta V_{\text{eff}}}{u} = \pi\beta \int \frac{d^2 p_\perp}{(2\pi)^2} dE\, n(E)[n(E)+1] \sum_{a,b} \left| \int dz\, f_\kappa^{a*}(z) \partial_z M^2(\phi_{\text{cl}}(z)) f_\kappa^b(z) \right|^2. \tag{2.12}
$$

This will reproduce the result from (1.8) provided the sum over (a, b) above can be appropriately related to the reflection and transmission coefficients $\mathcal{R}$ and $\mathcal{T}$.

8

Using the eigenequation (2.5) that defines the f_κ^a, it is straightforward to show that the z-integrand is a total derivative:

$$f_\kappa^{a*}(z)\partial_z M^2(\phi_{cl}(z))f_\kappa^b(z) = \partial_z\left[\frac{1}{2}f_\kappa^{a*}\overset{\leftrightarrow}{\partial}_z^2 f_\kappa^b\right], \tag{2.13}$$

where $\overset{\leftrightarrow}{\partial}_z^2$ is defined by

$$f\overset{\leftrightarrow}{\partial}_z^2 g \equiv (\partial_z^2 f)g - 2(\partial_z f)(\partial_z g) + f(\partial_z^2 g). \tag{2.14}$$

So the result (2.12) depends only on the values of $f_\kappa^{a*}\overset{\leftrightarrow}{\partial}_z^2 f_\kappa^b$ at infinity, where f_κ^a becomes a superposition of incoming and outgoing plane waves. On plane waves, note that $\overset{\leftrightarrow}{\partial}_z^2$ gives

$$\left(e^{ikz}\right)^*\overset{\leftrightarrow}{\partial}_z^2 e^{ikz} = -4k^2, \qquad \left(e^{-ikz}\right)^*\overset{\leftrightarrow}{\partial}_z^2 e^{ikz} = 0. \tag{2.15}$$

Now let us choose the basis for $f_\kappa^\pm$ depicted in fig. 3 and parameterize the asymptotic behavior as

$$f_\kappa^+(z) \to \begin{cases} \frac{1}{\sqrt{4\pi k_1}}\left(e^{ik_1 z} + S_{++}e^{-ik_1 z}\right), & x \to -\infty, \\ \frac{1}{\sqrt{4\pi k_2}}S_{+-}e^{ik_2 z}, & x \to +\infty, \end{cases}$$

$$f_\kappa^-(z) \to \begin{cases} \frac{1}{\sqrt{4\pi k_1}}S_{-+}e^{-ik_1 z}, & x \to -\infty, \\ \frac{1}{\sqrt{4\pi k_2}}\left(e^{-ik_2 z} + S_{--}e^{ik_2 z}\right), & x \to +\infty, \end{cases} \tag{2.16}$$

where the S are functions of κ and

$$\kappa^2 = k_1^2 + M^2(-\infty) = k_2^2 + M^2(+\infty). \tag{2.17}$$

The matrix of coefficients

$$S = \begin{pmatrix} S_{++} & S_{+-} \\ S_{-+} & S_{--} \end{pmatrix} \tag{2.18}$$

is symmetric and unitary.* The reflection and transmission coefficients are $\mathcal{R} = |S_{++}|^2 = |S_{--}|^2$ and $\mathcal{T} = |S_{+-}|^2 = |S_{-+}|^2$. The normalization condition (2.6) is verified in the

---

* This follows from examining the Wronskians W of f^+ with f^- or f^{-*} and equating $W(z= -\infty)$ with $W(z= +\infty)$.

appendix. (When κ^2 is too small to allow transmission from phase 1 to phase 2, then $\mathcal{R} = 1$, $\mathcal{T} = 0$, and the f_κ^+ solution should be dropped.)

Using the eigenfunctions (2.16), the properties of S, and the action (2.15) of $\overleftrightarrow{\partial}_z^2$, it is straightforward to find that

$$\sum_{a,b} \left| \frac{1}{2} f_\kappa^{a*} \overleftrightarrow{\partial}_z^2 f_\kappa^b \Big|_{-\infty}^{+\infty} \right|^2 = \frac{1}{2\pi^2} \left[2(k_1^2 + k_2^2)\mathcal{R} + (k_1 - k_2)^2 \mathcal{T} \right] . \tag{2.19}$$

[When κ^2 is too small to allow transmission from phase 1 to 2, this instead becomes simply $(2k_1^2)/2\pi^2$.] Now use this, together with (2.13), to rewrite the result (2.12) for the bubble wall velocity in terms of $\mathcal{R}$ and $\mathcal{T}$. As promised, the result is identical to the result (1.8) of the last section, where I reviewed the simple picture of friction caused by reflection and transmission at the bubble wall.

3. A Toy Example

I now want to specialize to Khlebnikov's toy example and understand his results in terms of reflection and transmission probabilities. Khlebnikov[3] took $M^2(\phi) = M_0^2 + 2g\phi$ in the limit $\Delta M^2 \ll M_0^2 \ll T^2$ where $\Delta M^2 = M^2(z{=}{+}\infty) - M^2(z{=}{-}\infty)$. Consider first the transmission term in the net wall pressure (1.8), and consider the contribution to the integral from momenta $\Delta M^2 \ll p_z^2 \ll M_0^2$. $\mathcal{T}$ is approximately 1 for such momenta, and it is easy to extract the behavior of the integrand in these limits. Taking $E^2 \sim M_0^2 + p_\perp^2$, $p_{1z} \sim p_{2z}$, $p_{1z} - p_{2z} \sim \Delta M^2/2p_z$, and $n(E) \sim T/E$, this contribution to the transmission term in (1.8) becomes

$$\beta u \int \frac{d^2 p_\perp dE}{(2\pi)^3} n(E)[n(E) + 1](p_{1z} - p_{2z})^2 \mathcal{T} \theta(p_{2z}^2)$$

$$\approx u(\Delta M^2)^2 \mathcal{T} \frac{1}{4(2\pi)^3} \int \frac{dp_z}{p_z} \int \frac{d^2 p_\perp}{(p_\perp^2 + M_0^2)^{3/2}} \tag{3.1}$$

$$\approx u \frac{(\Delta M^2)^2 \mathcal{T}}{32\pi^2 M_0} \ln\left(\frac{M_0^2}{\Delta M^2} \right) ,$$

where p_z has been integrated over the region $\Delta M^2 \ll p_z^2 \ll M_0^2$, and where I have ignored terms on the right-hand side that are not enhanced by a logarithm. It is easy to check that this contribution dominates (by the logarithm) over the other regions of integration, $p_z^2 \lesssim \Delta M^2$ and $M_0^2 \lesssim p_z^2$.

Now consider the reflection term in the net wall pressure (1.8). Generically, $\mathcal{R}$ falls to zero for $p_z^2 \gg \Delta M^2$, and so one might not expect to get a logarithmic enhancement as in (3.1). However, if the bubble wall were a *step* function, with zero width, then $\mathcal{R}$ falls slowly enough at large p_z that one still gets a logarithm. Specifically, for a step potential,

$$\mathcal{R} = \left| \frac{p_{1z} - p_{2z}}{p_{1z} + p_{2z}} \right|^2. \tag{3.2}$$

Considering the region $\Delta M^2 \ll p_z^2 \ll M_0^2$ as before (so that $p_{1z} \sim p_{2z}$ and $\mathcal{T} \sim 1$), then

$$2(p_{1z}^2 + p_{2z}^2)\mathcal{R} \approx (p_{1z} - p_{2z})^2 \mathcal{T}, \tag{3.3}$$

and one sees that the $\mathcal{R}$ and $\mathcal{T}$ terms in (1.8) contribute equally, each giving (3.1). If the spatial width δ of the wall is small but *non*-zero, then $\mathcal{R}$ will behave like (3.2) for $p_z \ll 1/\delta$ but will fall faster with p_z than (3.2) for $p_z \gg 1/\delta$. The logarithm found for the transmission term in (3.1) will then, for the reflection term, be cut off by $1/\delta$ rather than M_0 if $1/\delta < M_0$. At the level of the leading logarithm, the various cases are

$$\beta u \int \frac{d^2 p_\perp dE}{(2\pi)^3} n(E)[n(E) + 1] 2(p_{1z}^2 + p_{2z}^2)\mathcal{R}\theta(p_{2z}^2)$$

$$\approx u \frac{(\Delta M^2)^2 \mathcal{T}}{32\pi^2 M_0} \begin{cases} \ln\left(\frac{M_0^2}{\Delta M^2}\right), & M_0^2 \lesssim \delta^{-2}, \\ \ln\left(\frac{\delta^{-2}}{\Delta M_0^2}\right), & \Delta M^2 \lesssim \delta^{-2} \lesssim M_0^2, \\ 0, & \delta^{-2} \lesssim \Delta M^2, \end{cases} \tag{3.4}$$

where zero in the last case means that there is no term with a logarithmic enhancement. Putting the approximations (3.1) and (3.4) into the formula (1.8) for the net pressure, the result for the wall velocity in Khlebnikov's model is

$$u = \frac{32\pi^2 M_0 \Delta V_{\text{eff}}}{(\Delta M^2)^2 \mathcal{T} l} + O(u/l), \tag{3.5}$$

where

$$
l = \begin{cases} 2\ln\left(\frac{M_0^2}{\Delta M^2}\right), & M_0^2 \lesssim \delta^{-2}, \\ \ln\left(\frac{M_0^2}{\Delta M^2}\right) + \ln\left(\frac{\delta^{-2}}{\Delta M_0^2}\right), & \Delta M^2 \lesssim \delta^{-2} \lesssim M_0^2, \\ \ln\left(\frac{M_0^2}{\Delta M^2}\right), & \delta^{-2} \lesssim \Delta M^2 . \end{cases} \tag{3.6}
$$

For a specific form of $\phi_{cl}(z)$, one could also calculate the non-logarithmic correction to l by precisely computing the reflection and transmission coefficients.

Khlebnikov gave a more approximate analysis of this model where he (1) accidentally missed the transmission term,* and (2) computed the contribution of fig. 4b to the Π_R in the approximation that the background field ϕ is ignored, so that the Φ propagator is simply $1/(P^2 - M_0^2)$. This approximation turns out to be equivalent to taking $\Delta M^2 \to 0$ in the logarithms (3.6) and produces a logarithmic infrared divergence. He was forced to cut off this divergence by including rescattering effects, which come from resumming higher order of the loop expansion. However, we see now that there is indeed no logarithmic divergence in the simple one-loop result.

In the case of the electroweak phase transition, M_0 and ΔM^2 are not separate scales, and so no logarithmic enhancement of the friction should be expected. In electroweak models, the wall width is typically large compared to inverse particle masses, and then the reflection and transmission coefficients can be well approximated by simple theta functions, $\theta(-p_{2z}^2)$ and $\theta(p_{2z}^2)$, representing complete transmission when energetically allowed. This approximation was used in refs. [2,1].

I have shown that the one-loop application of Khlebnikov's more general fluctuation-dissipation result is equivalent to the simple physical picture of computing the friction on

---

* The intended step between eqs. (19) and (20) of ref. [3] requires the identity

$$
\delta(E - E_1) = \frac{E}{pk}\delta\left(\cos\theta - \frac{k}{2p}\right) + \frac{E}{p|\cos\theta|}\delta(k),
$$

but the last term was left out. There is also a factor of 2 mistake going from eq. (20) to (21).

the wall from particle collisions in the ideal gas limit. For the electroweak phase transition, however, one generally may *not* ignore the effects of rescattering, and the reader should consult refs. [1-3] for attempts to analyze rescattering effects.

Appendix A.

In this appendix, I check that the solutions (2.16) are normalized according to (2.6). For a complete set of states, this normalization condition is equivalent to

$$\int d\kappa^2 f_\kappa^{a*}(z) f_\kappa^a(z') = \delta(z - z'). \tag{A.1}$$

The normalization of the basis (2.16) may be checked by verifying (A.1) for any particular range of z. Let's focus then on $z, z' \to -\infty$. Using (2.16) and the properties of S, one finds

$$f_\kappa^{a*}(z) f_\kappa^a(z') \to \frac{1}{4\pi k_1} \left[e^{-ik_1(z-z')} + e^{ik_1(z-z')} + 2S_{++} e^{-ik_1(z+z')} + 2S_{++}^* e^{ik_1(z+z')} \right]. \tag{A.2}$$

Then, since $\kappa^2 \to k_1^2$,

$$\int d\kappa^2 f_\kappa^{a*}(z) f_\kappa^a(z') \to \delta(z - z') + \frac{1}{\pi} \int_0^\infty dk_1 [S_{++} + S_{++}^*] e^{-ik_1(z+z')}. \tag{A.3}$$

The last term vanishes due to two analytic properties of S_{++}: (1) $S_{++}^*(k_1) = S_{++}(-k_1)$, and (2) $S_{++}(k_1)$ is analytic in the upper half-plane provided the potential has no bound states (which, in applications of interest, is generally the case). The last term can therefore be written as an integral from $-\infty$ to $+\infty$, which vanishes by closing the contour in the

13

upper-half plane. The first property follows from the fact that, if $f_\kappa^+(z)$ is a solution, then $f_\kappa^{+\,*}(z)$ is also a solution. For insight into the second property, suppose that $S_{++}(k_1)$ has a *pole* in the upper-half plane. At this pole, $k_2 = \sqrt{k_1^2 - \Delta M^2}$ will also be in the upper-half plane. Now renormalize the solution (2.16) for f_κ^+ by dividing it by S_{++}, and then take k_1 to be the position of the pole so that $S_{++} \to \infty$. The "incident" wave term disappears. The solution falls exponentially as $x \to \pm\infty$ and therefore describes a bound-state, which contradicts the assumption that there are none. (If there are bound states, the contribution from the poles of S_{++} reflects the fact that the sum over bound states should have been included on the left side of (A.1).) The simple argument above doesn't prove that there aren't other types of singularities in the upper-half plane, and the reader should try ref. [14] for a more general discussion of the analytic properties of the S matrix.

References

[1] M. Dine, R. Leigh, P. Huet, A. Linde and D. Linde, Phys. Rev. **D46**, 550 (1992); Phys. Lett. **B283**, 319 (1992).

[2] B-H. Liu, L. McLerran and N. Turok, Phys. Rev. **D46**, 2668 (1992).

[3] S. Khlebnikov, Phys. Rev. **D46**, 3223 (1992).

[4] P. Huet, K. Kajantie, R. Leigh, B-H. Liu and L. McLerran, SLAC-PUB-5943.

[5] M. Shaposhnikov, JETP Lett. **44**, 405 (1986); Nucl. Phys. **B299**, 707 (1988),

[6] L. McLerran, Phys. Rev. Lett. **62**, 1075 (1989).

[7] N. Turok and J. Zadrozny, Phys. Rev. Lett. **65**, 2331 (1990); Nucl. Phys. **B358**, 471 (1991); L. McLerran, M. Shaposhnikov, N. Turok and M. Voloshin, Phys. Lett. **256B**, 451 (1991).

[8] A. Cohen, D. Kaplan and A. Nelson, Phys. Lett. **B245**, 561 (1990); Nucl. Phys. **B349**, 727 (1991); Phys. Lett. **B263**, 86 (1992).

[9] M. Dine, P. Huet, R. Singleton and L. Susskind, Phys. Lett. **B257**, 351 (1992); M. Dine, P. Huet and R. Singleton, Nucl. Phys. **B375**, 625 (1992).

[10] For a good review, see A. Cohen, D. Kaplan and A. Nelson, UCSD-PTH-93-02, to appear in *Ann. Rev. Nucl. Part. Sci.* **43**.

[11] A. Linde, Nucl. Phys. **B216**, 421 (1983).

[12] N. Turok, Phys. Rev. Lett. **68**, 1803 (1992).

[13] H.A. Weldon, Phys. Rev. **D28**, 2007 (1983).

[14] J. Taylor, "Scattering Theory" (John Wiley & Sons: New York, 1972).

DATE FILMED
FILMED
9/16/93

END