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HARMONIC ANALYSIS OF THE AGS BOOSTER IMPERFECTION*

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ABSTRACT

The harmonic content of magnetic field imperfections in the AGS Booster has been determined through careful measurements of the required field corrections of transverse resonances. An analysis of the required correction yielded amplitude and phase information which points to possible sources of imperfections. Dipole and quadrupole imperfections, which are proportional to the field of bending magnets (B), are mainly driven by any misalignment of the magnets. Quadrupole and sextupole imperfections, which are proportional to dB/dt, are driven by imperfections of the eddy-current correction system. The observations also suggest the presence of a remnant field.

INTRODUCTION

The AGS Booster[1] is a rapid cycling synchrotron which accelerates protons, polarized protons and heavy-ion beams. The basic parameters of the Booster are listed in Table I. One of the characteristics of this machine is a correction system of the eddy-current field.[2] Without this system the eddy current induced along the beam duct in the bending magnet would produce a strong sextupole field because (dB/dt)/B is as high as 40 sec^{-1}. Another characteristic is a large tune spread of the proton beam doe to the space-charge tune shift. The tune diagram and the expected tune spread at the designed intensity (1.5×10^{13}ppp) are shown in Fig.1. High-intensity operation required a careful correction of the stop-bands (transverse resonances) up to the 3rd order.[3,4] These measurements of the required field corrections gave harmonic contents of magnetic field imperfections in the AGS Booster, which contained information about possible sources of imperfections.

The observed harmonic imperfections and corresponding resonances were:

Table I Basic parameters of the AGS Booster.

focusing function		FODO
periodicity		24
super periodicity		6
circumference		202 m
physical aperture	H/V	±76 / ±35 mm
operation cycle	(proton)	5 Hz
typical tune	ν_H/ν_V	4.78/ 4.82
Proton energy	Inj/Ext.	0.2 / 1.5 GeV

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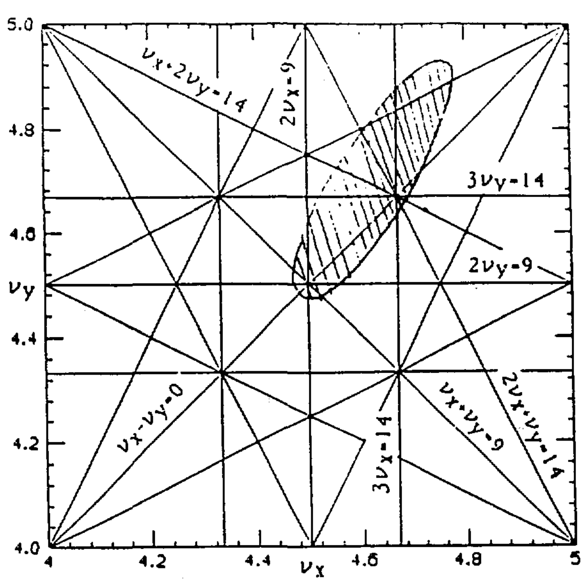

normal quadrupole
- 9th $2\nu_x=9$ and $2\nu_y=9$

skew quadrupole
- 0th $\nu_x-\nu_y=0$
- 9th $\nu_x-\nu_y=9$

normal sextupole
- 14th $3\nu_x=14$ and $\nu_x+2\nu_y=14$
- 9th $2\nu_x=9$ and $2\nu_y=9$ (down feeding)

skew sextupole
- 14th $2\nu_x+\nu_y=14$
- 9th $\nu_x+\nu_y=9$ (down feeding)

normal octupole
- 9th $2\nu_x=9$ and $2\nu_y=9$ (down feeding)
- 14th $3\nu_x=14$ (down feeding).

Fig. 1 Tune diagram of the AGS Booster.

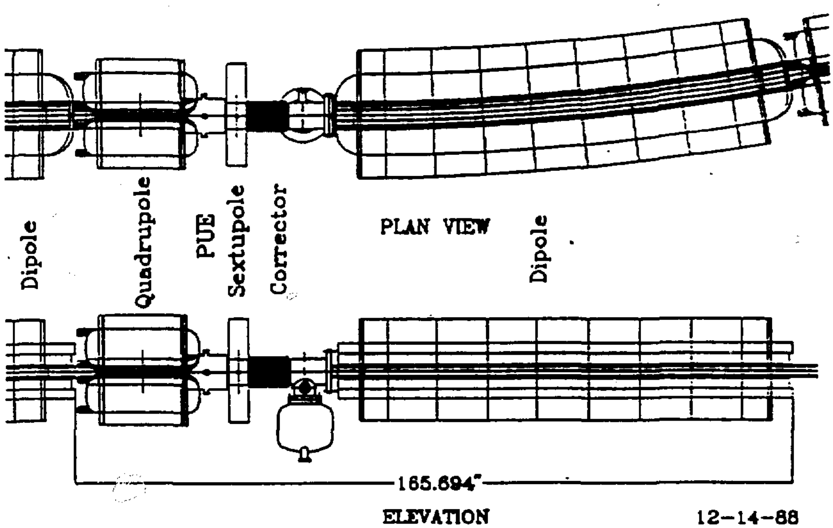

Fig. 2 Unit cell of the AGS Booster and correction elements[1].

Fig.2 shows one unit cell (half of a period) of the Booster. Normal quadrupole and normal sextupole corrections are applied through trim windings of the quadrupole magnets and the chromaticity control sextupoles, respectively. Dipoles, skew quadrupole and skew sextupole corrections are applied through each winding of the correction magnets.

The concept of down feeding is not common, but its principle is simple. To explain this effect we now consider the 9th-harmonic quadrupole imperfection, which produces half-integer stop-bands of

$2\nu_x=9$ and $2\nu_y=9$. A displacement of Xo at a sextupole magnet with the strength of S produces a perturbation quadrupole of 2S Xo

$$S\,(X+Xo)^2 = [S\,Xo^2] + [2S\,Xo]\,X + S\,X^2 \qquad (1)$$

It is important to distinguish between two kinds of down-feeding. One is a combination of C.O.D. and a systematic sextupole field (mainly due to the 9th C.O.D. and 0th harmonic sextupole field). The other is a combination of a dispersion function and a sextupole imperfection (mainly due to a displacement of the mean radius and the 9th harmonic sextupole imperfection). The first one changes the quadrupole imperfection, as schematically shown in Fig.3. An opposite harmonic quadrupole field can compensate for this imperfection. On the other hand, the second one produces a momentum (dP/P) dependence of the quadrupole imperfection. This effect is measured by determining the dependence of the quadrupole correction on the mean radius (or the mean momentum displacement). Unlike the first one, only a sextupole field correction can compensate for this effect, because the beam has a momentum spread. At the same time a measurement of the second kind of down-feeding enabled us to estimate the sextupole imperfection by observing the quadrupole imperfection.

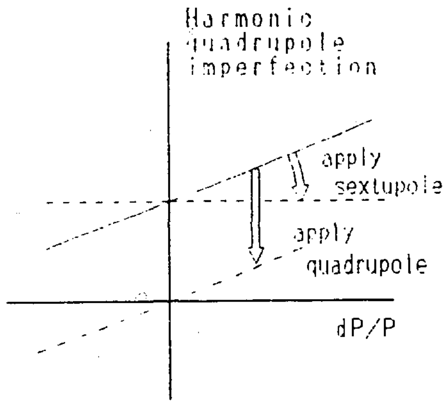

Fig. 3 Effect of two kinds of down-feeding.

In this report the observed strengths of harmonic imperfections are analyzed and compared with the estimated imperfection strengths from field measurements of the magnets.[5]

IMPERFECTION MEASUREMENT

Resonances, except for $\nu_x-\nu_y=0$, were observed by programming tunes so as to pass through each resonance at various timings during the magnet cycle, which meant at various B (bending field) and dB/dt. The amount of beam loss due to the resonance crossing was measured for several different correction settings in order to determine the setting necessary to minimize the loss. The strength of imperfection which drove $\nu_x-\nu_y=0$ was determined by W. van Asselt. He applied a skew quadrupole correction to decouple the horizontal and vertical betatron oscillations.

The correction strength was fitted with the following function:

$$N(xxx) = Co + Cb\,B + Cbt\,(dB/dt) \quad . \qquad (2)$$

The unit of correction strength (N(xxx)) was selected for convenience of control.[3] The xxx in the brackets will be replaced by a name indicating its harmonics. Co, Cb and Cbt are fitting parameters which correspond to a remnant field imperfection, misalignment and magnet production errors (we refer to it as the 'B-term') and an eddy-current imperfection, respectively. This parameterization enabled us to apply an appropriate correction for an arbitrary magnet cycle.

The second kind of down-feeding was measured by observing the dependence on the mean radius (dR). The coefficient between dR and the momentum displacement (dP/P) is (dR)/(dP/P)=319 cm.

The results of the measurement are summarized in Table II.

Table II Stop-band correction parameters. The values in the brackets are the results of measurements at the flat porch: B=1.7kG and dB/dt=0 kG/s.

resonances		Co	Cb(/kG)	Cbt(ms/G)
$2\nu_x=9$	N(cos9X)	33±130	101±31	5.5±3.8
	N(sin9X)	-12± 70	122±64	-1.5±1.3
	δN(cos9X)/δ[dR]	75± 40	-3±12	1.06±0.3
	δN(sin9X)/δ[dR]	52± 40	13±12	0.45±0.3
	δN(cos9X)/δ[dR²]	(-13± 7)		
	δN(sin9X)/δ[dR²]	(10± 7)		
$2\nu_y=9$	N(cos9Y)	138± 18	91± 7	3.36±0.11
	N(sin9Y)	-43± 26	39± 9	-6.30±0.20
	δN(cos9Y)/δ[dR]	49± 25	21± 9	0.94±0.18
	δN(sin9Y)/δ[dR]	-22± 9	1± 3	-0.44±0.06
	δN(cos9Y)/δ[dR²]	(-10± 9)		
	δN(sin9Y)/δ[dR²]	(15± 9)		
$\nu_x-\nu_y=0$	N(cos0XY)	-180	140	0
$\nu_x+\nu_y=9$	N(cos9XY)	35± 55	49.2±7.2	0.04±0.53
	N(sin9XY)	-111± 45	28.5±6.0	-0.11±0.41
	δN(cos9XY)/δ[dR]	-19.9± 1.0	-0.4±0.6	0.024±0.03
	δN(sin9XY)/δ[dR]	9.8± 1.0	1.6±0.6	0.044±0.03
$3\nu_x=14$	N(cos14XXX)	48± 70	-31±34	3.49±0.43
	N(sin14XXX)	-129± 34	40±16	6.00±0.20
	δN(cos14XXX)/δ[dR]	(69)		
	δN(sin14XXX)/δ[dR]	(-63)		
$\nu_x+2\nu_y=14$	N(cos14XYY)	5± 29	14±11	4.74±0.20
	N(sin14XYY)	-103± 24	17± 9	2.64±0.19
$2\nu_x+\nu_y=14$	N(cos14XXY)	720±120	-152±42	6.8±0.7
	N(sin14XXY)	604± 81	30±30	-0.3±0.6

HARMONIC ANALYSIS OF C.O.D.

The tune dependence of the 4th- and 5th-harmonic amplitudes of the horizontal and vertical C.O.D. were measured by K. Brown *et al.*[6] when dipole corrections were not applied. They calculated the harmonic amplitudes of the C.O.D. based on the beam positions at 22 PUEs for each direction. The n-th harmonic complex amplitude ($A_n + iB_n$) was fitted with the following function:

$$A_n+iB_n = (A_{no}+iB_{no})+(\nu^2/(\nu^2-n^2))(a_n+ib_n) . \qquad (3)$$

Here, ν is a tune and A_{no}, B_{no}, a_n and b_n are fitting parameters. The tune-independent term ($A_{no}+iB_{no}$) represents an off-set or displacement of the PUEs. The tune-dependent term (a_n+ib_n) represents the dipole field errors. There is a strong correlation between the tune-independent term and tune-dependent term, as shown in Fig.4. That correlation means that the dipole error was mainly produced by displacements of the quadrupole magnets; and the displacement of the quadrupole magnet and a PUE nearby had a strong correlation. When their displacements were the same (reasonable assumption since they were very close as shown in Fig.2) we expected the following correlation:[7]

$$(A_{no}+iB_{no}) = -1.46(1\pm0.56)(a_n+ib_n) . \qquad (4)$$

Here, ±0.56 presents the $\pm\sigma$ expected to be produced by random displacements of quadrupole magnets, with which no PUE for that direction accompanied. The observed correlation was roughly in the expected area.

There were an independent data that confirmed this result. The dependence of the quadrupole imperfection on the chromaticity (a measurement of the first kind of down-feeding) showed the existence of about 1mm horizontal displacements at the chromaticity sextupoles. On the other hand, the harmonic amplitude of the dipole errors ($|a_n+ib_n|$) expected from the magnet production errors were only 0.06 and 0.02mm for the horizontal and vertical direction, respectively.

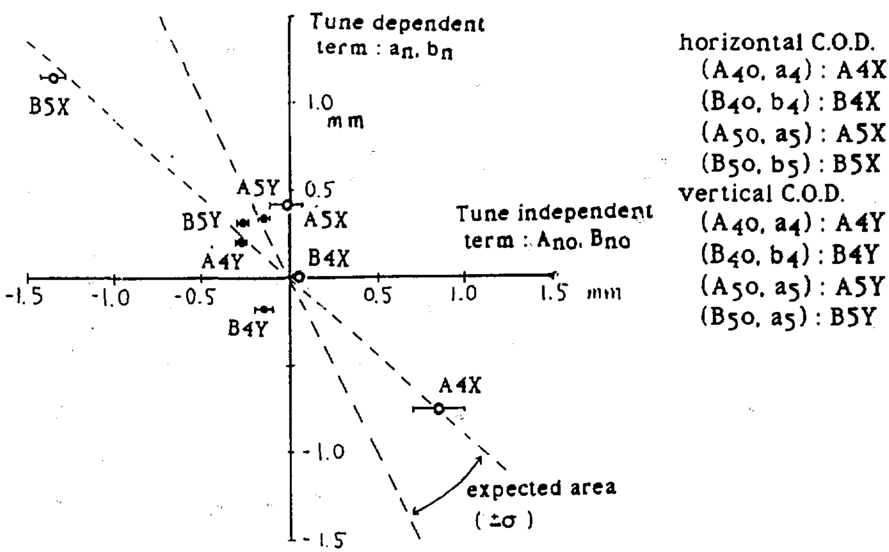

Fig. 4 Off-set and tune-dependent terms of the harmonic components of the C.O.D.

HARMONIC ANALYSIS OF B-TERMS

The B-terms can be estimated independently from the data produced by field measurements of the magnets.[5] We compare the observed strengths of the imperfections with the estimated values in Table III. The contribution of the magnet production error and the contribution of the down-feeding were calculated separately.

The observed strengths of the normal quadrupole imperfection were almost the same as the estimations, a main source of which was a combination of the 9th-harmonic C.O.D. and sextupole fields at the edge of the bending dipoles. When we need to correct the half-integer resonance with sufficient accuracy (stop-band width less than 10^{-4}) we will have to stabilize the C.O.D. to within 0.03 mm because of the first kind of down feeding. Here, although a very small C.O.D. is not required, the C.O.D. should be fixed. This is not easy with the present system.

The observed strength of the skew quadrupole imperfection was about 10-times larger than the observed strength of the normal quadrupole imperfection, even though the estimated strength is comparable, or even smaller. We have no explanation for this fact.

The normal and skew sextupole imperfections were roughly the same, and were about 3-times larger than the estimated strengths. There is no reason that the skew sextupole correction is not required, though the normal sextupole correction is required.

The observed strength of the octupole imperfections listed in Table III were over estimated because the values were the sum of the B-term and

Table III B-terms. The units of the quadrupole imperfections are the stop-band width. The units of sextupole imperfections are the harmonic integrated sextupole component. The units of the octupole imperfections are harmonic integrated octupole field strength at B=1.7kG.

Imperfection ‹ unit › / resonance	Observed	Estimation: magnet imperfection	Estimation: down feeding	Estimation: total
normal quadrupole ‹ × 10^{-3} ›				
$2\nu_x=9$	3.3	1.5	3.2	3.5
$2\nu_y=9$	3.1	1.5	2.7	3.1
skew quadrupole ‹ × 10^{-3} ›				
$\nu_x-\nu_y=0$	27	2.0	0.9	2.2
$\nu_x+\nu_y=9$	11	0.9	0.9	1.3
normal sextupole ‹ × 10^{-3} /m›				
$3\nu_x=14$, $\nu_x+2\nu_y=14$, $2\nu_x=9$, $2\nu_y=9$	40, 20, 40, 20	10	0.7	10
skew sextupole ‹ × 10^{-3} /m›				
$2\nu_x+\nu_y=14$, $\nu_x+\nu_y=9$,	20, 30	6	0.8	6
normal octupole ‹ T/m^2 ›				
$3\nu_x=14$, $2\nu_x=9$, $2\nu_y=9$	5.6, 4., 6.	0.4	0.1	0.4

the remnant field imperfection. If the remnant field imperfection was negligible the strength of the observed normal octupole imperfection was about 10-times larger than the estimated value

EDDY-CURRENT TERM

The eddy-current terms of normal quadrupole and normal sextupole imperfections were considerably large. On the other hand, the eddy-current terms of the dipole and skew quadrupole imperfections were undetectably small. The eddy-current term of the skew sextupole imperfection was much smaller than that of a normal sextupole imperfection. This result suggested an imperfection of the eddy-current correction system, which applied normal quadrupole and normal sextupole fields.

There is only one quadrupole eddy current correction coil, where the shape of the beam duct is special in order to inject the proton beam. That location is called C5. The phase of the observed normal quadrupole imperfection was just at the bending magnet of C5. Fig.5 shows the observed imperfection and the expected imperfection when the correction winding at C5 was misconnected with opposite polarity. Although the amplitude could have a considerable error,[8] the phase could not. We thus have sufficient reason to be suspicious about the connection of the correction coil at C5.

The strength of the observed normal sextupole eddy-current term was roughly the same as the expected strength when one of the correction coils was disconnected. If one of sextupole correction coils was misconnected we should have observed a large dipole dB/dt term, however we did not.[11] In addition, there is no one correction coil whose phase is the same as the observed imperfections. The imperfection should have produced by more than two coils or other reasons. The connections of the sextupole coils and the monitoring windings of dB/dt, except for 3 coils at A4, C5 and F7, were checked[9] after the experiment. Another possibility was a variation of the eddy current sextupole field of the beam duct. E. Blesser observed an uncorrected eddy-current sextupole field of about 15%.[5] The 15% random variation of 36 magnet makes 90%, which explains what we observed. However we are not sure about it, since we do not know any details concerning the measurements.

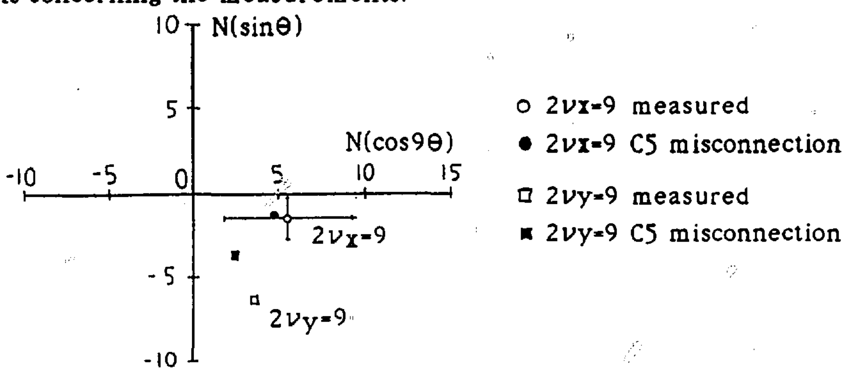

Fig. 5 Eddy-current terms of normal quadrupole imperfection.

REMNANT FIELD TERM

We observed a strong remnant field imperfection. There is no one location at which the phase fits the phases of all the observed imperfections. One possible error is a variation in the remnant fields of the magnets. E. Blesser observed about 0.9 mT field variation of the quadrupole magnets at low current,[10] which explains the strength of the observed normal quadrupole terms though that variation could be only a measurement error.

The strengths of the remnant field components are listed in Table V. At dR=70mm the field strength of each component is roughly the same, except for the normal quadrupole component. This means that the remnant field changed transversely with the scale of the beam duct. At dR=70mm the strength of each component was about 20 Gauss m.

To estimate the order of strength of the remnant field, we divided this by the circumference of the ring. We also assumed that the number of random remnant field error sources is roughly the same as the periodicity:24 (because this is also a number of locations with large β_x or β_y, which are the weight functions of the strengths of the resonances). The strength of the random remnant error field was estimated to be on the order of 0.5 Gauss, which is rather weak and is comparable to the Earth's field.

Table V Strength of harmonic imperfection of the remnant field.

field multipole	imperfection
normal quad.	3×10^{-3} T
skew quad.	3×10^{-2} T
normal sext.	4×10^{-1} T/m
skew sext.	5×10^{-1} T/m
normal oct.	5×10^{0} T/m^2

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