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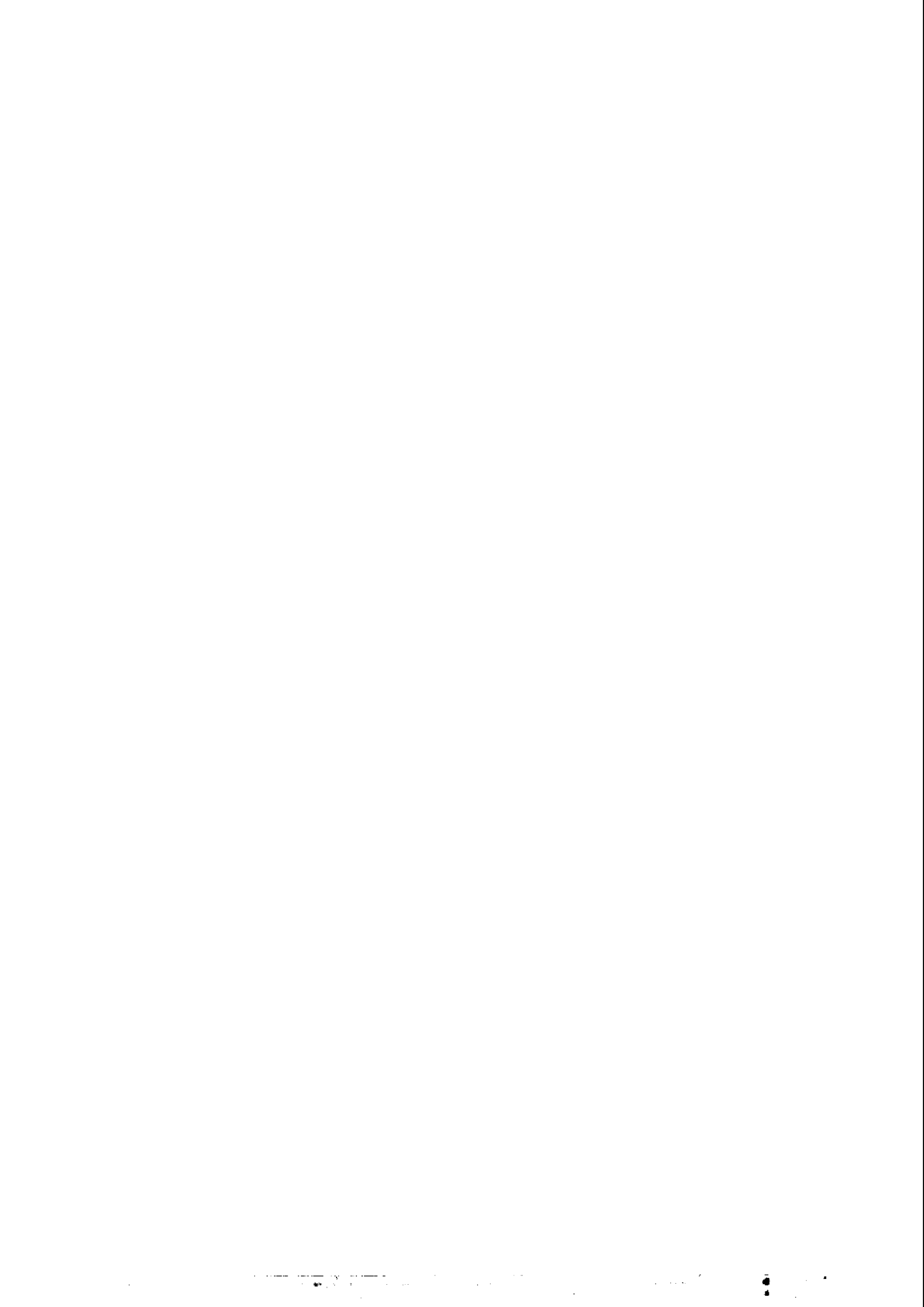**INTERNATIONAL
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BETHE ANSATZ AND QUANTUM DAVEY–STEWARTSON 1 SYSTEM WITH MULTICOMPONENT IN TWO DIMENSIONS

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ABSTRACT

The quantum 2-component DS1 system was reduced to two 1D many-body problems with $\delta-$function interactions, which were solved by Bethe ansatz. Using the ansatz in Ref.[1] and introducing symmetric and antisymmetric Young operators of the permutation group, we obtain the exact solutions for the system.

MIRAMARE – TRIESTE
January 1994

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1.   The Davey-Stewartson 1 (DS1) system is an integrable model in space of two spatial and one temporal dimensions ((2+1)D). The quantized DS1 system can be formulated in terms of Hamiltonians of quantum many-body problems in two dimensions, and some of them can be solved exactly[1][2]. Particularly, it has been shown in Ref.[2] that these 2D quantum many-body problems can be reduced to the solvable one-dimensional quantum many-body problems with two-body potentials[3]. Thus through solving the 1D many-body problems we can get the solutions of 2D's. In the present paper, we intend to generalize this idea to multicomponent DS1 system. Specifically, we will consider the case that the potential between two particles with two components in one dimension is delta-function. It is well known that the Bethe ansatz is useful and legitimate for solving 1D many-body problems with delta-function interactions[4][5]. Thus Bethe ansatz (including nested Bethe ansatz or Bethe-Yang ansatz[5]) will also play an important role for solving the 2D many-body problems induced from multicomponent DS1 system. Like 1D cases, the effects of multicomponent in 2D many-body problems cannot be trivially counted by summing single-component modes. Namely, a careful and non-trivial consideration is necessary. For definiteness, we will do so for a specific model of 2D quantum DS1 system with two components.

2.   Following usual DS1 equation[1][6], the equation for the DS1 system with two components reads

$$i\dot{\mathbf{q}} = -\frac{1}{2}(\partial_x^2 + \partial_y^2)\mathbf{q} + iA_1\mathbf{q} + iA_2\mathbf{q},\tag{1}$$

where $\mathbf{q}$ has two colour components,

$$\mathbf{q} = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix},\tag{2}$$

and

$$(\partial_x - \partial_y)A_1 = -ic(\partial_x + \partial_y)(\mathbf{q}^\dagger\mathbf{q})$$
$$(\partial_x + \partial_y)A_2 = ic(\partial_x - \partial_y)(\mathbf{q}^\dagger\mathbf{q})$$

where notation $\dagger$ means the hermitian transposition, and c is the coupling constant. Introducing the coordinates $\xi = x + y, \eta = x - y$, we have

$$A_1 = -ic\partial_\xi\partial_\eta^{-1}(\mathbf{q}^\dagger\mathbf{q}) - iu_1(\xi)\tag{3}$$
$$A_2 = ic\partial_\eta\partial_\xi^{-1}(\mathbf{q}^\dagger\mathbf{q}) + iu_2(\eta)\tag{4}$$

where

$$\partial_\eta^{-1}(\mathbf{q}^\dagger\mathbf{q}) = \frac{1}{2}(\int_{-\infty}^\eta d\eta' - \int_\eta^\infty d\eta')\mathbf{q}^\dagger(\xi,\eta',t)\mathbf{q}(\xi,\eta',t).\tag{5}$$

and u_1 and u_2 are constants of integration. According to Ref.[2], we choose them as

$$u_1(\xi) = \frac{1}{2}\int d\xi'd\eta' U_1(\xi - \xi')\mathbf{q}^\dagger(\xi',\eta',t)\mathbf{q}(\xi',\eta',t)\tag{6}$$

$$u_2(\eta) = \frac{1}{2}\int d\xi'd\eta' U_2(\eta - \eta')\mathbf{q}^\dagger(\xi',\eta',t)\mathbf{q}(\xi',\eta',t).\tag{7}$$

Thus Eq.(1) can be written as

$$i\dot{\mathbf{q}} = -(\partial_\xi^2 + \partial_\eta^2)\mathbf{q} + c[\partial_\xi \partial_\eta^{-1}(\mathbf{q}^\dagger\mathbf{q}) + \partial_\eta\partial_\xi^{-1}(\mathbf{q}^\dagger\mathbf{q})]\mathbf{q}$$
$$+\frac{1}{2}\int d\xi' d\eta'[U_1(\xi - \xi') + U_2(\eta - \eta')](\mathbf{q}'^\dagger\mathbf{q}')\mathbf{q}, \tag{8}$$

where $\mathbf{q}' = \mathbf{q}(\xi', \eta', t)$. We quantize the system with the canonical commutation relations

$$[q_a(\xi, \eta, t), q_b^\dagger(\xi', \eta', t)]_\pm = 2\delta_{ab}\delta(\xi - \xi')\delta(\eta - \eta'), \tag{9}$$

$$[q_a(\xi, \eta, t), q_b(\xi', \eta', t)]_\pm = 0. \tag{10}$$

where $a, b = 1$ or 2, $[,]_+$ and $[,]_-$ are anticommutator and commutator respectively. Then Eq.(8) can be written in the form

$$\dot{\mathbf{q}} = i[H, \mathbf{q}] \tag{11}$$

where H is the Hamiltonian of the system

$$H = \frac{1}{2}\int d\xi d\eta \left(-\mathbf{q}^\dagger(\partial_\xi^2 + \partial_\eta^2)\mathbf{q} + \frac{c}{2}\mathbf{q}^\dagger[(\partial_\xi \partial_\eta^{-1} + \partial_\eta\partial_\xi^{-1})(\mathbf{q}^\dagger\mathbf{q})]\mathbf{q} \right.$$
$$\left. +\frac{1}{4}\int d\xi' d\eta'\mathbf{q}^\dagger[U_1(\xi - \xi') + U_2(\eta - \eta')](\mathbf{q}'^\dagger\mathbf{q}')\mathbf{q} \right). \tag{12}$$

The N-particle eigenvalue problem is

$$H \mid \Psi\rangle = E \mid \Psi\rangle \tag{13}$$

where

$$\mid \Psi\rangle = \int d\xi_1 d\eta_1 \ldots d\xi_N d\eta_N \sum_{a_1 \ldots a_N} \Psi_{a_1 \ldots a_N}(\xi_1\eta_1 \ldots \xi_N\eta_N)q_{a_1}^\dagger(\xi_1\eta_1) \ldots q_{a_N}^\dagger(\xi_N\eta_N) \mid 0\rangle. \tag{14}$$

The N-particle wave function $\Psi_{a_1 \ldots a_N}$ is defined by Eq.(14), which satisfies the N-body Schrödinger equation

$$-\sum_i(\partial_{\xi_i}^2 + \partial_{\eta_i}^2)\Psi_{a_1 \ldots a_N} + c\sum_{i<j}[\epsilon(\xi_{ij})\delta'(\eta_{ij}) + \epsilon(\eta_{ij})\delta'(\xi_{ij})]\Psi_{a_1 \ldots a_N}$$
$$+\sum_{i<j}[U_1(\xi_{ij}) + U_2(\eta_{ij})]\Psi_{a_1 \ldots a_N} = E\Psi_{a_1 \ldots a_N} \tag{15}$$

where $\xi_{ij} = \xi_i - \xi_j, \delta'(\xi_{ij}) = \partial_{\xi_i}\delta(\xi_{ij})$, and $\epsilon(\xi_{ij}) = 1$ for $\xi_{ij} > 0, 0$ for $\xi_{ij} = 0, -1$ for $\xi_{ij} < 0$. Since there are products of distributions in Eq.(15), an appropriate regularization for avoiding uncertainty is necessary. This issue has been discussed in Ref.[7].

3.   Our purpose is to solve the N-body Schrödinger equation (15). The results in Ref.[2] remind us that we can make the following ansatz

$$\Psi_{a_1 \ldots a_N} = \sum_{\substack{a'_1 \ldots a'_N \\ b'_1 \ldots b'_N}} \prod_{i<j}(1 - \frac{c}{4}\epsilon(\xi_{ij})\epsilon(\eta_{ij}))$$

$$\times \mathcal{M}_{a_1 \ldots a_N, a'_1 \ldots a'_N} \mathcal{N}_{a_1 \ldots a_N, b'_1 \ldots b'_N} X_{a'_1 \ldots a'_N}(\xi_1 \ldots \xi_N) Y_{b'_1 \ldots b'_N}(\eta_1 \ldots \eta_N) \quad (16)$$

where $\mathcal{M}$ and $\mathcal{N}$ are matrices independent of the coordinates of ξ and η;
$X_{a_1 \ldots a_N}(\xi_1 \ldots \xi_N)$ and $Y_{b_1 \ldots b_N}(\eta_1 \ldots \eta_N)$ are required to satisfy the following one- dimensional N-body Schrödinger equations with two-body potentials

$$-\sum_i \partial^2_{\xi_i} X_{a_1 \ldots a_N} + \sum_{i<j} U_1(\xi_{ij}) X_{a_1 \ldots a_N} = E_1 X_{a_1 \ldots a_N} \quad (17)$$

$$-\sum_i \partial^2_{\eta_i} Y_{b_1 \ldots b_N} + \sum_{i<j} U_2(\eta_{ij}) Y_{b_1 \ldots b_N} = E_2 Y_{b_1 \ldots b_N} \quad (18)$$

where $E_1 + E_2 = E$. At this stage $\mathcal{M}$ and $\mathcal{N}$ are unknown temporarily. It is expected that after 1D many-body problems (i.e., Eqs.(17) (18)) are solved, we could construct the solutions $\Psi_{A_1 \ldots A_N}$ for 2D many-body problems Eq.(15) through constructing an appropriate $\mathcal{M} \times \mathcal{N}$- matrix.

Now, let us consider the case of $U_1(\xi_{ij}) = 2g\delta(\xi_{ij})$ and $U_2(\eta_{ij}) = 2g\delta(\eta_{ij})$ ($g > 0$, the coupling constant). Then Eqs.(17) and (18) become

$$-\sum_i \partial^2_{\xi_i} X_{a_1 \ldots a_N} + 2g \sum_{i<j} \delta(\xi_{ij}) X_{a_1 \ldots a_N} = E_1 X_{a_1 \ldots a_N} \quad (19)$$

$$-\sum_i \partial^2_{\eta_i} Y_{b_1 \ldots b_N} + 2g \sum_{i<j} \delta(\eta_{ij}) Y_{b_1 \ldots b_N} = E_2 Y_{b_1 \ldots b_N}. \quad (20)$$

If X and Y are wave functions of Fermions with two components, denoted by X^F and Y^F, the problem has been solved by Yang [6] (more explicitly, see Ref.[8] and Ref.[9]). According to the Bethe ansatz, the continual solution of Eq.(15) in the region of $0 < \xi_{Q_1} < \xi_{Q_2} < \ldots < \xi_{Q_N} < L$ reads

$$X^F = \sum_P \alpha_P^{(Q)} \exp\{i[k_{P_1}\xi_{Q_1} + \ldots + k_{P_N}\xi_{Q_N}]\}$$

$$= \alpha_{12 \ldots N}^{(Q)} e^{i(k_1\xi_{Q_1} + k_2\xi_{Q_2} + \ldots + k_N\xi_{Q_N})} + \alpha_{21 \ldots N}^{(Q)} e^{i(k_2\xi_{Q_1} + k_1\xi_{Q_2} + \ldots + k_N\xi_{Q_N})}$$

$$+(N! - 2) \text{ others terms} \quad (21)$$

where $X^F \in \{X^F_{a_1 \ldots a_N}\}$, $P = [P_1, P_2, \cdots, P_N]$ and $Q = [Q_1, Q_2, \cdots, Q_N]$ are two permutations of the integers $1, 2, \ldots, N$, and

$$\alpha^{(Q)}_{\ldots ij \ldots} = Y^{lm}_{ji} \alpha^{(Q)}_{\ldots ji \ldots}, \quad (22)$$

$$Y^{lm}_{ji} = \frac{-i(k_j - k_i)P^{lm} + g}{i(k_j - k_i) - g} \quad (23)$$

The eigenvalue is given by

$$E_1 = k_1^2 + k_2^2 + \ldots + k_N^2, \quad (24)$$

where $\{k_i\}$ are determined by the Bethe ansatz equations,

$$e^{ik_j L} = \prod_{\beta=1}^{M} \frac{i(k_j - \Lambda_\beta) - g/2}{i(k_j - \Lambda_\beta) + g/2} \tag{25}$$

$$\prod_{j=1}^{N} \frac{i(k_j - \Lambda_\alpha) - g/2}{i(k_j - \Lambda_\alpha) + g/2} = -\prod_{\beta=1}^{M} \frac{i(\Lambda_\alpha - \Lambda_\beta) + g}{i(\Lambda_\alpha - \Lambda_\beta) - g} \tag{26}$$

with $\alpha = 1, \ldots, M, j = 1, \ldots, N$. Through exactly same procedures we can get the solution Y^F and E_2 to Eq.(20).

In the Bosonic case, the wave-functions, denoted by X^B and Y^B, are given by

$$X^B = \sum_P \beta_P^{(Q)} \exp\{i[k_{P_1}\xi_{Q_1} + \ldots + k_{P_N}\xi_{Q_N}]\} \tag{27}$$

$$\beta_{\ldots ij\ldots}^{(Q)} = Z_{ji}^{lm}\beta_{\ldots ji\ldots}^{(Q)}, \tag{28}$$

$$Z_{ji}^{lm} = \frac{i(k_j - k_i)P^{lm} + g}{i(k_j - k_i) - g} \tag{29}$$

and the Bethe ansatz equations are as follows[9]

$$e^{ik_j L} = -\prod_{i=1}^{N} \frac{k_j - k_i + ig}{k_j - k_i - ig} \prod_{\beta=1}^{M} \frac{\Lambda_\beta - k_j + g/2}{\Lambda_\beta - k_j - g/2} \tag{30}$$

$$\prod_{\alpha=1}^{M} \frac{\Lambda_\beta - \Lambda_\alpha + ig}{\Lambda_\beta - \Lambda_\alpha - ig} = -\prod_{j=1}^{N} \frac{\Lambda_\beta - k_j + ig/2}{\Lambda_\beta - k_j - ig/2}. \tag{31}$$

Similarly for Y^B. It is well known that X^F and $Y^F (X^B$ and $Y^B)$ are antisymmetric (symmetric) when the coordinates and the colour-indices of the particles are interchanged simultaneously.

4.   For permutation group $S_N : \{e_i, i = 1, \cdots, N!\}$, the totally symmetric Young operator is

$$\mathcal{O}_N = \sum_{i=1}^{N!} e_i, \tag{32}$$

and the totally antisymmetric Young operator is

$$\mathcal{A}_N = \sum_{i=1}^{N!} (-1)^{P_i} e_i. \tag{33}$$

The Young diagram for $\mathcal{O}_N$ is $\boxed{1\ 2\ 3\ \cdots\ N}$, and for $\mathcal{A}_N$, it is $\begin{array}{|c|}\hline 1 \\ \hline 2 \\ \hline \vdots \\ \hline N \\ \hline \end{array}$. To S_3, for example, we have

$$\mathcal{O}_3 = 1 + P^{12} + P^{13} + P^{23} + P^{12}P^{23} + P^{23}P^{12}. \tag{34}$$

$$\mathcal{A}_3 = 1 - P^{12} - P^{13} - P^{23} + P^{12}P^{23} + P^{23}P^{12}. \tag{35}$$

Lemma 1: $(\mathcal{O}_N X_F)(\xi_1, \xi_2, \cdots, \xi_N)$ is antisymmetric with respect to the coordinate's interchanges of $(\xi_i \longleftrightarrow \xi_j)$.

Proof: From the definition of $\mathcal{O}_N$ (Eq.(32)), we have

$$\mathcal{O}_N P^{ab} = P^{ab}\mathcal{O}_N = \mathcal{O}_N. \tag{36}$$

To $N = 3$ case, for example, the direct calculations show $\mathcal{O}_3 P^{12} = P^{12}\mathcal{O}_3 = \mathcal{O}_3, \mathcal{O}_3 P^{23} = P^{23}\mathcal{O}_3 = \mathcal{O}_3$ and so on. Using Eqs.(36) and (23), we have

$$\mathcal{O}_N Y_{ij}^{lm} = (-1)\mathcal{O}_N. \tag{37}$$

From Eqs.(21) and (23), X^F can be written as

$$
\begin{aligned}
X^F =& \{ e^{i(k_1\xi_{Q_1}+k_2\xi_{Q_2}+\ldots+k_N\xi_{Q_N})} + Y_{12}^{12}e^{i(k_2\xi_{Q_1}+k_1\xi_{Q_2}+\ldots+k_N\xi_{Q_N})} \\
&+ Y_{13}^{23}Y_{12}^{12}e^{i(k_2\xi_{Q_1}+k_3\xi_{Q_2}+k_1\xi_{Q_3}+\ldots+k_N\xi_{Q_N})} + (N!-3)\text{ other terms }\}\alpha_{12\ldots N}^{(Q)}
\end{aligned}
\tag{38}
$$

Using Eqs.(37) and (38), we obtain

$$
\begin{aligned}
(\mathcal{O}_N X^F)(\xi_1, \cdots, \xi_N) =& \{ e^{i(k_1\xi_{Q_1}+k_2\xi_{Q_2}+\ldots+k_N\xi_{Q_N})} - e^{i(k_2\xi_{Q_1}+k_1\xi_{Q_2}+\ldots+k_N\xi_{Q_N})} \\
&+ e^{i(k_2\xi_{Q_1}+k_3\xi_{Q_2}+k_1\xi_{Q_3}+\ldots+k_N\xi_{Q_N})} + (N!-3)\text{ other terms }\}\mathcal{O}_N\alpha_{12\ldots N}^{(Q)} \\
=& \sum_P (-1)^P \exp\{i[k_{P_1}\xi_{Q_1} + \ldots + k_{P_N}\xi_{Q_N}]\}(\mathcal{O}_N\alpha_{12\ldots N}^{(Q)}).
\end{aligned}
\tag{39}
$$

Therefore we conclude that $(\mathcal{O}_N X^F)(\xi_1, \cdots, \xi_N)$ is antisymmetric with respect to $(\xi_i \longleftrightarrow \xi_j)$.

Lemma 2: $(\mathcal{A}_N X^B)(\xi_1, \xi_2, \cdots, \xi_N)$ is antisymmetric with respect to the coordinate's interchanges of $(\xi_i \longleftrightarrow \xi_j)$.

Proof: Noting (see Eqs.(33) (29) (27))

$$\mathcal{A}_N P^{ab} = P^{ab}\mathcal{A} = -\mathcal{A}_N, \tag{40}$$
$$\mathcal{A}_N Z_{ij}^{lm} = (-1)\mathcal{A}_N, \tag{41}$$

we then have

$$(\mathcal{A}_N X^B)(\xi_1, \cdots, \xi_N) = \sum_P (-1)^P \exp\{i[k_{P_1}\xi_{Q_1} + \ldots + k_{P_N}\xi_{Q_N}]\}(\mathcal{A}_N\beta_{12\ldots N}^{(Q)}). \tag{42}$$

Then the Lemma is proved.

5. The ansatz of Eq.(10) can be compactly written as

$$\Psi = \prod_{i<j}(1 - \frac{c}{4}\epsilon(\xi_{ij})\epsilon(\eta_{ij}))(\mathcal{M}X)(\mathcal{N}Y) \tag{43}$$

where $(\mathcal{M}X)$ and $(\mathcal{N}Y)$ are required to be antisymmetric under the interchanges of the coordinate variables. According to Lemmas 1 and 2, we see that

$$\mathcal{M}, \mathcal{N} = \begin{cases} \mathcal{O}_N & \text{for 1D Fermion} \\ \mathcal{A}_N & \text{for 1D Boson.} \end{cases} \tag{44}$$

If the DS1 fields $q_a(\xi, \eta)$ in Eq.(1) are (2+1)D Bose fields, the commutators ($[,]_-$, see (9) and (10)) must be used to quantize the system and the 2D many-body wave functions denoted by Ψ^B must be symmetric under the simultaneous colour-interchange ($a_i \longleftrightarrow a_j$) and the coordinate-interchange (($\xi_i \eta_i$) $\longleftrightarrow$ ($\xi_j \eta_i$)). Namely, the 2D Bose wave functions Ψ^B must satisfy

$$P^{a_i a_j} \Psi^B \mid_{\xi_i \eta_i \longleftrightarrow \xi_j \eta_j} = \Psi^B. \tag{45}$$

For q_a, (2+1)D Fermi fields, the anticommutators should be used, and Ψ^F must be antisymmetric under simultaneous interchange of ($a_i \longleftrightarrow a_j$) and (($\xi_i \eta_i$) $\longleftrightarrow$ ($\xi_j \eta_i$)), namely,

$$P^{a_i a_j} \Psi^F \mid_{\xi_i \eta_i \longleftrightarrow \xi_j \eta_j} = -\Psi^F. \tag{46}$$

Thus for the 2D Boson case, two solutions of Ψ^B can be constructed as following

$$\Psi_1^B = \prod_{i<j}(1 - \frac{c}{4}\epsilon(\xi_{ij})\epsilon(\eta_{ij}))[\mathcal{O}_N X^F(\xi_1 \cdots \xi_N)][\mathcal{O}_N Y^F(\eta_1 \cdots \eta_N)], \tag{47}$$

$$\Psi_2^B = \prod_{i<j}(1 - \frac{c}{4}\epsilon(\xi_{ij})\epsilon(\eta_{ij}))[\mathcal{A}_N X^B(\xi_1 \cdots \xi_N)][\mathcal{A}_N Y^B(\eta_1 \cdots \eta_N)]. \tag{48}$$

Using Eqs.(36),(39),(40) and (42), we can check Eq.(45) directly. In addition, from the Bethe ansatz equations (25) (26) (30) (31) and $E = E_1 + E_2$, we can see that the eigenvalues of Ψ_1^B and Ψ_2^B are different from each other generally, i.e., the states corresponding to Ψ_1^B and Ψ_2^B are non-degenerate.

For the 2D Fermion case, the desired results are

$$\Psi_1^F = \prod_{i<j}(1 - \frac{c}{4}\epsilon(\xi_{ij})\epsilon(\eta_{ij}))[\mathcal{O}_N X^F(\xi_1 \cdots \xi_N)][\mathcal{A}_N Y^B(\eta_1 \cdots \eta_N)], \tag{49}$$

$$\Psi_2^F = \prod_{i<j}(1 - \frac{c}{4}\epsilon(\xi_{ij})\epsilon(\eta_{ij}))[\mathcal{A}_N X^B(\xi_1 \cdots \xi_N)][\mathcal{O}_N Y^F(\eta_1 \cdots \eta_N)]. \tag{50}$$

Eq.(46) can also be checked directly. The eigenvalues corresponding to Ψ^F are also determined by the Bethe equations and $E = E_1 + E_2$.

The proof given in Ref.[2] can be extended to show that $\Psi_{1,2}^B$ and $\Psi_{1,2}^F$, shown above, are also the exact solutions of Eq.(15). Thus we conclude that the 2D quantum many-body problem induced from the quantum DS1 system with 2-component has been solved exactly.

6.   To summarize, we formulated the quantum multicomponent DS1 system in terms of the quantum multicomponent many-body Hamiltonian in 2D space. Then we reduced this 2D Hamiltonian to two 1D multicomponent many-body problems. As the potential between two particles with two components in one dimension is $\delta-$function, the Bethe ansatz was used to solve these 1D problems. By using the ansatz of Ref.[1] and introducing

some useful Young operators, we presented a new ansatz for fusing two 1D-solutions to construct 2D wave functions of the quantum many-body problem which is induced from the quantum 2-component DS1 system. There are two types of wave functions: Bosons and Fermions. Both of them satisfy the 2D many-body Schrödinger equation of the DS1 system exactly.

Acknowledgments

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