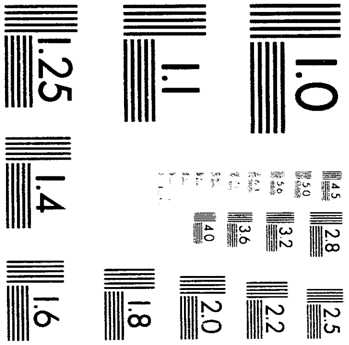
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Improved Semi-Analytic Algorithms for Finding the Flux from a Cylindrical Source

By

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ABSTRACT

Improved Semi-Analytic Algorithms for Finding the Flux from a Cylindrical Source

Hand calculation methods for radiation shielding problems continue to be useful for scoping studies, for checking the results from sophisticated computer simulations and in teaching shielding personnel. This paper presents two algorithms which give improved results for hand calculations of the flux at a lateral detector point from a cylindrical source with an intervening slab shield parallel to the cylinder axis. The first algorithm improves the accuracy of the approximate flux flux formula of Ono and Tsuro so that results are always conservative and within a factor of two. The second algorithm uses the first algorithm and the principle of superposition of sources to give a new approximate method for finding the flux at a detector point outside the axial and radial extensions of a cylindrical source. A table of error ratios for this algorithm versus an exact calculation for a wide range of geometry parameters is also given. There is no other hand calculation method for the geometric configuration of the second algorithm available in the literature.

Improved Semi-Analytic Algorithms for Finding the Flux from a Cylindrical Source

O. J. Wallace

I. Introduction

Hand calculation methods involving semi-analytic approximations of exact flux formulas continue to be useful in shielding calculations since they enable shield design personnel to make quick estimates of dose rates, check calculations made by more exact and time-consuming methods, and rapidly determine the scope of problems. They are also a valuable teaching tool.

The most useful approximate flux formula is that for the flux at a lateral detector point from a cylindrical source with an intervening slab shield. Such an approximate formula is given by Rockwell in Reference (a). An improved formula for this case is given by Ono and Tsuro in Reference (b). Shure and Wallace also give this formula together with function tables and a detailed survey of its accuracy in References (c) and (d). The second section of this paper gives an algorithm for significantly improving the accuracy of the formula of Ono and Tsuro.

The flux at a detector point outside the radial and axial extensions of a cylindrical source, again with an intervening slab shield, is another case of interest, but nowhere in the literature is this arrangement of source, shield, and detector point treated. In the third section of this paper an algorithm for this case is given, based on superposition of sources and the algorithm of Section II.

II. An Improved Algorithm Based on the Formula of Ono and Tsuro

The semi-analytic flux formula of Ono and Tsuro is given in References (b), (c) and (d). This formula substitutes a section of an annulus for the cylindrical source, as shown in Figure 1, and may be used to find the flux ϕ_{P_1} at the point P_1 in Figure 1 with a maximum conservative error factor of about 3.3. This formula is

$$\phi_{P_1} = S_v \frac{\phi_o}{2\pi \mu_s} L_o(\phi_o, b)\left[G(\theta_o, b) - G(\theta_o, b+\mu_s m_c R)\right], \qquad (1)$$

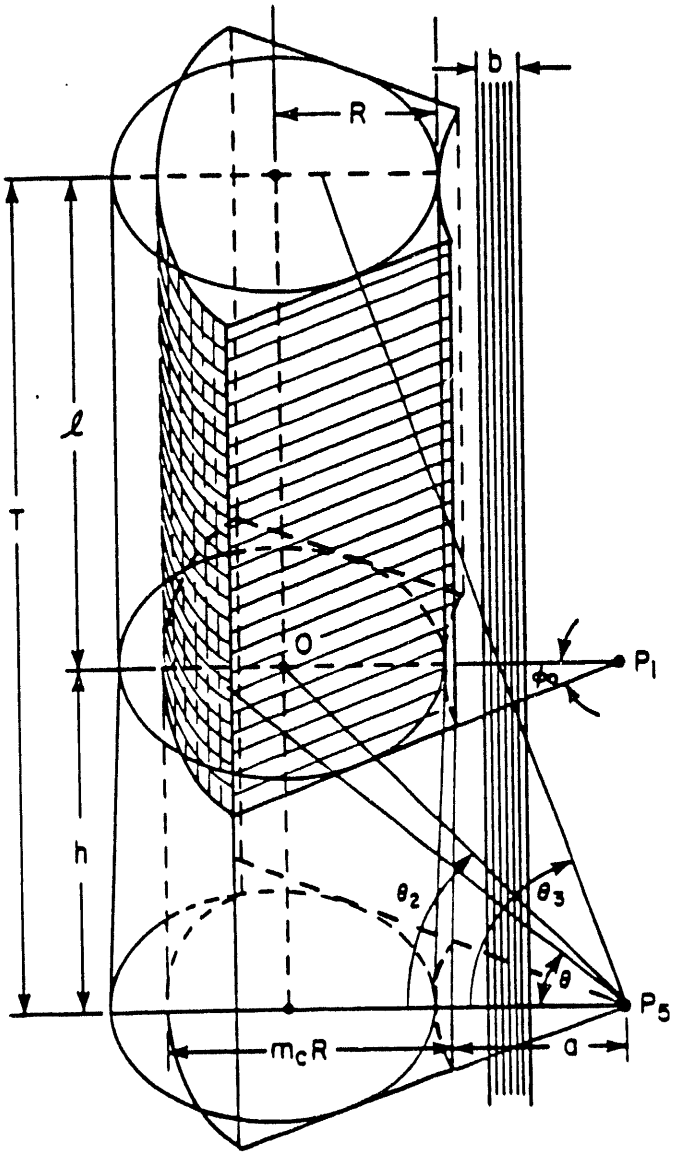

Figure 1

Cylindrical Source Approximation by Superposition
with a Slab Shield Parallel to the Source Axis

where

$$\phi_o = \sin^{-1} \frac{R}{a+R}\text{, and} \tag{2}$$

$$\theta_o = \tan^{-1} \frac{\ell}{a} \tag{3}$$

are the angles which define the base and height of the annular sector as shown in Figure 1. The units of all angles are radians.

S_v is the source strength in gamma-rays/cm^3-sec at an energy E MeV, b is the optical distance through the slab shield and μ_s is the attenuation coefficient of the source. Both b and μ_s are dependent on the gamma-ray energy E. b is a dimensionless quantity but is usually given the pseudo-units name of mean-free-paths. If the dimensions of the source, such as R, l, and a are given in centimeters, μ_s has units of cm^{-1}.

$$m_c = \sqrt{\left(\frac{a}{R}\right)^2 - \frac{\pi}{\phi_o}} - \frac{a}{R} \tag{4}$$

is a factor which preserves the area of the base of the source. Tables of the functions $L_o\,(\phi_o,\, b)$ and $G(\theta_o,\, b)$ are given in References (c) and (d). R, ℓ, and a are defined in Figure (1).

A modified algorithm has been devised from (1) which reduces the maximum conservative error in the calculated flux to a factor of about 2.1, while the calculated flux is conservative everywhere. The maximum errors occur for thin shields ($b < 0.2$) and for $a/R \leq 0.5$. This modified algorithm uses a more realistic "effective height" in (3). ℓ is the height of the cylinder and

$$\ell' = \frac{3}{2}\,\frac{\pi}{\phi_o}\,\frac{R^2 \ell a}{(a + m_c R)^3 - a^3} \tag{5}$$

is the height of the annular sector which preserves both basal area and volume. Then let $\ell'' = Min(2\ell', \ell)$ and $\ell''' = \frac{\ell'' + \ell}{2}$. If $\frac{a}{R} < 0.5$ and $\mu_s R < 5.0$ replace ℓ by ℓ''' in (3);

however, if $\frac{a}{R} \leq 0.5$ and $\mu_s R < 2.5$, replace ℓ by ℓ'' in (3). Otherwise, find θ_o, from (3) as written.

Then, using the value of θ_o found from (3), calculate ϕ_{P_1} using Equation (1).

The improved accuracy of this algorithm allows its use in a superposition method which gives an algorithm for finding the flux from the source with height ℓ at the point P_5 in Figure (1). P_5 lies outside the radial and axial extensions of the source, and no good approximation for the flux at such a point is given in the literature.

III. An Algorithm for the Flux at a Point Outside the Radial and Axial Extensions of a Cylindrical Source

The geometry of this case is given in Figure 1. The source height is ℓ. The variables defined in (2), (3), and (4) remain the same. The superimposed source height is $T = h + \ell$ in Figure 1. First find the height T' which will preserve both volume and basal area:

$$T' = \frac{3}{2}\frac{\pi}{\phi_o}\frac{R^2\,Ta}{(a + m_o R)^3 - a^3}$$

Let $T'' = Min(2T',\ T)$ and $T''' = \frac{T'' + T}{2}$.

Now if $\frac{a}{R} < 0.5$, $\mu_s R < 5.0$ and $\frac{\ell}{a} < 0.5$, replace ℓ by T''' in (3).

However, if $\frac{a}{R} < 0.5$, $\mu_s R < 2.0$ and $\frac{\ell}{a} < 0.75$, replace ℓ by T'' in (3).

For any other conditions, replace ℓ by T in (3). Then find the flux ϕ'_{P_5} for the source of height $T = h + \ell$ using (1). Next the flux ϕ''_{P_5} from the source of height h may be found by applying Algorithm I to this source, first finding h', h'', and h''' in place of ℓ', ℓ'', and ℓ'''. Finally, the desired flux at P_5 is given by

$$\Phi_{P_5} = \Phi'_{P_5} - \Phi''_{P_5}. \tag{6}$$

The error ratios for this approximate algorithm are shown in Table 1 for a wide range of problem parameters. The maximum conservative error factor is about 1.7, but some errors are non-conservative. Table 1 is therefore a valuable guide for using this algorithm. The values in Table 1 were calculated using an exact point kernel calculation and the approximate algorithm. The exact calculation method is given by Wallace in Reference (e) and was implemented in the SPAR1 program, Reference (f).

Nomenclature in Table 1 is the same as in Figure 1 except that $HP = h + \ell$ and Mus is the macroscopic attenuation coefficient of the source.

Table 1 gives error ratios for only one value of R. For $R = 10.0$ most of the error ratios decrease. The error ratios are relatively insensitive to the value of R for large b values.

As stated above, the author is aware of no analog of this algorithm anywhere in the literature. It therefore, represents a new hand calculation capability which was not heretofore available, and which extends the range of hand calculations of radiation flux.

TABLE 1

Approximate Flux/Exact Flux
from a Cylindrical Source at Detector Points Outside the Radial and Axial
Extensions of the Source with an Intervening Slab Shield Parallel to the
Source Axis, and for the Source Radius = 25.0

Mus = 0.10 and Mus*R = 2.50

a/R = 0.20

	b= .20			b= 1.00			b= 5.00			b= 10.00		
HP/L=	1.10	1.50	2.50	1.10	1.50	2.50	1.10	1.50	2.50	1.10	1.50	2.50
L/a												
.20	1.64	1.72	2.05	1.57	1.63	1.93	1.48	1.49	1.75	1.43	1.45	1.69
.50	1.60	1.64	2.32	1.52	1.55	2.07	1.40	1.40	1.57	1.36	1.32	1.33
1.00	1.49	1.74	1.65	1.41	1.56	1.45	1.28	1.21	1.09	1.18	1.08	.89
2.50	1.21	1.20	1.06	1.15	1.09	.92	1.03	.92	.52	.99	.82	.28

a/R = 0.50

	b= .20			b= 1.00			b= 5.00			b= 10.00		
HP/L=	1.10	1.50	2.50	1.10	1.50	2.50	1.10	1.50	2.50	1.10	1.50	2.50
L/a												
.20	1.76	1.82	1.95	1.71	1.76	1.87	1.60	1.61	1.61	1.53	1.50	1.43
.50	1.64	1.61	1.52	1.57	1.53	1.40	1.39	1.29	1.08	1.27	1.13	.89
1.00	1.41	1.31	1.12	1.32	1.21	1.00	1.12	1.00	.69	1.04	.90	.47
2.50	1.10	.99	.72	1.05	.92	.56	.98	.74	.16	.95	.56	.03

a/R = 1.00

	b= .20			b= 1.00			b= 5.00			b= 10.00		
HP/L=	1.10	1.50	2.50	1.10	1.50	2.50	1.10	1.50	2.50	1.10	1.50	2.50
L/a												
.20	1.45	1.48	1.50	1.43	1.45	1.46	1.37	1.37	1.31	1.33	1.30	1.17
.50	1.37	1.34	1.20	1.34	1.29	1.12	1.23	1.14	.87	1.15	1.02	.69
1.00	1.22	1.13	.91	1.18	1.07	.80	1.06	.91	.49	1.00	.82	.27
2.50	1.04	.89	.61	1.01	.82	.42	.97	.59	.06	.93	.38	.00

a/R = 2.50

	b= .20			b= 1.00			b= 5.00			b= 10.00		
HP/L=	1.10	1.50	2.50	1.10	1.50	2.50	1.10	1.50	2.50	1.10	1.50	2.50
L/a												
.20	1.22	1.22	1.20	1.21	1.21	1.18	1.19	1.18	1.10	1.17	1.15	1.02
.50	1.18	1.14	1.01	1.16	1.12	.96	1.11	1.03	.77	1.07	.96	.61
1.00	1.10	1.02	.82	1.08	.98	.72	1.02	.87	.41	.99	.78	.20
2.50	1.01	.84	.62	.99	.77	.41	.96	.51	.05	.93	.30	.00

Mus = 0.50 and Mus*R = 12.50

a/R = 0.20

	b= .20			b= 1.00			b= 5.00			b= 10.00		
HP/L=	1.10	1.50	2.50	1.10	1.50	2.50	1.10	1.50	2.50	1.10	1.50	2.50
L/a												
.20	1.54	1.53	1.50	1.48	1.45	1.41	1.34	1.30	1.22	1.29	1.24	1.13
.50	1.44	1.38	1.24	1.36	1.29	1.13	1.20	1.10	.94	1.13	1.02	.86
1.00	1.27	1.17	1.02	1.18	1.08	.93	1.04	.95	.77	1.00	.90	.66
2.50	1.05	.97	.80	1.00	.92	.71	.97	.81	.39	.95	.72	.20

a/R = 0.50

	b= .20			b= 1.00			b= 5.00			b= 10.00		
HP/L=	1.10	1.50	2.50	1.10	1.50	2.50	1.10	1.50	2.50	1.10	1.50	2.50
L/a												
.20	1.29	1.27	1.22	1.26	1.24	1.18	1.19	1.15	1.05	1.15	1.09	.96
.50	1.23	1.18	1.04	1.19	1.13	.98	1.10	1.01	.81	1.05	.94	.69
1.00	1.13	1.04	.86	1.09	.99	.78	1.00	.88	.54	.97	.81	.38
2.50	1.00	.87	.61	.98	.81	.46	.94	.63	.11	.91	.46	.02

a/R = 1.00

	b= .20			b= 1.00			b= 5.00			b= 10.00		
HP/L=	1.10	1.50	2.50	1.10	1.50	2.50	1.10	1.50	2.50	1.10	1.50	2.50
L/a												
.20	1.17	1.15	1.08	1.16	1.13	1.06	1.13	1.08	.97	1.10	1.04	.90
.50	1.14	1.07	.93	1.12	1.05	.88	1.06	.96	.71	1.02	.90	.58
1.00	1.07	.97	.76	1.04	.93	.67	.98	.82	.38	.96	.73	.19
2.50	.98	.80	.54	.96	.72	.35	.92	.48	.04	.88	.29	.00

a/R = 2.50

	b= .20			b= 1.00			b= 5.00			b= 10.00		
HP/L=	1.10	1.50	2.50	1.10	1.50	2.50	1.10	1.50	2.50	1.10	1.50	2.50
L/a												
.20	1.08	1.04	.97	1.08	1.04	.98	1.07	1.02	.91	1.06	1.00	.85
.50	1.06	1.00	.85	1.05	.99	.81	1.03	.93	.65	1.00	.88	.50
1.00	1.02	.92	.72	1.01	.89	.63	.98	.79	.33	.96	.69	.14
2.50	.97	.77	.57	.95	.69	.37	.91	.41	.04	.87	.22	.00

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