ORDER AND CHAOS IN NUCLEAR WAVE FUNCTIONS

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Abstract

An investigation of the order and chaos of the nuclear excited states has shown that there is order in the large and chaos in the small quasiparticle or phonon components of the nuclear wave functions. The order-to-chaos transition is treated as a transition from the large to the small components of the nuclear wave function. The analysis has shown that relatively large many-quasiparticle components of the wave function at an excitation energy $(4 \div 8)$ MeV may exist. The large many-quasiparticle components of the wave functions of the neutron resonances are responsible for enhanced E1-, M1- and E2- transition probabilities from neutron resonances to levels lying $(1 \div 2)$ MeV below them.

1. Introduction

Much attention has recently been paid to an interplay between order and chaos in nuclei [1]. Studies concerning the level spacing distribution in nuclei have usually identified chaos in nuclei via an agreement with Gaussian Orthogonal Ensemble (GOE) statistics. The GOE distribution of the level density cannot prove that the nuclear structure is chaotic, since the behavior of the level density gives only restricted information about the nuclear wave functions. The nuclear wave function of the excited state with an energy more than $(2 \div 4)$ MeV usually has many components with different quasiparticle numbers, with different isospin quantum numbers T_0 and $T_0 + 1$, with different K quantum numbers and so on. Such wave functions are superpositions of several interacting GOE spectra. In [2] this was demonstrated using a simple soluble model in which the appearance of a GOE-type distribution function for the nearest-neighbor level spacing does not directly correspond to a dissolution of the quantum numbers associated with the model.

The purpose of this paper is to discuss the order-to-chaos transition in terms of properties of the nuclear wave functions and to analyze how the structure of nuclear states changes with increasing excitation energy.

2. Nuclear Mean Field, Residual Interaction and General Form of Wave Function

The nuclear mean field is responsible for the order. The residual interaction plays a two-fold role:

1. The superconducting pairing interaction stabilizes the regularity of the nuclear mean field. The coherent interaction between quasiparticles leads to the formation of low-lying vibrational states and giant resonances, which generate regularity in the nuclei.

2. The quasiparticle-phonon interaction leads to fragmentation of the quasiparticle and phonon states. It generates the chaos in the nuclei.

To simplify the problem as much as possible, we confine ourselves to the low-spin bound and quasibound stationary nonrotation states of rigid nuclei. We treat all nonrotational states as small-amplitude collective or weakly collective vibrational states or quasiparticle states.

The ground, low-lying and high-lying nuclear states are very complex. We find a representation in which several states are described in the simplest way, though the wave functions of other states are very complex. If the density matrix is diagonal in a Hartree-Fock-Bogoliubov approximation, then the average nuclear field and superconducting pairing interactions can be separated [3]. A representation is usually used in which the density matrix is diagonal for the ground states of the doubly-closed shell or well-deformed nuclei. This representation has physical meaning, since the transitions from the ground to excited states are usually experimentally observed. We can choose another representation in which the density matrix is diagonal for a given excited state. In this case, though the wave function of the given state has a simple form but the wave functions of the other states are very complex.

If the density matrix is diagonal for the ground state of a doubly-closed shell or a well-deformed nucleus, then the wave function of an excited state can be represented as an expansion of a number of many-quasiparticle and many-phonon operators. In this representation there is a hierarchy of the components of wave functions with different numbers of quasiparticles. According to [4],[5], the wave function of an excited state with a fixed angular momentum and the parity of a doubly even-mass nucleus has the following form:

$$\begin{aligned}\Psi_n(J^\pi) \;=\; & \{\sum_{12} b^n_{12}\alpha^\dagger_1\alpha^\dagger_2 + \sum_{a} b^n_a Q^\dagger_a + \sum_{1234} b^n_{1234}\alpha^\dagger_1\alpha^\dagger_2\alpha^\dagger_3\alpha^\dagger_4 + \sum_{12a} b^n_{12a}\alpha^\dagger_1\alpha^\dagger_2 Q^\dagger_a + \\ & \sum_{aa'} b^n_{aa'}Q^\dagger_a Q^\dagger_{a'} + \sum_{123456} b^n_{123456}\alpha^\dagger_1\alpha^\dagger_2\alpha^\dagger_3\alpha^\dagger_4\alpha^\dagger_5\alpha^\dagger_6 + \ldots\}\Psi_0\,. \end{aligned} \quad (1)$$

Here $\alpha_i^\dagger$ denotes a quasiparticle creation operator. $Q_a^\dagger$ denotes a phonon creation operator for a collective vibrational state and Ψ_0 is the ground state wave function of a doubly even nucleus which is determined as a phonon vacuum. Also, $|b^n|^2$ defines the contribution of the corresponding quasiparticle or phonon component to the normalization of the wave function (1). To fulfil the orthogonality of the wave function (1) product $\alpha_i^\dagger \alpha_i^\dagger$ is replaced by the 0^+ phonon operator $Q_a^\dagger$ [4]. The wave function of the RPA one-phonon state $Q_a^\dagger \Psi_0$ comprises a coherent sum of many two-quasiparticle components.

The wave functions of the excited states of heavy nuclei with energies below 3 MeV have, as a rule, a dominating one-quasiparticle or two-quasiparticle or one-phonon component. With increasing excitation energy, the structure of the states becomes more complex and the wave function (1) has several relatively large components; the domination of a single component decreases. The complication of the nuclear states with excitation energy is a result of the coupling between the collective and non-collective degrees of freedom or of a quasiparticle-phonon interaction.

The role of the quasiparticle-phonon interaction increases with excitation energy. The structure of the nuclear states becomes more complex and the contribution of a few-quasiparticle components to the wave function strongly decreases with the excitation energy. The wave functions of the states with energies greater than $(3 \div 4)$ MeV are superpositions of many terms with different numbers of quasiparticles and phonons. In an energy region close to the neutron binding energy B_n the wave function (1) contains many thousands of components.

The common description of collective, weakly collective and quasiparticle states should be used for investigation of the complication of the state structure with increasing excitation energy.

3. Order in Large and Chaos in Small Components of Nuclear Wave Functions

Let us consider the order and chaos separately with respect to large and small quasiparticles as well as the phonon components of the excited states wave function.

There is experimental information that the wave functions of the low-lying states have one dominating one-quasiparticle or one-phonon component. They demonstrate the *regularity* in nuclei. A reasonably good description for the low-lying states has been obtained by means of the dominant component alone. The low-lying states show individuality.

The fragmentation (strength distribution) of the one-quasiparticle component increases as excitation energy increases. Experimental investigations on the fragmentation of the one-quasiparticle states in spherical nuclei have shown [6] that pronounced maxima of the strength distribution take place up to an excitation energy 10 MeV. This means that one-quasiparticle states with a relatively large angular momentum lying in a region rather far from the Fermi surface are not fully fragmented.

The available experimental data on the large components of the wave functions of low-lying and isobaric analog states have demonstrated a regularity in nuclei. The high-spin many-quasiparticle isomers have demonstrated a regularity of the model of independent quasiparticles. It is possible to conclude that the order is governed by the large components of the wave functions of the nuclear excited states.

Practically, there are no experimental data which tell information on the small components of the wave functions of the low-lying states. The experimental values of the reduced neutron and partial radiative widths were used in [4],[5] to estimate the average values of the one- and two-quasiparticle components of the wave functions of the neutron resonances. For nuclei with $50 < A < 250$ they were found to be $|\bar{b}|^2 = 10^{-6} - 10^{-8}$, and somewhat large for nuclei around closed shells.

The small components of a wave function manifest themselves in the distribution function of partial widths for the transition from a neutron resonance to the few-quasiparticle components of the wave function of the low-lying state. The distribution of the partial radiative widths of the neutron resonances are in good agreement with GOE statistics. This shows that the one- or two-quasiparticle components of the wave functions of neutron resonance have a chaotic character. This distribution, however, does not contain any information concerning the entire wave function.

One may say that chaos takes place in the small components of the wave function of the excited states.

The dependence of the s- or p-wave neutron strength function on the position of the $s_{1/2}$ or $p_{1/2}, p_{3/2}$ subshell relative to the neutron binding energy reflects the regularities of the nuclear mean field. The giant resonances demonstrate the regularities of the average values of the particle-hole components of the overlapping nuclear states. This means that an effect of the regularity of the average values takes place and that the chaos characterizes the small components of the wave function.

We consider the transition from order to chaos as a transition from large to small components of the nuclear wave function.

4. Neutron Resonances as a Key for Studying Order-to-Chaos Transitions

The order-to-chaos transition in terms of nuclear wave functions was formulated in [5],[7] in 1972 as "are there relatively large many-quasiparticle components in the wave function of neutron resonances?". We consider the transition from order to chaos as a transition from large to small components of the nuclear wave function. For this consideration it is necessary to investigate the fragmentation of the many-quasiparticle states.

In spherical and deformed nuclei there are many high-spin isomers whose main components are described by the many-quasiparticle configuration. They have long lif-times because there are no high-spin states whose main components are described by few quasiparticle configurations. The existence of isomers with many-quasiparticle components indicates a small fragmentation of such many-quasiparticle high-spin configurations. The many-quasiparticle low-spin configurations should be more strongly fragmented in comparison with high-spin states because the density of the low-spin state is much larger than the density of the high-spin states at intermediate excitation energies. The density of the low-spin states with the many-quasiparticle configuration are small in comparison with the total density of the low-spin states. We can therefore expect that the fragmentation of the low-spin many-quasiparticle configurations is much weaker than the fragmentation of the low -spin few-quasiparticle configurations.

The one-quasiparticle configuration with a relatively large angular momentum of spherical nuclei at an excitation energy 6-8 MeV is not fully fragmented. The one- and three- or two- and four-quasiparticle configurations at excitation energies close to the neutron binding energy are strongly fragmented. At these energies, the five- and seven- or six- and eight-quasiparticle configurations start to fragment. We can expect that the wave functions of the neutron resonance states contain large components of many-quasiparticle configurations.

Practically, no expeirmental data exist concerning the many-quasiparticle components of the wave function of the highly excited low-spin states. What experiments should be performed to answer the question concerning the existence of the many-quasiparticle components of the wave function of the highly excited states? In [7] it has been suggested that the most favourable way to observe the many-quasiparticle components of the wave functions is to study the gamma-transition from the neutron resonance states to the states lying $1 \div 2$ MeV below them.

Some information concerning the values of the many-quasiparticle components can be obtained by studying the $E1$, $M1$ and $E2$ transition probabilities from the neutron resonance states to the levels with energies lower than the neutron resonances energy by $1 \div 2$ MeV. A large contribution of the many-quasiparticle configuration to the normalization of the neutron resonance wave function would enhance in $E1$, $M1$ and $E2$ transitions. If the contribution of the many-quasiparticle component to the normalization of the neutron resonance wave function is equal to 20%, the corresponding reduced gamma-transition probabilities are $3 \div 4$ orders of magnitude larger than the reduced gamma-transition probabilities from the neutron resonance states to the low-lying states. It is possible to say that the state, whose largest components are described by a single configuration larger than $(10 \div 20)\%$, has its own individual characteristic feature.

It is very important to investigate such gamma-transitions from the neutron resonances with the correct subtraction of the background. The study of the gamma-transition after thermal neutron capture cannot give any information on the many-quasiparticle components of the highly excited states. Only if the neutron resonance state is very close to the neutron binding energy it may be possible to use the (n_{th},γ) reaction for detecting the many-quasiparticle components. The thermal and resonance neutron capture cross sections manifest the individuality of the atomic nuclei.

5. Conclusion

The above consideration allowed us to derive the following conclusions:

1. The order is governed by the large components of the wave function of the excited states.

2. Chaos takes place in the small components of the wave function of the nuclear excited states. The excited state is chaotic if its wave function is composed of only small components of few and many-quasiparticle or few and many-phonon configurations.

3. It is possible to consider the order-to-chaos transition as a transition from the large to the small components of the nuclear wave function.

4. An experimental investigation of the many-quasiparticle components of the wave function of the neutron resonance states may be carried out by using the new generation gamma-ray detector, which could observe the enhanced gamma-transition from neutron resonances to the levels lying $(1 \div 2)$ MeV below them.

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