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Algorithms for Orbit Control on SPEAR*

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Abstract

A global orbit feedback system has been installed on SPEAR to help stabilize the position of the photon beams. The orbit control algorithms depend on either harmonic reconstruction of the orbit or eigenvector decomposition. The orbit motion is corrected by dipole corrector kicks determined from the inverse corrector-to-bpm response matrix. This paper outlines features of these control algorithms as applied to SPEAR.

1. INTRODUCTION

A project has been initiated to stablize the electron beam orbit in SPEAR. The goal is to correct for current related and temperature related orbit motion in order to reduce motion at the 9 photon beam source points. In the first phase of this project, we utilize existing local servo loops that keep the photon beams fixed at monitors located several meters from the source [1]. Much like the original NSLS system [2], the job of the global feedback system is to complement the independent servo controllers to provide more stable photon beams. To reduce competition between the global and local feedback loops, the effect of the local servo bumps is subtracted from the global orbit perturbation before processing in the orbit control algorithm [3].

Presently, the global orbit control system operates on the SPEAR control computer with a cycle time of about 20 sec. Conversion to fast digital signal processor boards with PID (or more sophisticated) feedback compensation is underway [4]. To date, we have pursued two orbit correction algorithms, namely, harmonic orbit representation and direct decomposition of the orbit in terms of the eigenvectors of the corrector-to-bpm response matrix. In this paper, we discuss aspects of these algorithms that may be useful to other laboratories developing similar global orbit feedback systems. Specifically, practical questions related to estimation of harmonic orbit content from unequal beam position monitor (bpm) phase intervals [5], harmonic estimation in the presence of bpm readback noise, and a technique to decouple the local and global orbit feedback systems in the framework of eigenvector correction, are discussed. For SPEAR, we typically use 20 bpms and 30 correctors in each plane. The present horizontal and vertical betatron tunes in SPEAR are 6.834 and 6.714, respectively.

2. HARMONIC ORBIT CORRECTION

Harmonic correction is based on the fact that, in normalized betatron coordinates, a dipole field error generates an orbit perturbation with harmonic amplitudes

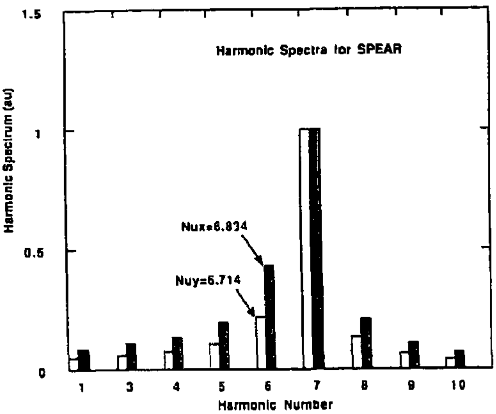

Figure 1. Harmonic spectrum for random orbit perturbations in SPEAR.

in ratio

$$h_n(\nu) \propto \frac{1}{n^2 - \nu^2} \qquad (1)$$

where ν is the tune and n is the harmonic number. In practice, there may be a distribution of field errors that superpose to make a more general spectrum; however, a random distribution produces a spectrum similar to that for a single dipole kick. Hence, as shown in Fig. 1, a large part of most orbit perturbations in SPEAR can be removed by canceling the dominant n=6,7,8 the harmonics.

Harmonic orbit correction proceeds in two stages. First, the orbit perturbation is approximated by a sum of harmonics (Fourier series). Then a corrector pattern is applied to cancel the corresponding harmonics. In the first stage of harmonic orbit correction, the harmonic amplitudes must be estimated from bpm readings, which, in SPEAR, are *not* distributed at uniform phase intervals. The harmonics can be estimated by writing each (normalized) bpm reading as a sum of sine and cosine terms, and solving the set of equations for the harmonic coefficients. We solve this linear system of equations with singular value decomposition (SVD), which automatically gives the least squares fit solution if the number of bpm readings exceeds the number of harmonic expansion coefficients.

If the number of harmonic coefficients is chosen to equal the number of bpms, the predicted orbit is correct at all bpm positions (n-equations in n-unknowns). However, due to the non-uniform grid, aliasing effects, and bpm readback noise, the harmonic representation of the orbit can be much larger than the actual orbit *between* the bpms. Selecting a subset of harmonics from this spectrum can also result in a

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poor approximation to the perturbed orbit. Another disadvantage is that large corrector kicks may be needed. For these reasons, given the non-uniform distribution of bpms in SPEAR, it is better to solve for a least-squares fit on a subset of harmonics to approximate the orbit.

For bpms separated by uniform phase, the harmonic coefficients can be found via a DFT. With non-uniform phase intervals, it is possible to make a linear transformation to a uniform phase grid. One method of transformation starts with the assumption that the bpm readings are samples of a periodic, band-limited function which may be represented by a finite sum of harmonics. In this case, the harmonic expansion is first written for the non-uniform phase bpm readings. Next, the same equations are written for the (unknown) bpm readings at uniform phase intervals, and the expansion coefficients are eliminated between the two sets of equations. The result is an interpolation formula for a linear transformation from the non-uniform to the uniform grid. Note, however, that the harmonic coefficients obtained from a DFT on the 'transformed' data will be the same as those obtained directly from the non-uniform grid data. Problems with aliasing and bpm noise on the original non-uniform grid cannot not be removed by the transformation to a uniform grid.

A difficulty can arise in the correction process when only a subset of harmonics is used to cancel the orbit perturbation; corrector errors introduce other (uncorrected) harmonics on each correction cycle. These components accumulate in a random-walk manner. This effect can occur on SPEAR under some conditions.

3. SINGULAR VALUE DECOMPOSITION

Singular value decomposition (or 'eigenvector' decomposition) of the orbit has the advantage that one operates directly on the corrector-to-bpm response matrix via SVD,

$$R = UWV^T \qquad (2)$$

to produce an orthonormal basis of eigenvectors (contained in the columns of U) that can be used to decompose the orbit perturbation [6,7,8]. The diagonal elements of W contain the corresponding singular values ('eigenvalues'), and the rows of V^T represent the corresponding (orthonormal) corrector patterns needed to produce the 'eigen-orbits', U. An example of the spectrum of singular values from the vertical component of the response matrix measured on SPEAR is shown in Fig. 2. The SVD algorithm automatically accounts for cases with more correctors than bpms, and otherwise rank-deficient response matrices. SVD also produces the minimum RMS corrector amplitudes required to correct the orbit. See Reference [9].

To apply the SVD orbit correction, we first project the orbit perturbation onto each eigenvector of the matrix **U**, and use the projection coefficients, divided by the singular values, to determine the corresponding strength of the corrector eigenvectors, **V**. Since small singular values indicate large corrector strengths, and their eigenvectors often correspond to local rather than global orbit perturbations, it is common practice to set a lower bound on the singular values to be used in the orbit correction. Similar to discarding

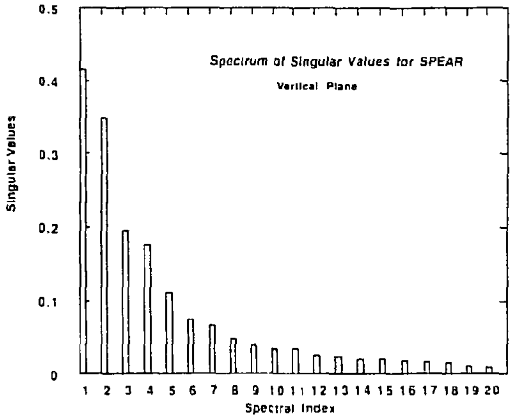

Figure 2. Spectrum of singular values for vertical response matrix measured on SPEAR.

Fourier components far from the tune in a harmonic orbit correction, the effect is to reduce sensitivity to bpm noise while accepting that the corrected orbit will not pass through each bpm exactly. For reasons discussed above, corrector errors can excite modes not covered by a truncated eigenspectrum. Figure 3 shows the effect of limiting the spectrum of harmonics, or SVD eigenvectors, used to represent the orbit. The data shown in this plot is an average value of the RMS orbit (evaluated over all bpms) produced from an ensemble of 300 perturbed orbits with 10 micron random quadrupole displacements and 10 micron random bpm readback noise. These plots show how the orbit correction is improved and the rms corrector strengths increase as the number or harmonics or eigenvectors is increased. The bpm readback noise eventually limits the level of orbit correction which can be achieved.

With a global harmonic or SVD orbit correction system operating in conjunction with local beamline steering systems, each global orbit correction cycle may cause the local system to respond in order to keep the photon beams on target. If a unified global/local feedback system is not available, an alternative approach to decoupling the systems is possible. In this case, one first measures the corrector-to-*photon bpm* response matrix. In SPEAR for instance, there are 30 correctors and 9 photon bpms. SVD will produce 9 eigenvectors that can control the orbit at the *photon* bpms, and 21 eigenvectors that perturb the electron beam orbit *but do not disturb the photon beam positions at the photon bpms*. If we use this set of 21 eigenvectors as an expansion basis for global orbit correction system, then the global and local systems are effectively decoupled.

In conclusion, we show in Fig. 4 an example of orbit control on SPEAR. Here, the SVD orbit correction algorithm was applied in a feedback loop with 15 of 20 vertical plane eigenvectors chosen to correct the electron beam orbit. The figure shows the vertical RMS orbit initially drifting before the feedback system is switched on. At time t=1hr, an orbit was measured to serve as the reference orbit for the global feedback system. This orbit was retained in

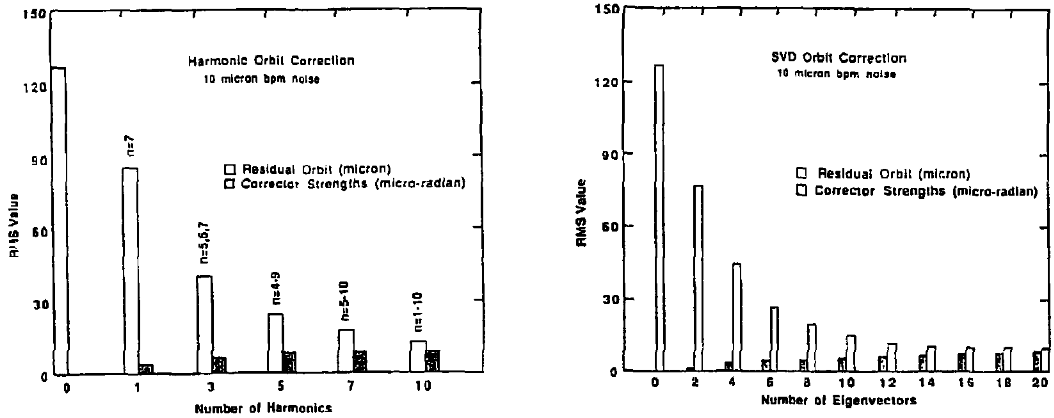

Figure 3. Comparison of harmonic and eigenvector orbit correction as a function of (a) number of harmonics and (b) number of eigenvectors used to represent the orbit. RMS values evaluated at bpm positions.

memory. Shortly after time t=2hr, the global orbit feedback system was switched on, and the orbit returned to the reference position (measured at t=1hr).

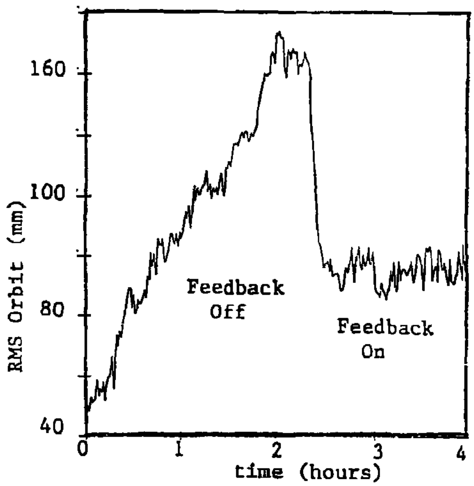

Figure 4. Example of SVD orbit control on SPEAR.

4. ACKNOWLEDGMENTS

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