Au9413957

UM-P-93/58, OZ-93/13

Anomalous WWZ couplings and $K_L \rightarrow \mu^+\mu^-$

Xiao-Gang He

Research Center for High Energy Physics

School of Physics

University of Melbourne

Parkville, Vic. 3052 Australia

(June, 1993)

Abstract

We study contributions to $K_L \rightarrow \mu^+\mu^-$ from anomalous WWZ interactions. There are, in general, seven anomalous couplings. Among the seven anomalous couplings, only two of them contribute significantly. The others are suppressed by factors like m_s^2/M_W^2, m_d^2/M_W^2, or m_K^2/M_W^2. Using the experimental data on $K_L \rightarrow \mu^+\mu^-$, we obtain strong bounds on the two anomalous couplings.

Typeset using REVTeX

In this paper we study contributions to $K_L \to \mu^+\mu^-$ from the anomalous WWZ interactions. The Minimal Standard Model of electroweak interactions is in very good agreement with present experimental data. However its structure should be tested in detail in order to finally establish the model. One of the important aspects is to test the structure of self-interactions of electroweak bosons. Such test will provide information about whether the weak bosons are gauge particles with interactions predicted by the MSM, or gauge particles of some extensions of the MSM which predict different interactions at loop levels, or even non-gauge particles whose self-interactions at low energies are described by effective interactions. In general there will be more self-interaction terms than the tree level MSM terms (the anomalous couplings) [1]. It is important to find out experimentally what are the allowed regions for these anomalous couplings. The process $K_L \to \mu^+\mu^-$ has been studied in the MSM extensively [2]. It has been used to study the allowed range for the top quark mass and the allowed ranges for some of the KM matrix elements. In this paper we show that $K_L \to \mu^+\mu^-$ also puts very strong constraints on some of the WWZ anomalous couplings.

The most general form for the anomalous WWZ interactions can be parametrized as

$$\begin{aligned} L = -g\cos\theta_W \Big[& i g_1^Z (W^{+\mu\nu} W^{-\mu} Z^\nu - W_\mu^+ W^{-\mu\nu} Z_\nu) \\ & + i\kappa^Z W_\mu^+ W_\nu^- Z^{\mu\nu} + i\frac{\lambda^Z}{M_W^2} W_{\sigma\rho}^+ W^{-\rho\delta} Z_\delta^\sigma \\ & + i\tilde{\kappa}^Z W_\mu^+ W_\nu^- \tilde{Z}_{\mu\nu} + i\frac{\tilde{\lambda}^Z}{M_W^2} W_{\sigma\rho}^+ W^{-\rho\delta} \tilde{Z}_\delta^\sigma \\ & + g_4^Z W_\mu^+ W_\nu^- (\partial^\mu Z^\nu + \partial^\nu Z^\mu) \\ & + g_5^Z \epsilon_{\mu\nu\alpha\beta} (W^{+\mu} \partial^\alpha W^{-\nu} - \partial^\alpha W^{+\mu} W^{-\nu}) Z^\beta \Big] , \end{aligned} \tag{1}$$

where $W_\mu^\pm$ and Z_μ are the W-boson and Z-boson fields, $W_{\mu\nu}$ and $Z_{\mu\nu}$ are the W-boson and Z-boson field strengths, respectively; and $\tilde{Z}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta} Z^{\alpha\beta}$. The terms proportional to g_1^Z, κ^Z, λ^Z and g_5^Z are CP conserving and $\tilde{\kappa}^Z$, $\tilde{\lambda}$ and g_4^Z are CP violating.

To obtain amplitude for the process $K_L \to \mu^+\mu^-$, we first evaluate the effective coupling for dsZ with the Z-boson off-shell. This coupling is induced at the one loop level. The effective Hamiltonian is given by

$$H_{eff} = -igcos\theta_W \frac{g^2}{2}\epsilon^{Z\mu} V_{ld} V_{ls}^* \bar{d}\gamma_\alpha\gamma_\nu\gamma_\beta \frac{1-\gamma_5}{2} s$$
$$\times \int \frac{d^4k}{(2\pi)^4} \frac{k^\nu (g^{\alpha\alpha'} - \frac{k^{+\alpha}k^{+\alpha'}}{M_W^2})(g^{\beta\beta'} - \frac{k^{-\beta}k^{-\beta'}}{M_W^2})\Gamma_{\mu\alpha'\beta'}(q,k^+,k^-)}{(k^2 - m_l^2)((p-k)^2 - M_W^2)((p'-k)^2 - M_W^2)} + H.C., \qquad (2)$$

where l is summed over u , s , and t, and

$$\Gamma_{\mu\alpha\beta}(q,k^+,k^-) = g_1^Z(g_{\alpha\beta}(k_\mu^- - k_\mu^+) + g_{\beta\mu}k_\alpha^+ - g_{\alpha\mu}k_\beta^-)$$
$$- \kappa^Z(g_{\alpha\mu}q_\beta - g_{\beta\mu}q_\alpha) - \tilde{\kappa}^Z \epsilon_{\mu\alpha\beta\rho} q^\rho$$
$$+ \frac{\lambda^Z}{M_W^2}(g_\alpha^\rho k^{+\delta} - g_\alpha^\delta k^{+\rho})(g_{\beta\delta}k_{-\sigma} - g_{\beta\sigma}k_{-\delta})(g_{\rho\mu}q^\sigma - g_\mu^\sigma q_\rho)$$
$$+ \frac{\tilde{\lambda}^Z}{M_W^2}(g_\alpha^\rho k^{+\delta} - g_\alpha^\delta k^{+\rho})(g_{\beta\delta}k^{-\sigma} - g_\beta^\sigma k_{-\delta})\epsilon_{\rho\sigma\mu\tau}q^\tau$$
$$+ ig_4^Z(g_{\beta\mu}q_\alpha + g_{\alpha\mu}q_\beta) + ig_5^Z\epsilon_{\mu\alpha\beta\sigma}(k^{+\sigma} - k^{-\sigma});,$$

where k, p, and p' are the internal, s-quark and d-quark momenta respectively, $q = p' - p$, $k^+ = p - k$ and $k^- = k - p'$, and $\epsilon^{Z\mu}$ is the Z-boson polarization vector. Performing the standard Feynman parametrization, we have

$$H_{eff} = -ig^3 cos\theta_W \epsilon^{Z\mu} V_{ls} V_{ld}^* \bar{d}\gamma_\alpha\gamma_\nu\gamma_\beta \frac{1-\gamma_5}{2} s$$
$$\times \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4k}{(2\pi)^4} \frac{k^\nu (g^{\alpha\alpha'} - \frac{k^{+\alpha}k^{+\alpha'}}{M_W^2})(g^{\beta\beta'} - \frac{k^{-\beta}k^{-\beta'}}{M_W^2})\Gamma_{\mu\alpha'\beta'}(q,k^+,k^-)}{(k^2 - 2k\cdot(xp+yp') - (m_l^2 + (M_W^2 - m_l^2)(x+y))^3}$$
$$+ H.C.. \qquad (3)$$

Due to the anomalous nature of the couplings, the loop integrals are in general cut-off Λ dependent. To calculate such dependence we use dimensional regularization with a (modified) minimal subraction renormalization scheme following the prescription in Ref. [3]. Substituting $k' = k - (xp + yp')$ into eq.(3), the terms in odd powers of k' vanish. We find that among all the even power terms in k', only terms proportional to g_1^Z and g_5^Z will produce terms with no powers in external momenta. All other terms will be at least with two powers in external momenta. Therefore their contributions to $K_L \to \mu^+\mu^-$ are suppressed by m_d^2/M_W^2, m_s^2/M_W^2 or m_K^2/M_W^2 compared with the contributions from the g_1^Z and g_5^Z terms. It is, then, obvious that the process $K_L \to \mu^+\mu^-$ can only put useful constraints on g_1^Z and g_5^Z but not the others. The g_1^Z and g_5^Z contributions to the effective dsZ coupling is given by

$$H_{eff}(dsZ) = -\frac{1}{32\pi^2} g^3 cos\theta_W V_{ls} V_{ld}^* F_A(x_l) Z^\mu \bar{d}\gamma_\mu \frac{1-\gamma_5}{2} s + H.C. , \tag{4}$$

where $x_l = m_l^2/M_W^2$ and the function $F_A(x)$ is given by

$$F_A(x) = -g_1^Z \frac{3}{2}\Big(x \ln\frac{\Lambda^2}{M_W^2} + \frac{x^2(2-x)}{(1-x)^2}\ln x + \frac{11x - 5x^2}{6(1-x)} - \frac{x}{6}\Big)$$
$$+ g_5^Z \Big(\frac{3x}{1-x} + \frac{3x^2 \ln x}{(1-x)^2}\Big) . \tag{5}$$

The amplitude for $K_L \to \mu^+\mu^-$ is obtained by exchanging a virtual Z-boson between ds and $\mu^+\mu^-$. At the quark level, we obtain

$$H_{eff} = \frac{G_F^2 M_W^2}{2\pi^2} cos^2\theta_W V_{ls} V_{ld}^* F_A(x_l) \bar{d}\gamma^\mu \frac{1-\gamma_5}{2} s \bar{\mu}\gamma_\mu \Big(\frac{1-\gamma_5}{2} - 2\sin^2\theta_W\Big)\mu + H.C. . \tag{6}$$

From this quark level effective Hamiltonian, we obtain the decay amplitude

$$M(K_L \to \mu^+\mu^-) = i\frac{G_F^2 M_W^2 f_K m_\mu}{2\sqrt{2}\pi^2} Re(V_{ls}V_{ld}^*) cos^2\theta_W F_A(x_l) \bar{\mu}\gamma_5\mu . \tag{7}$$

Here we have used: $< 0|\bar{s}\gamma^\mu\gamma_5 d|K^0 >= i f_K p_K^\mu$, $p_K^\mu \bar{\mu}\gamma_\mu\mu = 0$, and $p_K^\mu \bar{\mu}\gamma_\mu\gamma_5\mu = 2m_\mu\bar{\mu}\gamma_5\mu$. We note that the vector current part does not contribute. For the same reason the anomalous $WW\gamma$ interactions do not contribute to $K_L \to \mu^+\mu^-$.

Combining the constribution from the MSM, we obtain the total amplitude

$$M^t(K_L \to \mu^+\mu^-) = i\frac{G_F^2 M_W^2 f_K m_\mu}{2\sqrt{2}\pi^2} Re(V_{ls}V_{ld}^*)\eta_l F(x_l)\bar{\mu}\gamma_5\mu , \tag{8}$$

where η_l are the QCD correction factors which are of order one [4]. The function $F(x)$ is given by

$$F(x) = F_S(x) + cos^2\theta_W F_A(x) . \tag{9}$$

with the MSM contribution $F_S(x)$ given by [5]

$$F_S(x) = -\frac{2x}{1-x} + \frac{x^2}{2(1-x)} - \frac{3x^2\ln x}{2(1-x)^2} . \tag{10}$$

We are now ready to use experimental data to put constraint on g_5^Z. The total branching ratio Br^t for $K_L \to \mu^+\mu^-$ is $(7.3 \pm 0.4) \times 10^{-9}$ [6]. There are several different contributions to this decay which can be parametrized as $Br^t = R_{2\gamma} + R_{dis}$. Here $R_{2\gamma}$ is the absorptive contribution due to two real photons in the intermediate state and R_{dis} is the dispersive contribution which contains the weak contribution R_W from eq.(9) and long distance contribution R_{LD}. The absorptive part of the amplitude coming from real photons in the intermediate state has been unambiguously determined from the measured ratio $Br(K_L \to \gamma\gamma) = (5.7\pm0.27)\times10^{-4}$ [6]. This gives $R_{2\gamma} = (6.83\pm0.29)\times10^{-9}$. The dispersive contribution is then, $R_{dis} = (0.47 \pm 0.56) \times 10^{-9}$. When extracting the weak contribution from R_{dis}, one faces the problem of subtracting the long distance contribution. It has been argued that this contribution is small compared with the absorptive contribution by using data from $K_L \to e^+e^-\gamma$ [7]. The dispersive contribution may be solely due to weak contribution. At the present the long distance contribution is not well determined [8]. In our numerical analysis we will assume that R_{dis} is saturated by the weak contribution R_W.

To minimize uncertainties in f_K we scale the rate $\Gamma(K_L \to \mu^+\mu^-)_W$ due to the weak contribution by $\Gamma(K^+ \to \mu^+\nu_\mu)$. We have

$$Br(K_L \to \mu^+\mu^-) = \frac{\tau^0}{\tau^+}Br(K^+ \to \mu^+\nu_\mu)\frac{\Gamma(K_L \to \mu^+\mu^-)_W}{\Gamma(K^+ \to \mu^+\nu_\mu)}$$
$$= \frac{\tau^0}{\tau^+}Br(K^+ \to \mu^+\nu_\mu)\frac{G_F^2 M_W^4}{8\pi^4}\frac{(1-4m_\mu^2/m_K^2)^{1/2}}{(1-m_\mu^2/m_K^2)^2}\frac{|Re(V_{sl}V_{dl}^*\eta_l F(x_l)|^2}{|V_u s|^2}\,. \qquad (11)$$

The branching ratio $Br(K^+ \to \mu^+\nu_\mu)$ is 63.5%, and the lifetimes τ^0 of K_L and τ^+ of K^+ are $5.17 \times 10^{-8}s$ and $1.237 \times 10^{-8}s$, respectively [6]. We will use $|V_{us}| = 0.22$, and $\eta_l = 0.9$. The dominant contribution is from the top quark in the loop. We must know the value for $Re(V_{ts}V_{td}^*)$. Unfortunately this quantity is not well determined at present. We will use the most recent estimate for $|V_{td}|$ in Ref. [9] and take $Re(V_{ts}V_{td}^*)$ to be in the range 3.2×10^{-4} to 6.7×10^{-4}. In our analysis we will let the top quark mass and the anomalous couplings g_1^Z and g_5^Z vary.

If g_1^Z and g_5^Z is set to zero, we obtain the MSM result. Using the experimental data and allowing the relevant KM matrix to span the allowed region, we find that the top quark mass must be less than 240 GeV. This bound is weaker than the bound from LEP data [10]. In the following analysis, we consider the cases where one of g_1^Z and g_5^Z is not zero. In Tables 1, 2 and 3, we show the effects of non-zero g_1^Z. Table 1. shows how $R_W/R_{2\gamma}$ varies with g_1^Z for different cutoffs Λ. We see that depending on the sign of g_1^Z, the anomalous coupling g_1^Z can either increase or decrease R_W. Our results for the constraints on g_1^Z at 2σ level for two different cutoffs, $\Lambda = 1$ TeV and $\Lambda = 10$ TeV are shown in Table 2. and 3. The constraints on g_1^Z in Table 2. and 3. are for $Re(V_{ts}V_{td}^*)$ equal to 3.2×10^{-4} and 6.7×10^{-4}, respectively. If g_1^Z is positive the contribution from the anomalous interaction has the same sign as the MSM contribution. g_1^Z is constrained to be in the range -0.96 to 0.57 for $\Lambda = 1$ TeV. The constraints on g_1^Z become tighter when the top quark mass is increased. In Tables 4, 5, and 6, we show the effects of non-zero g_5^Z. This contribution is cutoff independent. If g_5^Z is positive, the contribution has the opposite sign as that of the MSM. g_5^Z is constrained to be between -3.36 to 5.67. Analysis with both g_1^Z and g_5^Z being non-zero can also be carried out. In this case cancellations between the anomalous contributions may happen. No significant additional constraints on g_1^Z and g_5^Z can be obtained using data only from $K_L \to \mu^+\mu^-$.

The same analysis can be carried out for $B \to \mu^+\mu^-$. In this case the long distance contribution is expected to be small. When experimental data for this decay will become available, one may obtain better constraints on g_1^Z and g_5^Z.

ACKNOWLEDGMENTS

I would like to thank G. Valencia for useful discussions. This work was supported in part by the Australian Research Council.

REFERENCES

[1] J.F. Gaemers and G. Gounaris, Z. Phys. **C1** 259(1979); K. Hagiwara, R. D. Peccei, D. Zeppenfeld and K. Hikasa, Nucl. Phys. **B282**, 253(1987).

[2] M.K. Gaillard and B.W. Lee, Phys. Rev. **D9**, 897(1974); V.A. Novikov, M.A. Shifman, A. I. Vainshtein and V.Z. Zakharov, Phy. Rev. **D16**, 223(1977); R.E. Schrock and M.B. Voloshin, Phys. Lett. **B87**, 375(1979); A.J. Buras, Phys. Rev. Lett. **46**, 1354(1981); V. Barger, W.F. Long, E. Ma and A. Pramudita, Phys. Rev. **D25**, 1860 (1982); A. Paschos, B. Stech and U. Turke, Phys. Lett. **B128**, 240(1983); C.G. Geng and J.N. Ng, Phys. Rev. **D41**, 2351(1990).

[3] C. Burgess and D. London, Preprint, McGill-92/05, UdeM-LPN-TH-84.

[4] V.A. Novikov, M.A. Shifman, A. I. Vainshtein and V.Z. Zakharov, Phy. Rev. **D16**, 223(1977); A.J. Buras, Phys. Rev. Lett. **46**, 1354(1981); C.O. Dib, I. Dunietz and F.J. Gilman, Phys. Rev. **D39**, 2639(1989).

[5] T. Inami and C.S. Lim, Prog. Theor. Phys. **65**, 297(1981).

[6] Review of Particle Properties, Phys. Rev. **D45**, 1992.

[7] L. Bergstrom, E. Masso and P. Singer, Phys. Lett. **B249**, 141(1990).

[8] L. Littenberg and G Valencia, Preprint, FERMILAB-PUB-93/004-T.

[9] A. J. Buras, M. James and P.H. Weisz, Nucl. Phys. **B347**, 491(1990).

[10] S.C.C. Ting, Plenary talk given at the Annual Meeting of the Division of Particles and Fileds of the APS. Fermilab, 1992.

TABLES

TABLE I. $R_W/R_{2\gamma}$ vs. g_1^Z for $m_t = 150GeV$ and $|Re(V_{ts}V_{td}^*)| = 5 \times 10^{-4}$.

g_1^Z	-1.0	-0.8	-0.6	-0.4	-0.2	-0.1	0.0	0.2	0.4	0.6
Λ = 1 TeV										
$R_W/R_{2\gamma}$	2.25	1.27	0.57	0.15	1.45×10^{-4}	0.03	0.13	0.54	1.22	2.18
Λ = 10 TeV										
$R_W/R_{2\gamma}$	12.35	7.51	3.86	1.41	0.17	7.3×10^{-4}	0.13	1.28	3.65	7.21

TABLE II. The constraints for g_1^Z with $|Re(V_{ts}V_{td}^*)| = 3.2 \times 10^{-4}$.

$m_t(GeV)$	100	125	150	175	200
Λ = 1 TeV					
g_1^Z	-0.99~ 0.59	-0.74~0.35	-0.59~ 0.21	-0.51~0.12	-0.45~0.06
Λ = 10 TeV					
g_1^Z	-0.51~0.30	-0.37~0.17	-0.29 ~0.10	-0.24~0.06	-0.20~0.03

TABLE III. The constraints for g_1^Z with $|Re(V_{ts}V_{td}^*)| = 6.7 \times 10^{-4}$.

$m_t(GeV)$	100	125	150	175	200
Λ = 1 TeV					
g_1^Z	-0.58~0.18	-0.45~0.06	-0.39~ -9.6×10^{-4}	-0.34~-0.04	-0.31~-0.07
Λ = 10 TeV					
g_1^Z	-0.30~0.09	-0.23~ 0.03	-0.19~-5.2×10^{-4}	-0.16~-0.02	-0.14~-0.03

TABLE IV. $R_W/R_{2\gamma}$ vs. g_5^Z for $m_t = 150GeV$ and $|Re(V_{ts}V_{td}^*)| = 5 \times 10^{-4}$.

g_5^Z	6.0	5.0	4.0	3.0	2.0	1.37	1.0	0.0	-1.0	-2.0	-3.0
$R_W/R_{2\gamma}$	1.49	0.92	0.48	0.19	2.8×10^{-2}	0.0	9.4^{-3}	0.13	0.39	0.79	1.33

TABLE V. The constraints for g_5^Z with $|Re(V_{ts}V_{td}^*)| = 3.2 \times 10^{-4}$.

m_t(GeV)	100	125	150	175	200
g_5^Z	5.67 ~ -3.36	4.73~ -2.21	4.2 ~ -1.47	3.91~ -0.94	3.74~-0.53

TABLE VI. The constraints for g_5^Z with $|Re(V_{ts}V_{td}^*)| = 6.7 \times 10^{-4}$.

m_t(GeV)	100	125	150	175	200
g_5^Z	3.32~ -1.00	2.92~ -0.40	2.73~ 0.008	2.64 ~ 0.32	2.63~0.59