APPLICATIONS OF A MAPPING PROCEDURE TO NUCLEAR AND NUCLEON STRUCTURE PROBLEMS

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ABSTRACT

We illustrate a mapping procedure which has been applied both for deriving boson images of fermion operators and for constructing nucleon images of quark operators within the framework of the non-relativistic quark model. We discuss in particular an application referring to the latter case. Starting from the one-body quark density operator, we calculate the one-body and two-body parts of its nucleon image and calculate their expectation values in the ground state of the doubly magic nuclei ^{16}O and ^{40}Ca. We analyze the role of quark exchanges between nucleons. We also investigate the effect on the quark density of short-range correlations in the nuclear wave functions as well as of variations in the nucleon size.

1. Introduction

Mapping procedures have found a great number of applications in the recent past especially in connection with the investigation of the microscopic foundations of the Interacting Boson Model[1]. Here, these procedures have attempted to establish a link between two different spaces: on one side, the subspace of the full shell model space spanned by "collective" pairs of nucleons of angular momentum J=0 and 2 (S and D pairs, respectively) and, on the other side, a space spanned by elementary bosons of angular momentum J=0 and 2 (s and d bosons, respectively). For any given fermion operator acting in the SD space, the mapping procedures have searched for an "image" operator in the sd space. In such a way, it has been possible to relate phenomenological parameters attached to IBM operators directly to the fermion countepart[2].

There is a well different scenario which shares, however, a strong analogy with the one just discussed. This is offered by the non-relativistic quark model of the baryons[3]. According to this model, baryons are clusters of three quarks, each of them carrying colour, spin and isospin degrees of freedom. Quarks interact via a potential whose main terms are a confining and a hyperfine term. The former is responsible for the confinement of the quarks within the baryons while the latter simulates the exchange of one gluon between quarks and is analogous to the electromagnetic potential describing the exchange of one photon in QED.

It is by now clear the analogy with the former IBM scenario. Once the structure of baryons in terms of quarks is known, one can attempt to establish a correspondence between two spaces: on the one side the space whose states are built in terms of clusters of three quarks and, on the other side, the space whose states are built in terms of the corresponding elementary baryons. It becomes then possible to construct baryon operators starting from the quark level.

In this paper, we will discuss a mapping procedure which has the interesting feature of being applicable to both the scenarios illustrated so far. For reasons of space, we will review the main points of it already in the quark case. However, it will be realized that with only some necessary changes the procedure is also suitable to describe the IBM case. Applications concerning this case can be found in Ref.4 . In the following, we will limit ourselves to illustrate the most recent application of the mapping procedure which refers to the case of the one-body quark density operator[5]. We will construct the one-body and two-body terms of its nucleon image. In terms of these, we will study the space distributions of quarks in doubly magic nuclei like ^{16}O and ^{40}Ca as they are predicted jointly by the shell model of the nucleus and the non-relativistic quark model of the nucleon. The comparison between the calculations with the one-body and the two-body nucleon terms will give us informations on the role of quark exchanges in different nuclear systems. We will also examine the effects on the quark distributions of short range correlations in the nuclear wave functions as well as of variations in the nucleon size.

The paper is organized as follows. In Sect.2, we will review the main lines of the mapping procedure to construct nucleon images of quark operators. In Sect.3, we will show an application of the procedure to the one-body quark density operator. In Sect.4, we will calculate quark distributions in nuclei and look into the effects of short range correlations and nucleon size. Finally, in Sect.5 we will summarize the results and draw some conclusions.

2. The procedure

The mapping procedure which will be used to construct the nucleon image of a quark operator is described in detail in Ref. 6. Here, we will only resume the main points.

We treat quarks by means of creation and annihilation operators $q^{\dagger}_{\eta}(\mathbf{r})$, $q_{\eta}(\mathbf{r})$, where $\eta \equiv \{c, s, t\}$ stands for the colour c, the spin and isospin projections s and t, respectively. They obey the usual fermion commutation relations

$$\{q_{\eta}(\mathbf{r}), q^{\dagger}_{\eta'}(\mathbf{r}')\} = \delta_{\eta\eta'}\delta(\mathbf{r}-\mathbf{r}') \quad , \tag{1a}$$

$$\{q_\eta(\mathbf{r}), q_{\eta'}(\mathbf{r}')\} = \{q^\dagger_\eta(\mathbf{r}), q^\dagger_{\eta'}(\mathbf{r}')\} = 0 \quad , \tag{1b}$$

where $\{A, B\} = AB + BA$.

Restricting ourselves to protons and neutrons, we only introduce *up* and *down* quarks which are characterized by isospin 1/2 and projections $t = 1/2$ and $t = -1/2$, respectively. By means of these operators, we construct operators $N^\dagger_{\sigma\tau}(\mathbf{R})$ which create clusters of three quarks in a colour singlet state, with total spin 1/2, isospin 1/2 and projections σ and τ, respectively, and with center of mass in $\mathbf{R}$. It is:

$$\begin{aligned} N^\dagger_{\sigma\tau}(\mathbf{R}) = & \frac{1}{3!\sqrt{2}} \sum_{c_1c_2c_3} \epsilon_{c_1c_2c_3} \\ & \times \sum_{s_1s_2s_3} \sum_S (\tfrac{1}{2}s_1 \tfrac{1}{2}s_2 \mid Ss_1 + s_2)(Ss_1 + s_2 \tfrac{1}{2}s_3 \mid \tfrac{1}{2}\sigma) \\ & \times \sum_{t_1t_2t_3} \sum_T (\tfrac{1}{2}t_1 \tfrac{1}{2}t_2 \mid Tt_1 + t_2)(Tt_1 + t_2 \tfrac{1}{2}t_3 \mid \tfrac{1}{2}\tau) \\ & \times \int d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{r}_3 \delta(\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 - 3\mathbf{R}) \Omega(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3; \mathbf{R}) q^\dagger_{\eta_1}(\mathbf{r}_1) q^\dagger_{\eta_2}(\mathbf{r}_2) q^\dagger_{\eta_3}(\mathbf{r}_3). \end{aligned} \tag{2}$$

Here, $\epsilon_{c_1c_2c_3}$ is the totally antisymmetric tensor of rank 3 and $\Omega(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3; \mathbf{R})$ describes the spatial distribution of the quarks in the cluster and is fully symmetric under permutation of $\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3$.

We take $\Omega(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3; \mathbf{R})$ to be the product of three gaussians in the coordinates of the quarks relative to the center of mass of the cluster

$$\Omega(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3; \mathbf{R}) = \frac{3^{9/4}\gamma^3}{\pi^{3/2}} e^{-\frac{\gamma^2}{2}[(\mathbf{r}_1-\mathbf{R})^2 + (\mathbf{r}_2-\mathbf{R})^2 + (\mathbf{r}_3-\mathbf{R})^2]} \quad . \tag{3}$$

The normalization is chosen such that

$$< 0_q | N_{\sigma\tau}(\mathbf{R}) N^\dagger_{\sigma'\tau'}(\mathbf{R}') | 0_q > = \delta_{\sigma\sigma'} \delta_{\tau\tau'} \delta(\mathbf{R} - \mathbf{R}') \quad , \tag{4}$$

where $|0_q>$ represents the vacuum of the quark space.

We notice that, although they describe nucleons, cluster operators (2) do not obey fermion commutation relations of the type (1) because of their composite nature.

We call N^A the space of nucleon clusters which is spanned by states of the form

$$N^\dagger_{\sigma_1\tau_1}(\mathbf{R}_1) N^\dagger_{\sigma_2\tau_2}(\mathbf{R}_2) ... N^\dagger_{\sigma_A\tau_A}(\mathbf{R}_A) | 0_q > \quad . \tag{5}$$

Similarly, we introduce creation and annihilation operators $n^\dagger_{\sigma\tau}(\mathbf{R})$, $n_{\sigma\tau}(\mathbf{R})$ for elementary nucleons. They do obey commutation relations of the type (1). We call n^A the space spanned by the states

$$n^\dagger_{\sigma_1\tau_1}(\mathbf{R}_1) n^\dagger_{\sigma_2\tau_2}(\mathbf{R}_2) ... n^\dagger_{\sigma_A\tau_A}(\mathbf{R}_A) | 0_n > \quad , \tag{6}$$

where $|0_n>$ is the vacuum of the nucleon space. States (6) are formally obtained from states (5) by replacing cluster creation operators $N^\dagger_{\sigma\tau}(\mathbf{R})$ with nucleon creation operators $n^\dagger_{\sigma\tau}(\mathbf{R})$ and the quark vacuum with the nucleon vacuum. This correspondence is such not to preserve the orthogonality relations between corresponding states.

The procedure to construct a nucleon image of a quark operator $\widehat{W}_q$ goes through two main steps. In the first, one defines a new quark operator $\widehat{W}_q^{(A)}$ exactly equivalent to $\widehat{W}_q$ within a given quark space $N^{(A)}$ and such not to lead out of this space. In the second step, one constructs a nucleon operator $\widehat{\widehat{W}}_n^{(A)}$ whose action on a state of $n^{(A)}$ is formally identical to that of the operator $\widehat{W}_q^{(A)}$ on the corresponding state of $N^{(A)}$. This guarantees that if $|\Psi_q>$ is an eigenstate of $\widehat{W}_q^{(A)}$ in $N^{(A)}$ with a given eigenvalue, the corresponding state $|\Psi_n>$ in $n^{(A)}$ is also an eigenstate of $\widehat{\widehat{W}}_n^{(A)}$ with the same eigenvalue.

As a consequence of the non-unitarity of the correspondence between states (5) and (6), the nucleon operator so constructed is, in general, non-hermitian. However, it was shown in Ref.(6) that, by means of an appropriate transformation, this undesired feature can be removed.

As a general result, the nucleon operator which is constructed is a sum of one-, two-, ..., A-body terms, if A is the number of cluster of the system under study, i.e.

$$\widehat{\widehat{W}}_n^{(A)} = \widehat{W}_n^{(1)} + \widehat{W}_n^{(2)} + ... + \widehat{W}_n^{(A)} \quad . \tag{7}$$

Such a complicated structure is determined by the need of simulating in a space of A elementary nucleons the complicated quark exchange dynamics within the A clusters. Each of the A terms contributing to form the nucleon image is linked to a different physical process, the one-body term reflecting only the quark dynamics within one cluster, the two-body term the quark exchanges between two clusters, etc... .

Of course, evaluating the exact nucleon image when A is large becomes quite difficult and therefore some approximations are required. By limiting ourselves to processes which involve at most exchanges of quarks between two nucleons, only the one-body and two-body terms of (7) are needed. These operators are characterized by the following matrix elements in the n^1 and n^2 spaces[6]:

$$< 0_n|n_{\sigma'\tau'}(\mathbf{R}')\widehat{W}_n^{(1)}n^\dagger{}_{\sigma\tau}(\mathbf{R})|0_n> = < 0_q|N_{\sigma'\tau'}(\mathbf{R}')\widehat{W}_q N^\dagger{}_{\sigma\tau}(\mathbf{R})|0_q> \tag{8}$$

and

$$< 1,2|\widehat{W}_n^{(2)}|1',2'>_n = \int_{1^*,2^*,\bar{1},\bar{2}} < 1,2|1^*,2^*>_q^{-1/2} < 1^*,2^*|\widehat{W}_q|\bar{1},\bar{2}>_q < \bar{1},\bar{2}|1',2'>_q^{-1/2}$$
$$- < 1,2|\widehat{W}_n^{(1)}|1',2'>_n \quad , \tag{9}$$

where $i = \{\mathbf{R}_i, \sigma_i, \tau_i\}$,

$$N^{\dagger}_{\sigma_1\tau_1}(\mathbf{R}_1)N^{\dagger}_{\sigma_2\tau_2}(\mathbf{R}_2)|0_q> \equiv |1,2>_q \quad , \tag{10}$$

$$n^{\dagger}_{\sigma_1\tau_1}(\mathbf{R}_1)n^{\dagger}_{\sigma_2\tau_2}(\mathbf{R}_2)|0_n> \equiv |1,2>_n \quad , \tag{11}$$

and where the symbol $\int_{i,...}$ means integration (summation) over continuous (discrete) variables $i, ...$.

In the following we will show practical realizations of these nucleon operators.

3. The nucleon image of the one-body quark density operator

Starting point for the applications of the mapping procedure discussed in the previous section will be the one-body quark density operator

$$\widehat{\rho}_q(\mathbf{r}) = \sum_{\eta} q^{\dagger}_{\eta}(\mathbf{r})q_{\eta}(\mathbf{r}) \quad . \tag{12}$$

According to eq.(8), the one-body nucleon image is

$$\widehat{\rho}^{(1)}_n(\mathbf{r}) = \sum_{\sigma\tau\sigma'\tau'} \int d\mathbf{R}d\mathbf{R}' < 0_q|N_{\sigma\tau}(\mathbf{R})\widehat{\rho}_q(\mathbf{r})N^{\dagger}_{\sigma'\tau'}(\mathbf{R}')|0_q > n^{\dagger}_{\sigma\tau}(\mathbf{R})n_{\sigma'\tau'}(\mathbf{R}') \tag{13}$$

and since

$$< 0_q|N_{\sigma\tau}(\mathbf{R})\widehat{\rho}_q(\mathbf{r})N^{\dagger}_{\sigma'\tau'}(\mathbf{R}')|0_q >= \delta_{\sigma,\sigma'}\delta_{\tau,\tau'}\delta(\mathbf{R}-\mathbf{R}')\frac{\gamma^3 3^{5/2}}{2^{3/2}\pi^{3/2}}e^{-3/2\gamma^2(\mathbf{R}-\mathbf{r})^2} \quad , \tag{14}$$

it is also

$$\widehat{\rho}^{(1)}_n(\mathbf{r}) = \sum_{\sigma\tau} \int d\mathbf{R}\frac{\gamma^3 3^{5/2}}{2^{3/2}\pi^{3/2}}e^{-3/2\gamma^2(\mathbf{R}-\mathbf{r})^2}n^{\dagger}_{\sigma\tau}(\mathbf{R})n_{\sigma\tau}(\mathbf{R}) \quad . \tag{15}$$

Eq.(14) gives the quark distribution in a free nucleon.

We notice that in the limit $\gamma \to \infty$, corresponding to point-like clusters,

$$\widehat{\rho}^{(1)}_n(\mathbf{r}) \to 3\sum_{\sigma\tau} n^{\dagger}_{\sigma\tau}(\mathbf{r})n_{\sigma\tau}(\mathbf{r}) \tag{16}$$

which is three times the usual one-body nucleon density operator. For any finite value of γ, instead, the quark distribution calculated with the operator (15) is given by the nucleon distribution folded with the quark distribution in a free nucleon.

The operator (15) does not take into account any quark exchange process between nucleons. The simplest of these processes, the exchange of quarks between two nucleons, can be described in terms of the two-body term

$$\widehat{\rho}^{(2)}_n(\mathbf{r}) = \frac{1}{4}\int_{1,2,1',2'} < 1,2|\widehat{\rho}^{(2)}_n(\mathbf{r})|1',2' >_n n^{\dagger}_1 n^{\dagger}_2 n_{2'} n_{1'} \quad . \tag{17}$$

The calculation of the two-body matrix elements appearing in this operator, to be done according to eq.(9), is rather involved and we will not discuss it here (for a deeper analysis, see Ref. 6). By limiting ourselves to quark exchange processes involving at most two nucleons, the nucleon image of the one-body quark density operator takes then the form

$$\hat{\rho}_n(\mathbf{r}) = \hat{\rho}_n^{(1)}(\mathbf{r}) + \hat{\rho}_n^{(2)}(\mathbf{r}) \quad . \tag{18}$$

4. Quark distributions in ^{16}O and ^{40}Ca

We have evaluated the expectation values of the operators (15) and (17) in the ground state of three doubly-magic nuclei: ^{4}He, ^{16}O and ^{40}Ca. Here, we will only show those corresponding to the last two. This ground state has been first described within a pure nuclear shell model with harmonic oscillator wave functions. Results are illustrated in Figs.1,2. where the solid line refers to the operator (18), while the dashed line refers to the one-body term (15) only. In these calculations, the harmonic oscillator parameter γ of the quark wave function (3) has been taken equal to 1.25 fm^{-1}, corresponding to a nucleon r.m.s.r. of 0.8 fm. The equivalent harmonic oscillator parameter for the nuclear wave functions has been chosen as follows: 0.63 fm^{-1} for ^{16}O and 0.50 fm^{-1} for ^{40}Ca. This choice guarantees the correct r.m.s.r. of these nuclei.

Figs. 1,2 clearly show the non-negligible effect of quark exchange processes in determining the quark distributions. This effect gets larger for the heavier nuclei and increases for decreasing values r of the distance from the origin of the system.

The calculations shown so far describe nuclei as systems of independent particles. However, being quark exchange intrinsically a short range process, one can expect that hard-core correlations between interacting nucleons can alter significantly the results. We have investigated to which degree this happens by introducing a correlation function of the type

$$f(\mathbf{R}_1,\mathbf{R}_2) = 1 - e^{-a|\mathbf{R}_1-\mathbf{R}_2|^2} \tag{19}$$

and replacing states (10) with states

$$f(\mathbf{R}_1,\mathbf{R}_2)N^{\dagger}_{\sigma_1\tau_1}(\mathbf{R}_1)N^{\dagger}_{\sigma_2\tau_2}(\mathbf{R}_2)|0_q\rangle \quad . \tag{20}$$

Obviously, the results which concern the one-body nucleon operator are not affected by these changes. With reference to the two-body term, instead, one observes the modifications which are quantified in Fig.3 for ^{16}O. Here, the solid line refers to the quark distributions calculated without hard-core correlations while the long-dashed and short-dashed lines are the results obtained by using two-cluster states with $a = 4$ and $a = 2$, respectively.

The short range repulsion is found to cause a decrease of the quark density at small r and a consequent increase at large r. This effect is the more evident the

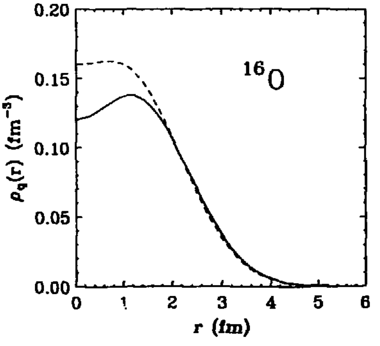

Fig. 1 - Quark distribution in ^{16}O as predicted by the non-relativistic quark model in conjunction with a pure nuclear shell model with harmonic oscillator wave functions. The solid line is the expectation value of the operator (28) in the nuclear ground state. The dashed line shows the contribution of the one-body part (15) only. The normalization is chosen such that $\int r^2 dr \rho_q(r) = 1$

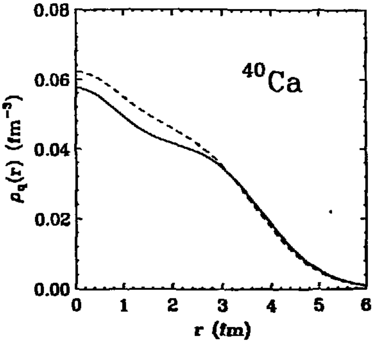

Fig. 2 - The same as in Fig.1 but for ^{40}Ca

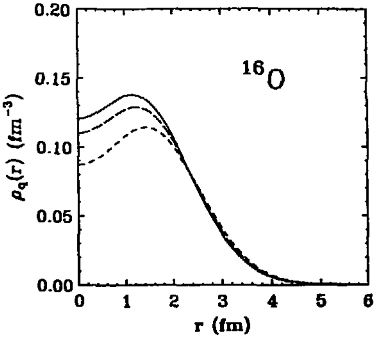

Fig. 3 - Effects of short-range nuclear correlations on the quark distribution of ^{16}O. Solid line: no correlations. Long-dashed line: a=4 fm^{-2} in the function (29). Short-dashed line: a=2 fm^{-2} in (29). The normalization is chosen such that $\int r^2 dr \rho_q(r) = 1$

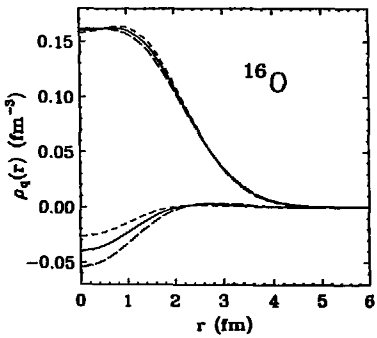

Fig. 4 - Expectation values of the one-body term (15) (positive sector) and of the two-body term (17) (negative sector) in the ground state of ^{16}O. Solid line: γ = 1.25 fm^{-1} ($< r^2 >^{1/2}$= 0.8 fm). Long-dashed line: $\gamma = 1.11$ fm^{-1} ($< r^2 >^{1/2}$= 0.9 fm). Short-dashed line: $\gamma = 1.43$ fm^{-1} ($< r^2 >^{1/2}$= 0.7 fm)

smaller is the parameter a of the correlation function (19), i.e. the more intense is the short range repulsion. Even for a rather intense repulsion, however, the effect is not such to alter drastically the quark distribution.

As a final point, we have calculated the variations of the quark density caused by changes in the nucleon size. In Fig.4, one sees the expectation values of the operators (15) (positive sector) and (17) (negative sector) for $\gamma = 1.11$ fm^{-1} (large-dashed line), $\gamma = 1.25$ fm^{-1} (solid line) and $\gamma = 1.43$ fm^{-1} (short-dashed line) in the case of ^{16}O. These cases correspond to nucleon r.m.s.r. of 0.9, 0.8 and 0.7 fm, respectively. For increasing nucleon radii (and constant nuclear radius) one observes a growing of the exchange effects at small values of r, while the expectation of the one-body term stays rather constant. Thus, the global effect is a reduction of the quark density similar to that observed in correspondence to the introduction of the correlation factor (19) simulating a short-range repulsion (Fig.3). In this case, however, the effect is only due to the antisymmetrization of the nuclear wave functions with respect to quarks.

5. Summary and conclusions

In this paper, we have discussed a method to construct nucleon images of quark operators. As an application, we have studied the one-body quark density operator. We have derived the one-body and two-body terms of its nucleon image and we have shown the space distributions of quarks in ^{16}O and ^{40}Ca as they are predicted jointly by the shell model of the nucleus and the quark cluster model of the nucleon. The quark distribution that one calculates in correspondence to the one-body term turns out to be equal to the distribution of elementary nucleons predicted by the nuclear shell model folded with the distribution of quarks inside a free nucleon. The contribution of the two-body term takes into account exchanges of quarks between two different nucleons.

We have performed two series of calculations, the first assuming an independent particle approacn and, the second, introducing short-range correlations between nucleons. We have found that quark exchange produces sizeable effects on the quark distributions, larger for the heavier elements and increasing for decreasing values of the distance r from the origin of the reference system. We have also seen that short range correlations can appreciably modify the quark distributions in the sense of shifting them torwards large values of r, without however causing drastic alterations. Finally, we have analyzed the variations of the quark distributions caused by changes in the nucleon size and found that an increase of this size gives rise to effects similar to those produced by the short-range correlations.

To our knowledge, one can find in literature only another calculation of quark observables in nuclear systems of comparable "size" as those studied in this paper. This is contained in the recent work of Yamauchi, Buchmann, Faessler and Arima[7] on quark exchange currents in nuclei. Their inspiring philosophy has been the same as ours[5,6], namely that of constructing an effective nucleon operator starting from a quark one and their procedure has been based on the Resonating Group Method.

So far, we have looked into space distributions of quarks in nuclei. With only a few changes in the formalism we are in a position to examine also quark momentum distributions. The importance of quark exchanges in modifying these distributions and the consequences that these can have on the nucleon structure function have already been pointed out in connection with the EMC effect[8,9]. However, these calculations have concerned either very small systems, namely A=3 systems[8], or nuclear matter[9]. Our formalism allows the extension of this analysis to more interesting intermediate situations. We expect that the not negligible role of quark exchanges found in the coordinate representation will be confirmed in the momentum representation and we therefore hope that these calculations can provide a significant contribution to the understanding of this interesting nuclear phenomenon.

6. References

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