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**EFFECTIVE LAGRANGIAN FOR GLUONS AND HIGGS
PSEUDOSCALARS*****Russel P. Kauffman and Wendy Schaffer**Physics Department
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Models of electroweak symmetry breaking with more than a single doublet of Higgs scalars contain a neutral pseudoscalar boson. The production of such a pseudoscalar in hadron collisions proceeds primarily via gluon fusion through a top-quark loop (except for those models in which the pseudoscalar coupling to bottom quarks is strongly enhanced). We compute the QCD corrections to this process in the heavy-quark limit, using an effective Lagrangian derived from the axial anomaly.

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1. Introduction

The primary mission of the next generation of colliders is the study of the physics of electroweak symmetry breaking. Since the Higgs mechanism provides the most promising scenario for the symmetry breaking, the search for the particles of the Higgs sector is of primary importance. The minimal model of electroweak symmetry breaking contains one complex scalar doublet, three components of which become the longitudinal degrees of freedom of the W^\pm and Z . The remaining component of the doublet is the so-called Higgs boson. However, the minimal model with one doublet has no *a priori* justification and there are several motivations for considering models with enlarged Higgs sectors, either containing more doublets, singlets or more exotic representations. For example, supersymmetric models require at least two doublets. Likewise, at least two doublets are required to produce CP violation in the Higgs sector.¹

Models with enlarged Higgs sectors have a richer particle content than the minimal model; in general, neutral pseudoscalars (with respect to their fermion couplings) and charged scalars as well as extra neutral scalars are present. In what follows we will study the QCD corrections to the production of a Higgs pseudoscalar (A^0) via gluon fusion. This process proceeds primarily through a top-quark loop (unless the coupling to bottom quarks is greatly enhanced). We will focus on the case of a light pseudoscalar and work in the heavy-top-quark limit : $m_t \gg M_\lambda$.

To fix our normalization we take the coupling of the pseudoscalar to top quarks to be $\hat{m}_t \gamma_5 / v$ where $v = 246$ GeV. The lowest-order amplitude for $gg \rightarrow A^0$ is well known and takes the form²

$$\mathcal{M}(g(k_{1\mu}^a), g(k_{2\nu}^b) \rightarrow A^0) = -ig_A \delta^{ab} \epsilon^{\mu\nu\rho\sigma} k_{1\rho} k_{2\sigma} \tau f(\tau), \quad (1)$$

with $\tau = 4m_t^2/M_\lambda^2$, $g_A = \alpha_s/2\pi v$, and

$$f(\tau) = \begin{cases} \left[\sin^{-1} \left(\sqrt{1/\tau} \right) \right]^2, & \text{if } \tau \geq 1, \\ -1/4 [\ln(\eta_+/\eta_-) - i\pi]^2, & \text{if } \tau < 1, \end{cases} \quad (2)$$

where $\eta_\pm = (1 \pm \sqrt{1-\tau})$. We see that in the large m_t limit the amplitude for $gg \rightarrow A^0$ is independent of the top quark mass, just as in the scalar case: $\tau f(\tau) \rightarrow 1$ as $\tau \rightarrow \infty$. The heavy-top approximation of the amplitude is accurate to within 5% for $m_t^2 > 2M_\lambda^2$ and to within 10% for $m_t^2 > M_\lambda^2$.

2. Effective Lagrangian

The amplitude in Eq. 1 can be computed by evaluating the triangle diagram with a top quark in the loop and taking the limit $m_t \rightarrow \infty$ or instead by noticing that this amplitude is related to the axial anomaly.³ Let $j_5 = \bar{\psi} \gamma_5 \psi$ be the axial current and $j_5^\mu = \bar{\psi} i \gamma_\mu \gamma_5 \psi$ be the axial vector current. The anomaly equation reads:

$$\partial_\mu j_5^\mu = -2m_t j_5 + i \frac{\alpha_s}{8\pi} G_{\mu\nu}^a \tilde{G}_{\mu\nu}^a, \quad (3)$$

where $G_{\mu\nu}^a$ is the field-strength tensor for SU(3) and $\tilde{G}_{\mu\nu}^a$ is its dual, $\tilde{G}_{\mu\nu}^a = \epsilon^{\mu\nu\rho\sigma} G_{\rho\sigma}^a$. In the heavy quark limit the left side of Eq. 3 vanishes; it is proportional to more powers of external momenta than the other terms.⁴ Therefore, in the heavy quark limit the matrix element of j_5 between gluon states is given

$$2m_t \langle g | j_5 | g \rangle = \frac{i\alpha_s}{8\pi} \langle g | G_{\mu\nu}^a \tilde{G}_{\mu\nu}^a | g \rangle. \quad (4)$$

Equivalently, one may treat the interactions of gluons with the A^0 in the heavy-quark limit as arising from the effective Lagrangian⁵

$$\mathcal{L}_{\text{eff}} = \frac{\alpha_s}{8\pi v} G_{\mu\nu}^a \tilde{G}_{\mu\nu}^a A^0. \quad (5)$$

The power of Eq. 5 for our problem comes from the Adler-Bardeen theorem⁶ which states that Eq. 3 is true *to all orders in perturbation theory*. Therefore, Eq. 5 provides the correct effective Lagrangian with which to compute radiative corrections. The advantage is that amplitudes which correspond to two-loop diagrams in the original theory are one-loop diagrams in the effective theory. A similar Lagrangian can be written for the interaction of photons with the pseudoscalar;⁷ in that case one sees immediately that the $\mathcal{O}(\alpha_s)$ corrections to $\gamma\gamma \rightarrow A^0$ vanish in the $m_t \rightarrow \infty$ limit.

The effective Lagrangian, Eq. 5, leads to two- and three-gluon vertices with the A^0 (as well as a four-gluon vertex which is irrelevant to the current study):

$$\begin{aligned} V_3(k_{1\mu}^a, k_{2\nu}^b) &= -ig_A \delta^{ab} \epsilon^{\mu\nu\rho\sigma} k_{1\rho} k_{2\sigma}, \\ V_4(k_{1\mu}^a, k_{2\nu}^b, k_{3\rho}^c) &= gg_A f^{abc} \epsilon^{\mu\nu\rho\sigma} (k_1 + k_2 + k_3)_\sigma, \end{aligned} \quad (6)$$

where the k_i are the gluon momenta directed inward.

3. Real and Virtual Corrections

We use dimensional regularization in computing the radiative corrections. However, the ϵ -tensors in Eq. 6 are intrinsically four-dimensional objects and must be treated as such. The product of two ϵ -tensors can be written in terms of $\bar{g}^{\mu\nu}$ the metric tensors in the four-dimensional sub-space:

$$-\epsilon^{\alpha\lambda\mu\nu} \epsilon_{\alpha\rho\sigma\tau} = \bar{g}_\rho^\lambda (\bar{g}_\sigma^\mu \bar{g}_\tau^\nu - \bar{g}_\tau^\mu \bar{g}_\sigma^\nu) - \bar{g}_\sigma^\lambda (\bar{g}_\rho^\mu \bar{g}_\tau^\nu - \bar{g}_\tau^\mu \bar{g}_\rho^\nu) + \bar{g}_\tau^\lambda (\bar{g}_\rho^\mu \bar{g}_\sigma^\nu - \bar{g}_\sigma^\mu \bar{g}_\rho^\nu), \quad (7)$$

where $\bar{g}^{\mu\nu} \bar{g}_{\mu\nu} = 4$. For simplicity, we take the incoming momenta to be in four dimensions (this is simply a choice of frame). The lowest order cross section is then, averaged over colors and polarizations in $n = 4 - 2\epsilon$ dimensions,

$$\begin{aligned} \sigma^{(0)}(g(k_1)g(k_2) \rightarrow A^0) &= \frac{\pi g_A^2}{8(N_c^2 - 1)(1 - \epsilon)^2} \delta(1 - z) \\ &\equiv \sigma_{(\epsilon)} \delta(1 - z) \equiv \frac{1}{(1 - \epsilon)^2} \sigma_0 \delta(1 - z), \end{aligned} \quad (8)$$

where $z = M_\lambda^2/s$ and $s = (k_1 + k_2)^2$.

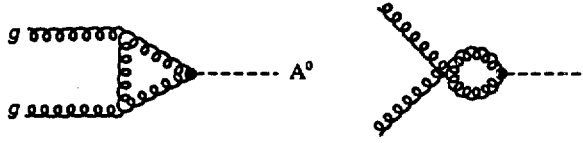


Figure 1: Feynman diagrams for the virtual corrections to $gg \rightarrow A^0$.

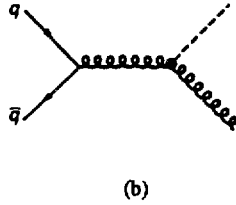
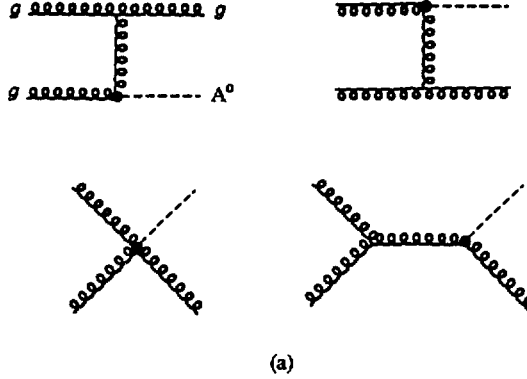


Figure 2: Feynman diagrams for the processes a) $gg \rightarrow gA^0$ and b) $q\bar{q} \rightarrow gA^0$. The diagram for $q\bar{q} \rightarrow qA^0$ is a crossing of diagram b.

The virtual radiative corrections are given by the diagrams in Figure 1, where the amplitude involving the four-gluon vertex vanishes due to the antisymmetry of the ggA^0 -vertex. The other diagram yields the result for the virtual corrections

$$\sigma_v = \frac{N_c \alpha_s \sigma(\epsilon)}{\pi} \eta \left(-\frac{1}{\epsilon^2} + \frac{2}{3} \pi^2 + 2 \right) \delta(1-z), \quad (9)$$

where

$$\eta = \Gamma(1+\epsilon) \left(\frac{4\pi\mu^2}{M_\Lambda^2} \right)^\epsilon. \quad (10)$$

The real corrections are given by the diagrams in Figure 2. The amplitudes squared for the various processes are, averaged over colors and spins,

$$|\mathcal{M}(gg \rightarrow gA^0)|^2 = 8N_c \alpha_s \sigma(\epsilon) \left\{ \frac{s^4 + t^4 + u^4 + M_\Lambda^8}{stu} - 2\epsilon s \left(\frac{u}{t} + \frac{t}{u} \right) + 2\hat{k}_3 \cdot \hat{k}_3 s^2 \left(\frac{1}{t^2} + \frac{1}{u^2} \right) \right\},$$

$$\begin{aligned}
|\mathcal{M}(gg \rightarrow qA^0)|^2 &= 8C_F\alpha_s\sigma(\epsilon)(1-\epsilon) \left\{ -\frac{s^2+u^2}{t} + 2\frac{s^2}{t^2}\hat{k}_3 \cdot \hat{k}_3 \right\}, \\
|\mathcal{M}(q\bar{q} \rightarrow gA^0)|^2 &= 16C_F^2\alpha_s\sigma_0 \frac{(t^2+u^2)}{s},
\end{aligned} \tag{11}$$

where s , t , and u are the familiar Mandelstam variables and k_3 is the momentum of the final state quark or gluon, with the hat denoting the $(n-4)$ -dimensional components. Since the initial state particles are taken to be in four dimensions $\hat{k}_3 \cdot \hat{k}_3$ is the only quantity which depends on the $(n-4)$ -dimensional components. Under integration over the angular variables transverse to the incoming particles we find⁸

$$\int d\Omega_T \hat{k}_3 \cdot \hat{k}_3 = -\frac{(n-4)}{(n-2)} k_T^2 \int d\Omega_T, \tag{12}$$

where k_T is the transverse momentum of the outgoing gluon. So the terms with $\hat{k}_3 \cdot \hat{k}_3$ will contribute terms of $\mathcal{O}(1)$, but only when multiplied by double poles, t^{-2} or u^{-2} (this was used in simplifying Eq. 11).

Integration over n -dimensional phase space gives the cross sections

$$\begin{aligned}
\sigma(gg \rightarrow gA^0) &= \frac{\alpha_s N_c \sigma(\epsilon)}{\pi} \eta' \left\{ \delta(1-z) \left(\frac{1}{\epsilon^2} - \frac{\pi^2}{3} \right) - \frac{2z}{\epsilon} \left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z) \right] \right. \\
&\quad \left. + 2 \left(\frac{\log(1-z)}{1-z} \right)_+ [1 + (1-z)^4 + z^4] - \frac{11}{6}(1-z)^3 \right\}, \\
\sigma(qg \rightarrow qA^0) &= \frac{\alpha_s C_F \sigma(\epsilon)}{2\pi} (1-\epsilon) \eta' \left\{ \left(-\frac{1}{\epsilon} + 2 \log(1-z) \right) [1 + (1-z)^2] \right. \\
&\quad \left. - \frac{1}{2}(1-z)(7-3z) \right\}, \\
\sigma(q\bar{q} \rightarrow gA^0) &= \frac{2\alpha_s C_F^2 \sigma_0}{3\pi} (1-z)^3,
\end{aligned} \tag{13}$$

where

$$\eta' = \Gamma(1+\epsilon) \left(\frac{4\pi\mu^2}{s} \right)^\epsilon. \tag{14}$$

It is interesting to note that the real corrections, when written in terms of the lowest-order cross section are identical in form to those for the scalar case.⁵

We see that the terms of $\mathcal{O}(1/\epsilon^2)$ cancel between the real and virtual diagrams. The terms of $\mathcal{O}(1/\epsilon)$ may be absorbed into redefinitions of the parton distribution functions in the usual factorization procedure. We use the $\overline{\text{MS}}$ prescription which fixes the subtraction terms as

$$\begin{aligned}
\sigma_{\text{AP}}^{gg} &= \frac{\alpha_s}{2\pi} \sigma(\epsilon) 2z P_{gg}(z) (4\pi)^\epsilon \Gamma(1+\epsilon) \frac{1}{\epsilon}, \\
\sigma_{\text{AP}}^{qg} &= \frac{\alpha_s}{2\pi} \sigma(\epsilon) z P_{gq}(z) (4\pi)^\epsilon \Gamma(1+\epsilon) \frac{1}{\epsilon},
\end{aligned} \tag{15}$$

where the splitting functions are defined

$$\begin{aligned}
P_{gg}(z) &= 2N_c \left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z) \right] + \beta_0 \delta(1-z) \\
P_{gq}(z) &= C_F \frac{[1 + (1-z)^2]}{z},
\end{aligned} \tag{16}$$

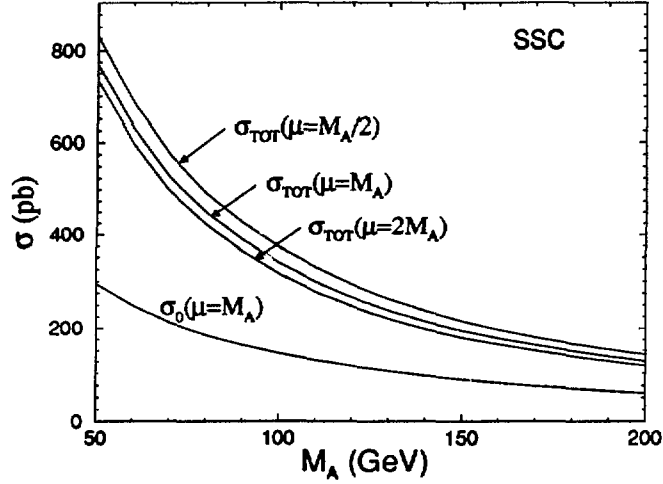


Figure 3: Radiatively corrected results for A^0 production in proton-proton collisions at the SSC, for several values of the factorization/renormalization scale μ . Shown for comparison is the lowest order cross section.

where $\beta_0 = \frac{11}{6}N_c - \frac{1}{3}N_f$ with N_f being the number of light quark flavors. The charge renormalization subtraction term is

$$\sigma_{\text{ch}} = -\frac{\alpha_s}{\pi}(4\pi)^\epsilon \Gamma(1+\epsilon) \sigma_{(\epsilon)} \delta(1-z) \frac{\beta_0}{\epsilon}. \quad (17)$$

When all the contributions are included the final results are

$$\begin{aligned} \hat{\sigma}_{gg} &= \frac{\alpha_s N_c \sigma_0}{\pi} \left\{ \delta(1-z) \left(\frac{\pi^2}{3} + 2 \right) + 2z \ln \left(\frac{M_A^2}{z\mu^2} \right) \left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z) \right] \right. \\ &\quad \left. + 2 \left(\frac{\ln(1-z)}{1-z} \right)_+ [1 + (1-z)^4 + z^4] - \frac{11}{6}(1-z)^3 \right\}, \\ \hat{\sigma}_{qg} &= \frac{\alpha_s C_F \sigma_0}{2\pi} \left\{ \left[1 + \ln \left(\frac{M_A^2(1-z)^2}{z\mu^2} \right) \right] [1 + (1-z)^2] \right. \\ &\quad \left. - \frac{1}{2}(1-z)(7-3z) \right\}. \end{aligned} \quad (18)$$

Since the real corrections in our result are the same as for the scalar case, the only difference between the scalar and pseudoscalar results is the coefficient of the $\delta(1-z)$ term coming from the virtual result: $N_c(\pi^2/3 + 2)$ for the pseudoscalar and $(N_c\pi^2/3 + 5/2N_c - 3/2C_F)$ for the scalar. The numerical similarity of the two constant terms [an accident of SU(3)] means that the ratio between the next-to-leading order result and the lowest-order result, or the 'K-factor,' will be almost equal for the two processes.

The radiatively corrected cross section is plotted for proton-proton collisions at the SSC, $\sqrt{S} = 40$ TeV, and at the LHC, $\sqrt{S} = 15.4$ TeV in Figure 3 and Figure 4 for several choices of renormalization scales and using HMRSB parton distributions.⁹

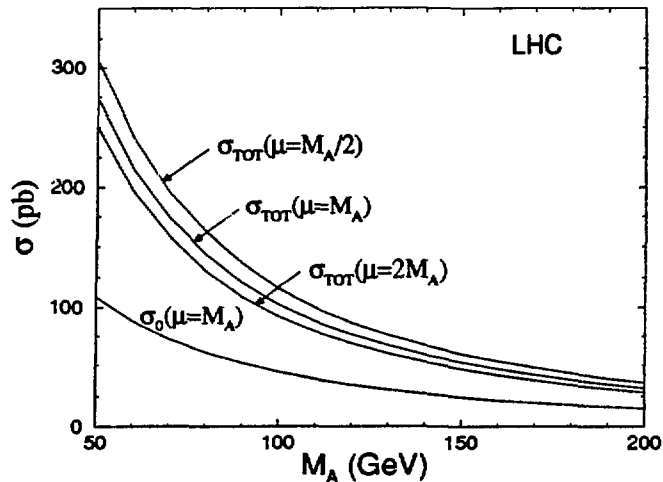


Figure 4: Same as Figure 3 for the LHC.

The ratio between the radiatively corrected result and the lowest order result (computed with the two-loop α_s) ranges from about 2.6 for $M_A = 50$ GeV to about 2.2 for $M_A = 200$ GeV for both values of \sqrt{S} . As is the case for the scalar Higgs, the contribution from the gg initial state dominates with about half the correction coming from the $\delta(1-z)$ term.

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