CHIRAL MODEL AND THE RELATIVISTIC SELF-CONSISTENT CALCULATION FOR ATOMIC NUCLEI

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Abstract:

The properties of finite nuclei are investigated in the framework of a chiral field theory using relativistic mean field approximation. It is shown that, in contrast with the conventional point of view, the normal type solutions for finite nuclei do exist. We have obtained these solutions and used them to calculate the nuclear bulk properties with two adjustable parameters. The mean field normal solutions are not so far from the observed nuclei. Theory simultaneously contains also configurations of abnormal type which in some cases may appear to be much more strongly bound than those of normal type. However, for the conventional values of the scalar meson mass m_σ (~500 MeV) only one possibility remains - that of a normal configuration.

1. Introduction

Relativistic quantum field theory is widely used now to describe nuclear matter and finite structures, the Lagrangian density of the system including nucleon and meson degrees of freedom on the same grounds [1,2]. Many essential results were obtained in the framework of this approach, in particular, self-consistent calculations were carried out for finite nuclei both in the Hartree [3,4] and Hartee-Fock approximations [5].

Relativistic quantum hadrodynamics has been used also to study πN scattering (the additional mesons (π, ρ...) being included). As appeared, in this framework it is possible to obtain successfully low-energy pion dynamics in free space. However, the same dynamics, extrapolated to the nuclear medium, cannot be reasonably described. To ensure reasonable pion dynamics at finite density, it is necessary to impose some kind of chiral symmetry.

So it appears to be essential to consider consequences connected with the invariance of the Lagrangian density describing the interacting meson and nucleon fields relative to the chiral symmetry in addition to the conventional isotopic symmetry. After papers [6,7] interest to investigations of chiral models [8-11] considerably increased. From the theoretical point of view the chiral invariance introduces essential limitations on the processes with strong interactions both in the empty space and in the nuclear medium. In particular, it is believed now that chiral models may provide a new treatment of saturation problem in nuclear matter.

There are several different realizations of the chiral symmetry. Here we shall restrict ourselves to investigation of a chiral model including nucleon, scalar and vector fields. In this case it is possible to establish the relationship between the chiral approach and the conventional Dirac phenomenology for atomic nuclei.

The standard chiral $\sigma - \omega$ theory does not have a saturating normal matter ground state, the theory being considered in a mean field approximation (MFA) [7,12,13]. It is possible to avoid this difficulty introducing the vector field ω_μ by Higg's mechanism [12]. If one allows for a coupling between the meson fields, bifurcations disappear and a saturating equation of state can be obtained. In [12] the ω-meson mass is determined dynamically: it appears (as well as the nucleon mass) as a result of spontaneously broken chiral symmetry.

We shall start from the following Lagrangian density

$$L = \overline{\psi}(i\gamma^\mu\partial_\mu - g_\omega\gamma^\mu\omega_\mu - M^*(r) - ig_\sigma\gamma_5\vec{\tau}.\vec{\pi})\psi + \frac{C}{2}\overline{\psi}\gamma^\mu[\gamma_5\vec{\tau}(\sigma\partial_\mu\vec{\pi} - \vec{\pi}\partial_\mu\sigma) + \vec{\tau}.\vec{\pi}\times\partial_\mu\vec{\pi}]\psi$$
$$+\frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec{\pi}\partial^\mu\vec{\pi}) - \frac{m_\sigma^2 - m_\pi^2}{8f_\pi^2}(\sigma^2 + \vec{\pi}^2 - f_\pi^2\frac{m_\sigma^2 - 3m_\pi^2}{m_\sigma{}^2 - m_\pi^2})^2 \quad (1)$$
$$+f_\pi m_\pi^2\sigma - \frac{1}{4}(\partial_\mu\omega_\nu - \partial_\nu\omega_\mu)(\partial^\mu\omega^\nu - \partial^\nu\omega^\mu) + \frac{1}{2}G_\omega^2(\sigma^2 + \vec{\pi}^2)\omega^2,$$

with $C = (g_A - 1)f_\pi^{-2}$, g_A being the nucleon axial-vector form-factor. f_π is the vacuum value of the scalar field which determines the weak interaction pion decay rate. The upper arrows refer to isotopic space. Note that the terms proportional to "C" are considered to account for deviation of the axial-vector form-factor g_A from unity. Higg's type treatment of the ω-meson used by Boguta [12] may be considered as a simple and elegant way to ensure saturation within the linear σ-model. As far as we know, such approach does not contradict any physical principle. Thus we employ this model for practical reason.

Lagrangian (1) contains a symmetry-breaking term $f_\pi m_\pi^2\sigma = \varepsilon\sigma$ introduced into the chiral approach to obtain the observed value of the pion mass. The ρ-meson is not included into consideration in eq.(1) since we study the mean-field approximation to N=Z nuclei, and ρ-meson should not be taken into account in this case.

Lagrangian density (1) was considered in [12,13] (we use the conventional notations here) for the case $G_\omega = g_\omega$, which will be referred to in what follows as Model I. The authors [12,13] succeeded to describe the ground-state properties of nuclear matter (E/A=-16 MeV, $\rho_0 = 0.145 fm^{-3}, M^*$=0.78M) with the following values of the parameters g_σ=8.35, m_σ=650 MeV (in the framework of the theory considered there is a relation connecting two coupling constants g_ω and g_σ: $g_\sigma^2 = G_\omega^2 \frac{M^2}{m_\omega^2}$, $m_\omega = 783$ MeV).

Two points must be outlined. The first one is that the value of the nuclear matter compressibility at saturation predicted by the theory considered here is rather large (K=650 MeV). But a large value of the compressibility is a common feature of relativistic models used in MFA. The second point is that the value of the coupling constant g_σ in Model I does not correspond to the value obtained from the Goldberger-Treiman relation ($g_\sigma = \frac{g}{g_A} = 10.6$, g_A being the axial form-factor). To overcome the latter difficulty, in papers [14] there was introduced another version of the theory which allows $G_\omega \neq g_\omega (\eta = \frac{G_\omega}{g_\omega})$ (Model II). The ground-state nuclear matter properties were also reproduced in Model II (E/A=-16 MeV, $\rho_0 = 0.17 fm^{-3}$) with two adjustable parameters: m_σ=883.6 MeV, $\eta = 1.385$ (g_σ was taken equal to 10.6 which corresponds to the correct value of g_A=1.25).

2. General theory and results

In [12,13] finite nuclei were investigated in the framework of the Lagrangian (1) though only abnormal states with properties drastically different from the observed ones could be found by the authors for finite structures.

As our calculations have shown, Lagrangian (1), due to strongly non-linear character of the model, may contain simultaneously self-consistent normal and abnormal solutions.

Let us consider the Lagrange-Euler equations corresponding to the Lagrangian density (1) (for $\vec{\pi}=0$), the Coulomb potential $A_0(r)$ being added for protons [14]:

$$[\vec{\alpha}.\vec{p}+\beta M^*(r)+V(r)+\frac{1}{2}e(1+\tau_3)A_0(r)]\psi_\lambda=E_\lambda\psi_\lambda, \tag{2a}$$

$$\nabla^2 M^*(r)+\frac{M^2-M^{*2}(r)}{2M^2}[m_\sigma^2 M^*(r)+m_\pi^2(M-M^*(r))\frac{M^*(r)+2M}{M^*(r)+M}]=$$

$$=g_\sigma^2\rho_s(r)-M^*(r)\frac{G_\omega^2}{g_\omega^2}V^2(r), \tag{2b}$$

$$\nabla^2 V(r)-\frac{G_\omega^2}{g_\sigma^2}M^{*2}(r)V(r)=-g_\omega^2\rho(r), \tag{2c}$$

$$\nabla^2 A_0(r)=-e\rho_p(r), \tag{2d}$$

where $M^*(r)=g_\sigma\sigma$ =M+S(r) is the nucleon effective mass, $\rho(r)$ is the vector - and $\rho_s(r)$ the scalar - density. It should be noticed that the pion field equals zero in the mean field approximation. However, even in this case the role of chiral symmetry is essential. It is revealed by the fact that the strength of the σN coupling as well as the non-linear σ-terms are uniquely determined by the πN coupling and is also manifested by the structure of the field eqs. (2), in particular, for the σ-field, the chiral partner of the pion field.

Both normal and abnormal solutions satisfy eqs. (2a)-(2d). To be complete, we start from discussing some properties of abnormal solutions. Let us consider the simplest nucleus - ^4He. Abnormal solutions in this case have been investigated in [13]. The authors established two types of abnormal solutions for ^4He: abnormal solution with negative value of the total binding energy (T.B.E.) and abnormal solution with positive value of T.B.E. We shall not discuss the latter case any more. As for the first type of abnormal solutions, it demonstrates a shell-like structure, most of its mass being concentrated on the nucleus surface. The existence of this solution is connected with possibility for a scalar field to have a kinky configuration. In all these points our calculations reproduce the results of [13] (in the latter case only Model I was considered).

It should be emphasized that while the authors of papers [12,13] have succeeded to obtain the abnormal solutions for ^4He, they have missed the case (probably more interesting) corresponding to the normal nuclear configurations. Let us consider this case in more detail. Our self-consistent results for the normal structure of ^4He are given in Figs. 1a,b (Model I). The charge density distribution and values of V(r) and $M^*(r)$ are reproduced in Fig. 1a while in Fig. 1b the upper component $G_N(r)$ and the component $F_N(r)$ of the wave function of normal $1s_{1/2}$ state of ^4He are

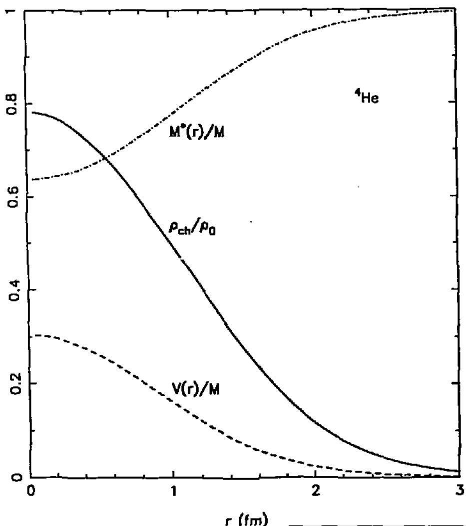


Fig. 1a. Charge density distribution over ρ_0, V(r)/M and $M^*(r)$/M for the normal solution in ^4He for Model I

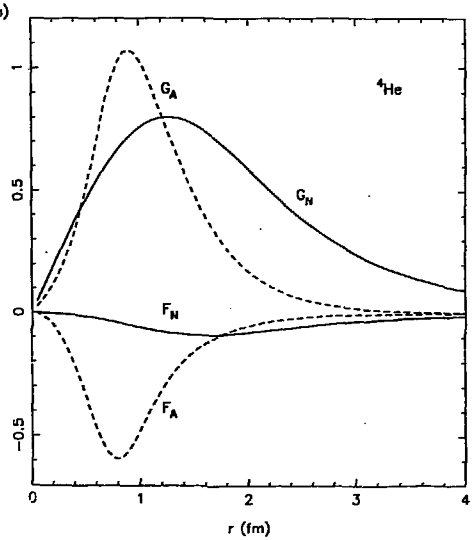


Fig. 1b. The upper component $G_N(r)$ and the lower component $F_N(r)$ of the wave function of the normal $1s_{1/2}$ state of ^4He are compared with the components $G_A(r)$ and $F_A(r)$ of the same state but for the kinky abnormal structure

compared with the functions $G_A(r)$ and $F_A(r)$ of the same state but for the kinky abnormal structure. The r.m.s. charge radius for the normal solution in ^4He is equal to 1.84 fm (Model I) and to 1.62 fm (Model II).

Let us emphasize that the results presented above for normal states are in obvious contradiction to the conclusions of [13] where no normal solutions had been obtained for finite nuclei. Moreover, normal solutions are stated in [13] to be completely impossible in the framework of the chiral field theory based on the Lagrangian density (1). This assertion could be considered valid if the non-linear boundary problem treated in [13] had a single solution only. But this is not the case since, as our calculations have shown there exist self-consistent normal type solutions as well.

In this situation the question arises which of the solutions obtained (normal or abnormal one) may be considered to describe the nuclear ground state. It should be answered basing on the energetics argument. To receive the answer, we have calculated the total binding energy corresponding to the normal and abnormal solutions using the following formula:

$$\frac{E_B}{A} = \frac{1}{A}\left\{\sum_{\lambda<F}\epsilon_\lambda + 4\pi\int_0^\infty r^2 dr\left[\frac{1}{2g_\sigma^2}\left(\frac{dM^*}{dr}\right)^2 + \frac{m_\sigma^2}{2g_\sigma^2}(M^*-M)^2\right.\right.$$

$$-\frac{1}{2g_\omega^2}\left(\frac{dV}{dr}\right)^2 - \frac{m_\omega^2}{2g_\omega^2}\left(\frac{M^*}{M}\right)^2 V^2(r) + \frac{m_\sigma^2 - m_\pi^2}{2M}\frac{1}{g_\sigma^2}(M^*-M)^3 \tag{3}$$

$$\left.\left.+\frac{m_\sigma^2 - m_\pi^2}{8M^2}\frac{1}{g_\sigma^2}(M^*-M)^4 - \frac{1}{2e^2}\left(\frac{dA_0}{dr}\right)^2\right]\right\}; \quad \epsilon_\lambda = E_\lambda - M.$$

Table 1. Single-particle binding energies (in MeV) in ^{16}O for protons and neutrons (normal solutions)

	^{16}O					
	Model I		*Model II*		*Experimental* [15]	
	p	n	p	n	p	n
$1s_{1/2}$	33.22	38.04	40.72	45.93	40 ± 8	47
$1p_{3/2}$	17.38	21.81	23.19	28.10	18.4	21.8
$1p_{1/2}$	14.29	18.72	18.95	23.92	12.1	15.7

Table 2. RMS charge radii for ^{16}O

	^{16}O		
	Model I	*Model II*	*Exp* [16]
r_{ch}	2.56	2.33	2.73

The following results for the total binding energies have been obtained for ^4He: -27.5 MeV (Model I) and -57.5 MeV (Model II) for normal solution, +17.4 MeV (Model I) and -551 MeV (Model II) for abnormal one. Let us mention here that the chiral symmetry breaking term $m_\pi^2 f_\pi \sigma$ in eq. (1) may drastically influence the behaviour of the abnormal solutions, the total binding energies, in particular (for example in the case $m_\pi^2 = 0$ the result of Boguta [12] -80 MeV for T.B.E. has been reproduced in our calculations for the kinky abnormal solution in Model I (Model II was not considered in [12,13])). We have solved eqs. (2a)-(2d) self-consistently and calculated $\frac{E_B}{A}$, the term $m_\pi^2 f_\pi \sigma$ in eq.(1) always being taken into account.

It must be emphasized also that the precise value of T.B.E. is rather sensitive to the value of the step of integration, the latter has been chosen to be equal to 0.05 fm in the calculations mentioned above.

Now we consider our results for ^{16}O with the values of parameters fitted to reproduce the nuclear matter ground-state properties. In this case we have received three solutions: two types of abnormal solutions and one normal solution (while the authors of [13] have succeeded to receive only one solution in this case).

As for normal solution for ^{16}O (missed in [12,13]), we have ohtained the single-particle energies, density distributions, r.m.s. charge radii (see Tables 1, 2 and Figures 2, 3) which qualitatively reproduce the corresponding values for observed nuclei. For example, the total binding energy corresponding to the normal solution in ^{16}O is equal to -139.3 MeV (Model I) and -200 MeV (Model II) for the physical values of the coupling constants. The deficiencies of some of these results are evident, however, we shall outline them: the calculated charge densities do not reproduce

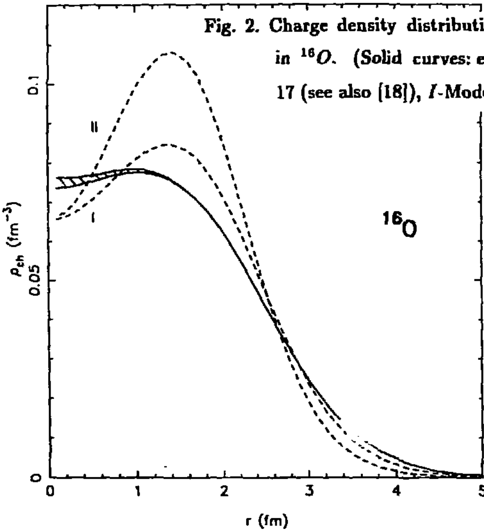


Fig. 2. Charge density distributions for the normal solution in ^{16}O. (Solid curves: experimental values from ref. 17 (see also [18]), *I*-Model I, *II*-model II)

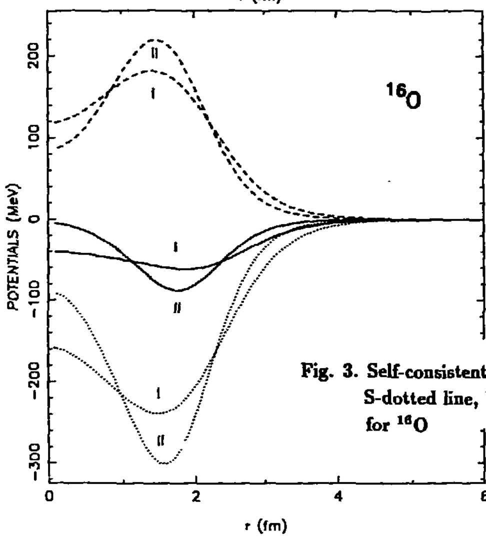


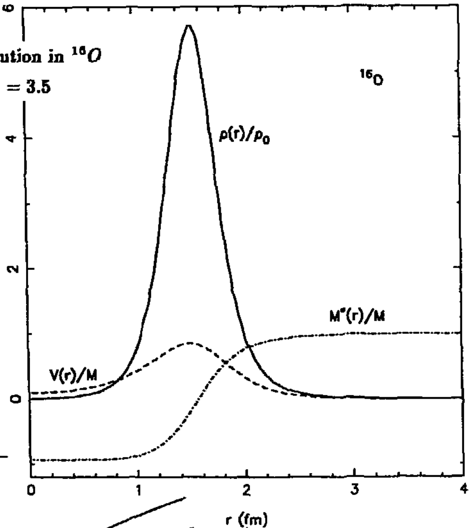
Fig. 3. Self-consistent potentials (V-dashed line, S-dotted line, V+S -continuous line) for ^{16}O

the data well, especially for Model II; the spin-orbit splittings are too small, by a factor of 2, and worse for Model I; the single-particle binding energies are too large, especially for Model II. We have received results with normal solutions also for ^{40}Ca but we do not reproduce them here since qualitatively they are quite similar to those for ^{16}O.

For ^{16}O we have obtained two types of abnormal solutions: 1) the abnormal solution with a barrier at the origin for the scalar field, and 2) the abnormal solution with kinky configuration of the scalar field. As for abnormal self-consistent solution of the first type in ^{16}O we have just reproduced the result presented in Fig. 6 of [13]. (Let us mention that in this case we used $m_\pi = 0$ to compare our results directly with those of ref. [13]). For small values of g_ω this solution is very strongly bound, Fig. 6 in [13] corresponds to $g_r = \frac{g_\omega}{g_\sigma} = 0.3$, in this case T.B.E. for ^{16}O is equal to -9302 MeV, however this solution ceases to exist for physical strength of vector repulsion $g_r = 0.83$. In this point we agree with the results of [13].

For ^{16}O we have obtained also the abnormal solution with a kinky configuration of the scalar field (see Fig. 4). This solution was also overlooked in [13] (moreover, it was considered there as non-existing one for all values of g_ω). Probably in this case the statement of paper [12] concerning the matching condition for the wave function on the boundary of a square well was misleading. As seen from Fig. 5, the total binding energy of abnormal structure may be very large. This structure may be much more strongly bound than the normal configuration. However, at the present stage the case with the physical value of $\frac{g_\omega^2}{4\pi}$ is not completely unambiguous (see the discussion below).

Fig. 4. Kinky abnormal solution in ^{16}O for Model I with $\frac{g_\omega^2}{4\pi} = 3.5$



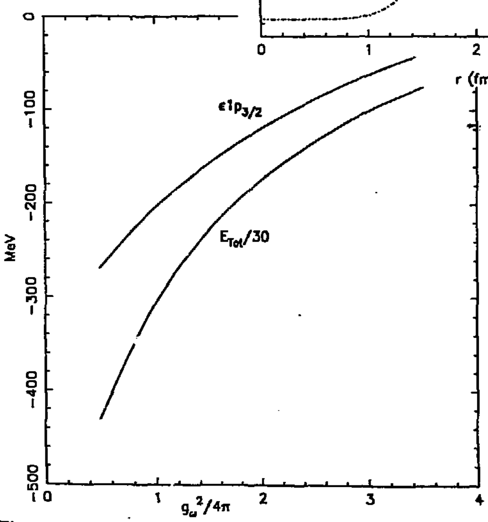


Fig. 5. Dependence of the total binding energy and that of the energy of $1p_{3/2}$ level in the kinky abnormal structure of ^{16}O (the latter level being the highest one for the abnormal configuration) on the value of $\frac{g_\omega^2}{4\pi}$

Now we discuss some computational aspects of the problem considered. In all calculations presented here we use a standard fourth-order Runge-Kutta method to resolve the Dirac equation (2a). In a numerical iterative treatment of the system of nonlinear differential equations (2a)-(2b) special care was taken with respect to convergence, dependence on start value, etc.

As starting point for the normal solutions, we approach S(r) and V(r) by two Woods-Saxon potentials with S(0)=-180 MeV and V(0)=+140 MeV. At each iteration we only take 20% from new potentials. Self-consistency was achieved at ~60 iterations for Model II, the procedure converges more rapidly in the case of Model I. Convergence is very good for both cases.

As for kinky abnormal solutions, in this case it is possible, for example, to take the initial approximation for the scalar field prompted by the corresponding configuration obtained in [13], and the initial approximation for the vector field - in the conventional Woods-Saxon form. Very good convergence was achieved in our calculations for abnormal solutions in ^4He at less than 60 iterations.

As for kinky abnormal solutions in ^{16}O, we must emphasize the following points. We have established unambiguously that in this case self-consistent solutions do exist and we have obtained them (see Fig. 4) for values of $\frac{g_v^2}{4\pi}$ smaller than the physical value (up to ~3.5). Self-consistency is achieved, the convergence being very well established. To obtain the results with higher accuracy, one needs only to further increase the precision of the computer facilities (in the present calculations we have used WAX-8350, with double precision). As for values of $\frac{g_v^2}{4\pi}$ close to the physical value 3.858, regrettably, the result is not so unambiguous, convergence is not so good in this case and it is hardly to say if this fact is connected with the computer precision available or with the actual absence of the kinky abnormal solution in ^{16}O for the physical value of g_ω.

It should be noticed also that behaviour of kinky abnormal solution is much more dependent on the details of calculations than that of a normal one. In particular, the T.B.E., position of $1p_{3/2}$-level in ^{16}O (this level is the highest for the abnormal structure) are rather sensitive to the value of $\frac{g_\omega^2}{4\pi}$. This fact is illustrated by Fig. 5. The T.B.E. for abnormal solutions are sensitive also to the value of the step of integration, the latter value was taken to be equal to 0.05 fm in calculations for abnormal solutions.

Up to the present we have carried out investigations in the framework of two chiral models which have been used recently. We would like to emphasize the points in which previous investigators [12,13] were far from being complete. 1. The authors of [12,13] obtained kinky abnormal solutions only for ^4He and rejected existence of abnormal solutions of this type in ^{16}O. We have obtained kinky abnormal solutions in the latter case also (see Fig. 4) undoubtedly for the values of the $\frac{g_\omega^2}{4\pi}$ up to ~ 3.5. 2. While the authors of ref.[12,13] have succeeded in some cases to obtain the abnormal solutions, for all nuclei considered they have missed the case (probably more interesting) corresponding to the normal configurations. 3. The chiral models considered in this paper have a common feature of a very large scalar mass value. However the ground-state properties of nuclear matter in the framework of the same approach may be obtained with smaller values of m_σ, for example, m_σ=500 MeV. Our calculations have shown that in this case the kinky abnormal solutions cease to exist both for ^4He and ^{16}O even for rather small values of g_ω. So for more conventional values of m_σ only one possibility remains - that of the normal configuration.

3. Summary

Some properties of a chiral model have been investigated in the framework of a relativistic mean field approximation for finite nuclei. Following conclusions may be formulated.

1) It is for the first time that in the framework of a relativistic chiral model a normal-type solution is shown to exist and is obtained for finite structures.

2) The normal type solution was used to calculate the nuclear bulk properties: charge density distributions, r.m.s. radii, single-particle energies, total binding energies, the values obtained are not so far from the observed nuclei.

3) We have shown that in the case of ^4He the normal-type solution for Model I corresponds to the ground state of this nucleus.

4) Theory simultaneously contains also configurations of abnormal type which for large values of m_σ in some cases may be much more strongly bound than those of normal type.

5) However, for more conventional values of m_σ (~500 MeV) only one possibility remains - that of a normal configuration.

6) The model considered in this paper presents one of many possible ways to take into account chiral symmetry. In paper [19] the authors have developed a linear chiral σ-model where the scalar field includes two contributions - one is the chiral

partner of the pion and the other is due to the correlated two-pion exchange in the scalar-isoscalar channel; this approach is of considerable interest and deserves further investigation.

7) It is natural that this paper is not a complete investigation of nuclear structure. Particularly, exchange effects are to be included. For this reason, the results obtained in this paper are not ultimate and a further development of the model is necessary what will be subject of future papers.

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