B_c

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1. MOTIVATION

Mesons with beauty and charm hold a special fascination for the theorist. A bound state of two heavy quarks, B_c admits reliable calculations of both spectroscopy and decay. A rich spectrum of extremely narrow $c\bar{b}$ states awaits discovery. The deep binding of the heavy quarks has a pronounced influence upon the B_c lifetime and the pattern of weak decays. The decays of B_c may open new ways of measuring the Kobayashi-Maskawa matrix element V_{cb}.

These are timely issues because the first observation of B_c is imminent. More than 10^9 b quarks are produced in a year's run of the Tevatron Collider. They materialize not only as B_u and B_d, but also as B_s, b-baryons, and still rarer birds. CDF has successfully operated a silicon vertex detector in the Tevatron environment, and has shown that subtle spectroscopy of the χ_c states is possible in a hadron collider. The $\psi\pi$ decay mode offers a reasonable tag for discovery and subsequent study of the B_c. For all these reasons, Estia Eichten and I are filling in a portrait of the B_c and its excited states.[1] In this talk, I will summarize what we have learned.[2]

2. THE B_c SPECTRUM

The $c\bar{b}$ quarkonium system occupies a region of space already probed by the ψ and Υ families, so the Schrödinger equation will provide a reliable description of the bound-state spectrum. Calculating the B_c mass is straightforward: (i) Choose a quarkonium potential with c- and b-quark masses determined from ψ and Υ; (ii) Compute the 1S center of gravity; (iii) Add the hyperfine splitting,

$$M(B_c) = M(1S) - \tfrac{3}{4}\Delta M; \quad M(B_c^*) = M(1S) + \tfrac{1}{4}\Delta M \;,$$

where $\Delta M = \alpha_s|\Psi(0)|^2/m_b m_c$. After surveying a variety of potentials that provide good descriptions of the ψ and Υ spectra, we estimate that $M(B_c) = 6256 \pm 20$ MeV/c^2, and that $M(B_c^*) - M(B_c) \approx 72$ MeV/c^2.

We have used standard methods to calculate the entire spectrum, including fine-structure and hyperfine-structure splitting, in the Buchmüller-Tye potential.[3] Figure 1 shows that approximately fifteen narrow states will lie below the BD flavor threshold.

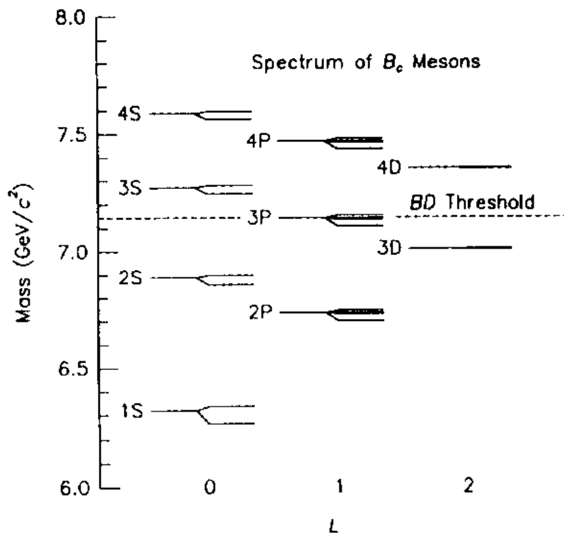

Figure 1. The spectrum of $c\bar{b}$ states.

3. TRANSITIONS WITHIN THE B_c SPECTRUM

The $c\bar{b}$ system is the true hadronic analog of the hydrogen atom because, in contrast to $c\bar{c}$ or $b\bar{b}$, its constituents cannot annihilate into gluons. All the excited $c\bar{b}$ states below flavor threshold decay by electromagnetic or hadronic transitions that cascade to the ground-state B_c, which decays weakly. Only for the 2S levels do hadronic decays ($\rightarrow 1\text{S} + \pi\pi$) dominate over the electromagnetic transitions. All the $c\bar{b}$ levels are extraordinarily narrow, with total widths ranging between 21 and 173 keV.

We believe that, in time, it will be possible to map out part of the $c\bar{b}$ spectrum by detecting γ or $\pi\pi$ in coincidence with weak decays of B_c. That would be a wonderful triumph of experimental technique and an opportunity to test our understanding of the force between quarks.

4. THE LIFETIME OF B_c

Weak decays of B_c proceed through the decay of either heavy quark or by $c\bar{b}$ annihilation into a virtual W^+. To estimate the semi-inclusive decay rates of the c and $\bar{b}$ quarks, we modify the spectator picture to take account of the deep binding of the heavy quarks. The influence of confinement has been overlooked in previous work on B_c decays, in which c and $\bar{b}$ have been regarded as free. It has a decisive effect on the systematics of B_c decays.

For the (Cabibbo-Kobayashi-Maskawa–suppressed) decays of the $\bar{b}$ antiquark, the binding of the $\bar{b}$ to c reduces the $\bar{b} \rightarrow \bar{c}$ decay rate by about 40 percent compared with the decay rate of a free $\bar{b}$ antiquark, to $\Gamma(B_c \rightarrow \bar{b}s + W^\star) = 3.9 \times 10^{11}\ \text{s}^{-1}$.

For the (CKM-favored) decays of the c quark, binding suppresses the $c \rightarrow s$ transition by a factor of 4.8 compared to the decay rate of an isolated c quark. This compensates for the favorable quark mixing ($|V_{cs}| \gg |V_{cb}|$) and makes the $c \rightarrow s$ decay, at $\Gamma(B_c \rightarrow \bar{b}s + W^\star) = 0.7 \times 10^{11}\ \text{s}^{-1}$, less important than $\bar{b} \rightarrow \bar{c}$.

Table 1. Partial decay rates and branching fractions for semi-inclusive B_c decays.

Channel	Partial Width ($10^{10}\ \text{s}^{-1}$)	Branching Fraction (%)
$(c\bar{c})e\nu_e$	6.90	10
$(c\bar{c})\mu\nu_\mu$	6.86	10
$(c\bar{c})\tau\nu_\tau$	1.33	2
$(c\bar{c})u\bar{d}$	18.79	27
$(c\bar{c})c\bar{s}$	5.56	8
$(s\bar{b})e\nu_e$	2.91	4
$(s\bar{b})\mu\nu_\mu$	2.67	4
$(s\bar{b})u\bar{d}$	1.57	2
$\tau\nu_\tau$	6.70	10
$c\bar{s}$	15.77	23
Total	69.06	

The compact size of the $c\bar{b}$ system means that the pseudoscalar decay constant f_{B_c} will be large, so that annihilation decays into massive final states are also prominent. We adopt the value $f_{B_c} = 500$ MeV, suggested by quarkonium calculations, to estimate decay rates. Decays into light products are helicity-suppressed, but decays into $c\bar{s}$ and $\tau^+\nu_\tau$ proceed with significant rates, so that $\Gamma(B_c \rightarrow W^\star) = 2.2 \times 10^{11}\ \text{s}^{-1}$.

The decay rates computed using constituent-quark masses for the decay products of the virtual W-boson are collected in Table 1. Adding up the decay rates presented there, we arrive at a total rate that corresponds to a lifetime $\tau(B_c) = 1.44$ ps. Annihilations account for 33% of the total rate, c-decays for 11%, and b-decays for 56%. Using current-quark masses instead of constituent-quark masses for the decay products of the virtual $W^\star$, we find $\tau(B_c) = 1.28$ ps. In this case, c-decays increase to 19% at the expense of annihilations. We adopt 1.35 ± 0.15 ps as our best *semi-inclusive* estimate for the B_c lifetime. The uncertainty reflects the broad range of experimentally allowed values of $|V_{cb}|$, as well as limitations of the modified spectator approximation that we shall bring to light at once.

To the extent that f_{B_c} is known—and we estimate that the uncertainty from potential model calculations is no more than 20% today—a measurement of the annihilation decay rate constitutes an independent determination of $|V_{cb}|$. It is worth thinking about how such measurements would be made, and normalized.

5. EXCLUSIVE DECAYS

Only a small number of final states are available for the decay of the charmed quark in B_c. Among the Cabibbo-favored decays, the list

$$B_c \rightarrow \pi \begin{Bmatrix} B_s \\ B_s^\star \\ B_s^{\star\star} \\ B_u K \end{Bmatrix}; \quad K \begin{Bmatrix} B_u \\ B_u^\star \\ B_u \pi \end{Bmatrix}; \quad \rho \begin{Bmatrix} B_s \\ B_s^\star \end{Bmatrix}; \quad K^\star \begin{Bmatrix} B_u \\ B_u^\star \end{Bmatrix}$$

is nearly exhaustive. We can use a combination of heavy-quark methods[4] and the nonrelativistic wave functions to calculate the exclusive rates for these decays. We have found the semi-inclusive decay rate $c \rightarrow s + W^+$ calculated in the spectator model to be significantly inaccurate for transitions like $B_c \rightarrow \rho B_s^{(\star)}$ that lie very close to the kinematical limit. This is because inclusive-exclusive duality is not local, but represents an averaging over a "typical hadronic" scale of energies. When the allowed phase space for decay is concentrated near threshold, semi-inclusive methods cannot be trusted. We estimate the exclusive decay rates $\Gamma(B_c \to B_s\pi) \approx \Gamma(B_c \to B_s\rho) \approx 10 \times 10^{10}\ \mathrm{s}^{-1}$, far larger than the semi-inclusive estimate for $B_c \to (s\bar{b})u\bar{d}$ that should include them. A similar conflict would arise in calculations of the tau decay rates, if the tau mass were around 1 GeV/c^2. Increasing the exclusive decays of the charmed quark will decrease $\tau(B_c)$ to perhaps 1.1 to 1.2 ps.

For the $\bar{b} \to \bar{c}$ transitions, the kinematical situation is favorable for trustworthy calculations of exclusive decay rates. The $\bar{c}$ moves with a velocity close to that of the spectator c, so it is a good approximation to treat the wave functions of the B_c initial state and the $c\bar{c}$ final state nonrelativistically. The rates for the semileptonic decays to ψ, ψ', χ_c, and η_c, as well as the lepton spectra in the rest frame of the decaying B_c, can then be calculated in terms of a quantum-mechanical overlap integral.

With an adventurous spirit, it takes just a little imagination to consider B_c decays as a source of tagged B_s for the study of B_s-$\overline{B}_s$ mixing and CP violation in the B_s system. The decay

$$\begin{array}{ccc} B_c^+ & \to & B_s^{(*)}\ell^+\nu \ , \\ (c\bar{b}) & & (\bar{b}s) \end{array}$$

which occurs with a branching fraction about four percent for either electrons or muons, identifies the flavor of B_s at the time of its production. An ℓ^+ signals the decay $c \to s$, yielding $\bar{b}s = B_s$, while an ℓ^- tags $b\bar{s} = \overline{B}_s$. The subsequent decay of the B_s into $D_s\ell\nu$ or D^-K^+, etc. constitutes a second flavor measurement at the time of decay. Event rates at the SSC and LHC, or at the Tevatron with the Main Injector, may make this a practical technique, particularly if displaced-vertex triggering becomes a reality.

6. PRODUCTION AT THE TEVATRON

Just how rare will B_c be in high-energy collisions? A number of estimates, most recently a perturbative calculation of the rate at which energetic b quarks fragment into B_c and B_c^*, suggest that B_c should be produced at about 10^{-3} of the b rate in the Tevatron collider.[5] We would therefore expect about 10^6 B_c to be produced in the next collider run at Fermilab. Similar estimates suggest that a few hundred B_c should be produced per million hadronic events at LEP.

Promising signatures for the discovery of B_c include

$$B_c \to \psi\pi \quad \llcorner\!\!\to e^+e^- \text{ or } \mu^+\mu^- \ ,$$

for which we estimate a branching fraction of 0.4% times 6.3%, or 2.5×10^{-4}, and

$$B_c \to \psi\ell\nu \quad \llcorner\!\!\to e^+e^- \text{ or } \mu^+\mu^- \ ,$$

for which we estimate 1.5% times 6.3%, or 9.4×10^{-4}. The semi-inclusive branching fraction for $B_c \to \psi +$anything will be around 10%. The ψa_1 mode may also be detected with good efficiency. Decay modes that involve two short tracks may also be appealing targets. The $D_s^*\psi$ ($\approx 5\%$) and $B_s\pi$ modes are of special interest.

7. SUMMARY

Mesons with beauty and charm are an interesting theoretical laboratory that should soon become accessible to experiment. We can predict the properties of $c\bar{b}$ states with confidence. In particular, we expect the mass of the ground-state B_c to lie close to 6256 MeV/c^2. More than a dozen narrow states lie below flavor threshold. They cascade to the ground state by making electromagnetic or hadronic transitions within the $c\bar{b}$ spectrum. The excited states are extraordinarily narrow. They may be observable in γ or $\pi\pi$ coincidences with B_c.

Weak decays of B_c are affected by the strong binding of the $\bar{b}$ and c constituents. Our estimates of the semi-inclusive decay rates lead to a lifetime $\tau(B_c) \approx 1.35 \pm 0.15$ ps that is appreciably longer than the guess $\tau(B_c) \approx (1/\tau_b + 1/\tau_c)^{-1} \approx 0.71$ to 0.88 ps, which neglects binding. The semi-inclusive analysis understates the importance of $B_s +$light hadron modes. Our best lifetime estimate is 1.1 to 1.2 ps.

Finally, while B_c is an exotic—indeed, undiscovered—state today, it may have important practical applications in the future. The decay $B_c \to B_s\ell\nu$ may provide a clean and efficient flavor tag for B_s.

8. ACKNOWLEDGEMENT

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9. REFERENCES

1. Estia J. Eichten and Chris Quigg, in preparation. This paper includes complete references to earlier work.
2. For earlier work on B_c, see for example, D. Du and Zi Wang, *Phys. Rev. D* **39**, 1342 (1989); S. S. Gershteĭn, A. K. Likhoded, and S. R. Slabopitskiĭ, *Int. J. Mod. Phys. A* **6**, 2309 (1991); M. Lusignoli and M. Masetti, *Z. Phys. C* **51**, 549 (1991); Chao-Hsi Chang and Yu-Qi Chen, *Phys. Lett.* **B284**, 127 (1992).
3. W. Buchmüller and S.-H. H. Tye, *Phys. Rev. D* **24**, 132 (1981).
4. For a closely related application, see Elizabeth Jenkins, Michael Luke, Aneesh V. Manohar, and Martin J. Savage *Nucl. Phys.* **B390**, 463 (1993).
5. Eric Braaten, Kingman Cheung, and Tzu-Chiang Yuan, Northwestern preprint NUHEP-TH-93-6 (unpublished).