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# VERTICAL DISPLACEMENT AND POSITION CONTROL IN TOKAMAKS

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# VERTICAL DISPLACEMENTS AND POSITION CONTROL IN TOKAMAKS

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## ABSTRACT

Free-boundary nearly rigid displacements are considered in a plasma confined by a magnetic field consisting of one part generated by the plasma current density, and one part being due to steady currents in fixed external conductors. An induced surface current effect and a related force on the plasma arise when the externally applied field is inhomogeneous in the direction of displacement. This additional force has not been taken into account in conventional MHD theory.

In the particular case of tokamaks, the induced surface current effect has two impacts on vertical nearly rigid displacements. First, there arises an additional restoring force and a positive contribution to the change in potential energy when the externally applied field is inhomogeneous in the vertical direction. A special design of poloidal field coils can thus provide new means for vertical position control in tokamaks, also in the case of strongly elongated cross-sections. Second, an earlier simplified model, in which the plasma is represented by a line current, has to be modified since the plasma is a highly conducting body of finite size.

*Key words:* Magnetic confinement, energy principle, position control, tokamaks.

# I. INTRODUCTION

Position control of a magnetized plasma constitutes one of the major problems in fusion physics. An important question concerns the stability with respect to nearly rigid displacements in the axial (vertical) direction of tokamak geometry.[1] This applies in particular to axially elongated plasma cross-sections. From an analysis by Laval et al.[2] based on the conventional energy principle[3], it has been found that the eigenfunction at marginal stability is a rigid displacement in the direction of elongation, i.e. in the special case of a constant-current elliptical plasma cross-section at infinite aspect ratio. As described in a review by Wesson[1], several authors have also treated the stability of vertical displacements in terms of a simplified model in which the plasma is thought of as a small current-carrying conductor placed in an externally applied magnetic mirror field.

Recently a modified magnetohydrodynamic (MHD) theory[4] has been developed for a plasma confined in a magnetic field, part of which is generated by external constant current sources. For free-boundary modes the inhomogeneity of the externally imposed magnetic field part has then been shown to generate an induced surface current effect which gives rise to an additional force on the plasma body. The plasma thus responds to the free-boundary motion somewhat like the passive feedback from a conducting wall, but without the intermediate step of induced external wall currents.

This paper has two purposes. The first is to use the special case of vertical displacements in toroidal geometry as a concrete and straight-forward demonstration of the induced surface current effect and its role in respect to the energy principle. The second purpose is to show that this effect provides additional means for vertical position control which have so far not been taken into account. All the present deductions are made in terms of ideal MHD theory.

## II. THE EQUILIBRIUM STATE

A plasma is assumed to be confined in equilibrium by a magnetic field $\mathbf{B}_0 = \mathbf{B}_{j0} + \mathbf{B}_{c0}$ where $\mathbf{j}_0 = \text{curl } \mathbf{B}_{j0}/\mu_0$ is the current density within the plasma volume and $\mathbf{B}_{c0}$ is the field produced by steady currents in a set of external fixed conductors. Thus curl $\mathbf{B}_{c0} = 0$ in the plasma and *in vacuo* near its boundary. The balance of the plasma volume forces in equilibrium is given by

$$\mathbf{F}_{V0} \equiv \mathbf{j}_0 \times \mathbf{B}_0 - \nabla p_0 = 0 \tag{1}$$

where $p_0$ is the unperturbed pressure. For the sake of simplicity, we assume $\mathbf{j}_0$ to vanish at the plasma boundary which forms a magnetic surface. No surface current is assumed to flow at the plasma-vacuum boundary in the unperturbed state.

## III. RIGID PLASMA DISPLACEMENTS

### III.A. The Plasma Volume Forces

For any incompressible displacement $\tilde{\xi}$ of small amplitude the perturbation of the plasma volume force becomes[4]

$$\tilde{\mathbf{F}}_V = \tilde{\mathbf{j}} \times \mathbf{B}_0 + \mathbf{j}_0 \times \tilde{\mathbf{B}} - \nabla\tilde{p} =$$

$$= \left\{ \operatorname{curl} \left[ (\mathbf{B}_0 \cdot \nabla)\,\tilde{\xi} \right] - \tilde{\mathbf{D}} \times \mathbf{B}_0 \right\} \times \mathbf{B}_0/\mu_0 + (\operatorname{curl} \mathbf{B}_0) \times \left[ (\mathbf{B}_0 \cdot \nabla)\tilde{\xi} \right]/\mu_0 + \tilde{\mathbf{D}} p_0 \qquad (2)$$

Here $\tilde{\mathbf{B}}$, $\tilde{\mathbf{j}}$ and $\tilde{p}$ are the perturbations of the magnetic field, current density, and pressure,

$$\tilde{\mathbf{D}} = \left[ (\partial\tilde{\xi}/\partial x) \cdot \nabla,\ (\partial\tilde{\xi}/\partial y) \cdot \nabla,\ (\partial\tilde{\xi}/\partial z) \cdot \nabla \right] \qquad (3)$$

and use has been made of Eq. (1). The corresponding change in potential energy is obtained from integration over the plasma volume V, i.e.

$$\delta W_V = -\frac{1}{2}\iiint \tilde{\xi} \cdot \tilde{\mathbf{F}}_V dV \qquad (4)$$

The perturbed part (2) of the volume force and the associated change (4) in energy are identical with the original forms used in conventional theory[3].

In the case of rigid displacements $\tilde{\xi} = \tilde{\xi}_c = \mathbf{const.}$, Eqs. (2) - (4) yield the result $\tilde{\mathbf{F}}_V = 0$ and $\delta W_V = 0$. This is understandable from the physical point of view, because there is no internal work performed by the pressure and Lorentz forces within a rigidly displaced plasma body. The corresponding eigenvalue equation of the conventional theory becomes $\partial^2\tilde{\xi}_c/\partial t^2 = \tilde{\mathbf{F}}_V(\tilde{\xi}_c) = 0$ which yields an eigenfunction $\tilde{\xi}_c = \tilde{\mathbf{c}}_1 + \tilde{\mathbf{c}}_2 t$ where $\tilde{\mathbf{c}}_1$ and $\tilde{\mathbf{c}}_2$ are constant vectors. This solution represents an unaccelerated (translatory) rigid motion. However, as will be shown in Section III.B., such an eigenfunction does not satisfy the boundary condition of the tangential magnetic field component.

A theory which is based on the volume force (2) only, will in other words allow rigid displacements to proceed freely across any inhomogeneous external field $\mathbf{B}_{c0}$ without any resulting force on the plasma body. Obviously this is a paradoxical and physically irrelevant result. The resolution of the paradox is due to the generation of an induced surface current, to be considered in the following sections. The corresponding eigenvalue equation then has to include an extra force term which modifies the resulting eigenfunctions as compared to those of the conventional theory. As demonstrated elsewhere[4], the problem of solving this modified eigenvalue equation is not simple, but will not be further discussed in this context.

### III.B. The Induced Surface Current Effect

In the unperturbed state the total magnetic field strength is $\mathbf{B}_0(\mathbf{r}_b) = \mathbf{B}_{j0}(\mathbf{r}_b) + \mathbf{B}_{c0}(\mathbf{r}_b)$ at the plasma and vacuum sides near a point $\mathbf{r}_b$ at the plasma boundary. For a rigid displacement $\tilde{\boldsymbol{\xi}}_c$ of the plasma and its boundary the perturbations $\tilde{\mathbf{B}}$ and $\tilde{\mathbf{j}}$ of the magnetic field and current density within the plasma body become

$$\tilde{\mathbf{B}} = \mathrm{curl}(\tilde{\boldsymbol{\xi}}_c \times \mathbf{B}_0) = -(\tilde{\boldsymbol{\xi}}_c \cdot \boldsymbol{\nabla})\mathbf{B}_0 \tag{5}$$

and

$$\tilde{\mathbf{j}} = \mathrm{curl}\,\tilde{\mathbf{B}}/\mu_0 = -(\tilde{\boldsymbol{\xi}}_c \cdot \boldsymbol{\nabla})\,\mathbf{j}_0 \tag{6}$$

in the laboratory frame. Eq. (5) implies that there is a rigid displacement of the frozen-in magnetic field inside the plasma. At the plasma side of the displaced boundary, near the point $\mathbf{r}_b + \tilde{\boldsymbol{\xi}}_c$, the total field then has the unchanged value $\mathbf{B}_0(\mathbf{r}_b)$, as well as each of its parts $\mathbf{B}_{j0}(\mathbf{r}_b)$ and $\mathbf{B}_{c0}(\mathbf{r}_b)$. Eq. (6) further implies that the entire volume current system is displaced rigidly by the distance $\tilde{\boldsymbol{\xi}}_c$. After the displacement the magnetic field generated by this current system has the unchanged value $\mathbf{B}_{j0}(\mathbf{r}_b)$ immediately outside the plasma boundary, near the point $\mathbf{r}_b + \tilde{\boldsymbol{\xi}}_c$. At the same point the externally imposed magnetic field has changed from $\mathbf{B}_{c0}(\mathbf{r}_b)$ to $\mathbf{B}_{c0}(\mathbf{r}_b + \tilde{\boldsymbol{\xi}}_c)$. For small $\tilde{\boldsymbol{\xi}}_c$ this tends to produce a magnetic field discontinuity

$$[\tilde{\mathbf{B}}] = (\tilde{\boldsymbol{\xi}}_c \cdot \boldsymbol{\nabla})\,\mathbf{B}_{c0} \tag{7}$$

An alternative way in verifying this result is to use a frame which follows the rigid plasma motion. In such a frame the plasma and its magnetic field and current system remain unchanged, whereas the external conductor system is displaced by the distance $-\tilde{\boldsymbol{\xi}}_c$. The only change in magnetic field then occurs outside the plasma boundary, as being due to a rigid displacement of the magnetic field configuration generated by the external conductor currents. The resulting magnetic field discontinuity is again given by Eq. (7).

The boundary condition of the tangential magnetic field component at the plasma-vacuum surface requires an induced surface current

$$\tilde{\mathbf{K}} = \hat{\mathbf{n}} \times [\tilde{\mathbf{B}}]/\mu_0 \tag{8}$$

to flow, where $\hat{\mathbf{n}}$ is the outward directed normal of the surface. The results (7) and (8) are independent of the question whether or not there exists a volume current density in the plasma. The same results are thus obtained for displacements of a solid body of high but finite conductivity.

The tangential part of the field discontinuity (7) is balanced by the magnetic field $\tilde{\mathbf{B}}_K$ generated at the vacuum side near $\mathbf{r}_b$ by the induced surface current $\tilde{\mathbf{K}}$. From the equality between the tangential components of $[\tilde{\mathbf{B}}]$ and $\tilde{\mathbf{B}}_K$ it then follows that also the normal components of $[\tilde{\mathbf{B}}]$ and $\tilde{\mathbf{B}}_K$ should become equal at the plasma boundary, as is further required by $\mathrm{div}\mathbf{B} = 0$. This condition also becomes consistent with the fact that the lines of the frozen-in magnetic field within the plasma run parallel with the boundary in the perturbed state. In analogy with the case of a superconductor, the field lines in the vacuum region immediately outside the boundary have to do so as well, and this is provided by the induced surface current. Otherwise the field lines would intersect at the boundary.

The induced surface current produces a surface force

$$\bar{\mathbf{f}}_S = \tilde{\mathbf{K}} \times \mathbf{B}_o = -\hat{\mathbf{n}}\left\{(\mathbf{B}_{jo} + \mathbf{B}_{co}) \cdot \left[(\tilde{\xi}_c \cdot \nabla)\mathbf{B}_{co}\right]\right\}/\mu_o \equiv \bar{\mathbf{f}}_{Sj} + \bar{\mathbf{f}}_{Sc} \tag{9}$$

and a corresponding change in potential energy

$$\delta W_S = -\frac{1}{2}\iint \tilde{\xi}_c \cdot (\bar{\mathbf{f}}_{Sj} + \bar{\mathbf{f}}_{Sc})dS \equiv \delta W_{Sj} + \delta W_{sc} \tag{10}$$

where S is the plasma surface. Here $\delta W_{Sj}$ could have either sign, whereas $\delta W_{Sc}$ can be rewritten as

$$\delta W_{Sc} = (1/2\,\mu_o)\iiint \left[(\tilde{\xi}_c \cdot \nabla)\mathbf{B}_{co}\right]^2 dV \geq 0 \tag{11}$$

The surface integral (10) can formally be rewritten as

$$\delta W_S = -\frac{1}{2}\iiint \tilde{\xi}_c \cdot \bar{\mathbf{F}}_S\, dV \tag{12}$$

where

$$\bar{\mathbf{F}}_S = \left\{\mathrm{curl}\left[(\tilde{\xi}_c \cdot \nabla)\mathbf{B}_{co}\right]\right\} \times \mathbf{B}_o/\mu_o - \left\{\left[(\tilde{\xi}_c \cdot \nabla)\mathbf{B}_o\right] \cdot \nabla\right\}\mathbf{B}_{co}/\mu_o \tag{13}$$

Since a finite surface force acting on an infinitesimally thin surface layer would give rise to an infinite acceleration, the force $\tilde{\mathbf{f}}_S$ will be transferred into a volume force by "retarded" MHD signals which propagate into the plasma, as discussed elsewhere[4]. The detailed form of this volume force will not be discussed in this context. Here we shall only assume the displacement $\tilde{\xi}_c$ to proceed slowly as compared to the characteristic time for the MHD signals to traverse the plasma body. Expressions (9) – (10) or (12) – (13) then provide two equivalent options for determining the total forces and energy changes due to the induced surface current effect and its force on the plasma.

### III.C. The Total Forces and Changes in Potential Energy

Conventional theory and its extended energy principle take only the volume force effects of Eqs. (2) – (4) into account, also when this is done in terms of a rewritten form of Eq. (4) which contains a volume integral over the plasma, a surface integral, and a volume integral over the external vacuum region[3].

In cases where there is an induced surface current effect, the expression for the total change $\delta W$ in potential energy has to be modified to include all forces acting on the plasma, i.e.,

$$\delta W = \delta W_V + \delta W_S \tag{14}$$

For incompressible displacements $\tilde{\xi} = \tilde{\xi}_c + \tilde{\eta}$ of moderately large nonuniformity as given by $|\tilde{\eta}(x,y,z)| << |\tilde{\xi}_c|$, $\tilde{\xi}_c$ can be replaced by $\tilde{\xi} = \tilde{\xi}_c + \tilde{\eta}$ in Eqs. (5) – (13) with good approximation[4]. An order-of-magnitude comparison between the volume forces $\tilde{\mathbf{F}}_V$ and $\tilde{\mathbf{F}}_S$ of Eqs. (2) and (13) then shows that $|\delta W_S(\tilde{\xi})| >> |\delta W_V(\tilde{\eta})|$ when $|\tilde{\mathbf{B}}| \neq 0$. This implies that the induced surface current effect has a much larger impact on nearly rigid displacements than the volume force effect of the conventional theory.

## IV.RIGID VERTICAL DISPLACEMENTS IN TOKAMAKS

Rigid vertical displacements $\boldsymbol{\xi}_c = \xi_c\,\hat{\mathbf{z}}$ where $\hat{\mathbf{z}} = (0,0,1)$ are now studied for tokamak geometry in a cylindrical frame $(r,\varphi,z)$ with z along the axis of symmetry. The externally imposed field becomes

$$\mathbf{B}_{co} = \mathbf{B}_{cot} + \mathbf{B}_{cop} = (0,\ B_{cot},\ 0) + (B_{copr},\ 0,\ B_{copz}) \qquad (15)$$

where $B_{cot} \propto 1/r$ is the toroidal and $\mathbf{B}_{cop}$ the poloidal parts, being generated by external field coils. Then

$$\tilde{\mathbf{K}} = \tilde{\xi}_c\,\hat{\mathbf{n}} \times (\partial\mathbf{B}_{cop}/\partial z)/\mu_0 \equiv (0,\ \tilde{K},\ 0) \qquad (16)$$

With a field $\mathbf{B}_{jo} = (B_{jor},\ 0,\ B_{joz})$ due to the plasma volume currents we have

$$\tilde{\mathbf{f}}_S = \tilde{\mathbf{K}} \times (\mathbf{B}_{jo} + \mathbf{B}_{cop}) = \tilde{K}(B_{joz} + B_{copz},\ 0,\ -B_{jor} - B_{copr}) \qquad (17)$$

When $\partial\mathbf{B}_{cop}/\partial z \neq 0$, and since $\partial\mathbf{B}_{cop}/\partial z$ does not in general become parallel with the surface normal $\hat{\mathbf{n}}$, there is a non-vanishing surface current $\tilde{\mathbf{K}}$. An extreme situation where the externally imposed poloidal field $\mathbf{B}_{cop}$ cancels the field $\mathbf{B}_{jo}$ generated by the plasma volume current, can further be readily excluded. Consequently, the surface force (17) never vanishes when the externally imposed poloidal field has a derivative in the axial direction. This implies that the surface current cannot become force-free, and that there is a contribution $\delta W_S$ from the induced surface current to the change in potential energy.

To investigate the influence of the induced surface current more in detail, we first turn to the contribution $\delta W_{Sc}$ from the surface force component $\tilde{\mathbf{f}}_{Sc}$ of Eqs. (9) and (10). Using Eq. (16), two cases are identified:

— In configurations with a homogeneous externally imposed vertical magnetic field, no extra poloidal field, and a nearly circular plasma cross-section, the field $\mathbf{B}_{co}$ has no derivative along $\hat{\mathbf{z}}$. Then $\delta W(\tilde{\xi}_c\hat{\mathbf{z}}) = 0$ and there is no effect which counteracts vertical displacements.

— More advanced configurations with inhomogeneities of the field $\mathbf{B}_{co}$ in the vertical direction yield $\delta W_{Sc}(\tilde{\xi}_c\hat{\mathbf{z}}) > 0$ according to Eq. (11). The induced surface current effect then generates a restoring foce. Specially designed poloidal field coils can therefore provide additional means for vertical position control, also in tokamaks with strongly elongated cross-sections.

We next consider the contribution $\delta W_{Sj}$ which can be regarded as the result of an interaction between the total plasma current and the external conductor currents, with the induced surface current interaction as a mediatorial link. This can be illustrated by the limiting case of a small and nearly circular plasma cross-section of radius a, traversed by a total toroidal current $J_0$ in the plane $z = 0$. The force $\tilde{f}_{Sj}$ can then be integrated over the plasma surface to obtain a corresponding total force

$$\tilde{F}_{Sj} \equiv \int_0^{2\pi} \hat{n}(\hat{n} \cdot \tilde{C})d\alpha = \int_0^{2\pi} \left[ \tilde{C}_r(\cos\alpha)^2, 0, \tilde{C}_z(\sin\alpha)^2 \right] d\alpha = \pi(\tilde{C}_r, 0, \tilde{C}_z) \qquad (18)$$

per unit length, where

$$\tilde{C} = J_0 \times \left[ (\tilde{\xi}_c \cdot \nabla)B_{c0} \right] / 2\pi \qquad (19)$$

and $\alpha$ is the angle between the normal $\hat{n}$ and the plane $z = 0$. In the case of a magnetic mirror field $B_{c0p}$ having its plane of symmetry at $z = 0$, the derivative of this force in the axial direction becomes

$$\partial \tilde{F}_{Sjz} / \partial z = -\frac{1}{2} J_{0\varphi}(\partial B_{c0pr} / \partial z) \qquad (20)$$

for small $\tilde{\xi}_c$.

As in the case of the simplified model where the plasma is substituted by a small current-carrying conductor[1], the force $\tilde{F}_{Sj}$ thus contributes to vertical stability. However, the result (20) only yields half as large a force as that obtained from such a model. The discrepancy can be understood as follows. During the displacement it has first to be noticed that the plasma interior is screened from the influence of external magnetic perturbations. The plasma body therefore "feels" the change $(\tilde{\xi} \cdot \nabla)B_{c0}$ of the external field only in the form of an induced surface current density $\tilde{K}$. The force $\tilde{K} \times B_{j0}$ at the plasma surface then becomes the relevant link in the interaction between the plasma current $J_0$ and the external field $B_{c0}$. The types of interaction to be understood in this connection can be further demonstrated by two cases in simple line-current geometry:

— The first case is given by two parallel line currents, each of the strength $J_0$ and originally being separated by the distance L. The mutual attraction force becomes $F_0 = \mu_0 J_0^2/2\pi L$. When the currents are displaced towards each other by the small distance $\tilde{\xi}_c$, the attraction force increases by the small amount $\tilde{F} = \mu_0 J_0^2 \tilde{\xi}_c / 2\pi L^2$.

— The second case is one in which the same line currents each become surrounded by and mechanically connected with a highly conducting cylindrical shell of small radius $\bar{a} << L$. From the beginning the shells are current-less and the mutual force between the currents is again equal to $F_0$. When the separation distance is suddenly being decreased by the amount $\tilde{\xi}_c$, surface currents are induced in the shells. With the magnetic field $B_{j0} = \mu_0 J_0 / 2\pi \bar{a}$ due to the line current inside a shell, the induced surface current is then easily shown to result in an increase in the attraction force by the amount $\tilde{F}/2$, and not $\tilde{F}$.

This change in force lasts as long as undamped surface currents are flowing. In the case of finite shell conductivity, the same currents begin to decay. The external field will then penetrate into the shells containing the line currents, thereby making the increase in the attraction force approach the value $\tilde{F}$ as obtained from the simplified line current model.

## V. A SIMPLE TECHNICAL APPLICATION

The effect of induced surface currents due to nearly rigid displacements in tokamak configurations can be illustrated by a simple concrete example. A plasma is considered the cross section of which has the average minor radius $\bar{a}$ and is traversed by a total toroidal plasma current $J_0$ which generates a poloidal field $B_{j0}$ of the strength $B_{ja} = \mu_0 J_0 / 2\pi\bar{a}$ at the plasma surface. The imposed toroidal field $B_{cot}$ has the strength $B_{ta}$ at the plasma surface. This corresponds to the safety factor

$$q_a = \bar{a}B_{ta} / RB_{ja} \tag{21}$$

where $R$ is the major plasma radius. In addition, a transverse octupole field $B_{cop}$ is now superimposed for stabilizing purposes.

In a first estimate the approximation of a corresponding straight plasma column is adopted with the axis along $z$ in a frame $(x,y,z)$. Here $z$ corresponds to the toroidal direction. The octupole field is generated by four conductors, each carrying a current $J_c$ in the negative $z$ direction, and being placed at the corners $(\pm a_c, \pm a_c)$ of a square in the $xy$ plan. Introducing the variables $\rho = x / \bar{a}$ and $\lambda = y / \bar{a}$, the unpertubed poloidal magnetic field components at the plasma surface become

$$\mathbf{B}_{j0} = B_{ja}(-\lambda, \rho, 0) \tag{22}$$

$$\mathbf{B}_{cop} = B_{cop}(\lambda^3 - 3\rho^2\lambda, -\rho^3 + 3\rho\lambda^2, 0) \tag{23}$$

where $B_{cop} = 2\mu_0 J_c \bar{a}^3 / \pi a_c^4$. Here the current $J_0$ is in the positive $z$ direction. We further define the ratio $g_0 = B_{cop} / B_{ja}$ between these components. For a closed magnetic separatrix at the plasma surface we have $g_0 \leq 1$.

A nearly rigid displacement $\tilde{\boldsymbol{\xi}} = (0, \tilde{\xi}_c, 0)$ in the vertical direction generates an induced surface force (9) which becomes

$$\tilde{f}_s = \tilde{f}_0 h_s \qquad \tilde{f}_0 = 3\tilde{\xi}_c B_{ja} B_{cop} / \mu_0 \bar{a} \tag{24}$$

$$\mathbf{h}_s = \lambda(3\rho^2 - \lambda^2)\left[(\rho, -\lambda, 0) + g_0(\rho^3 - 3\rho\lambda^2, \lambda^3 - 3\rho^2\lambda, 0)\right] \tag{25}$$

With $\rho = \cos\varphi$, $\lambda = \sin\varphi$ and $n = (\rho, \lambda, 0)$ the force (24) can be integrated over the plasma surface to yield a total force

$$\tilde{F}_S = \int_0^{2\pi} \tilde{f}_S \bar{a} d\varphi = (0, \tilde{F}_{Scy}, 0) \qquad (26)$$

per unit length in the axial direction, where

$$\tilde{F}_{Scy} = -2\pi\bar{a}g_0\tilde{f}_0 < 0 \qquad (27)$$

Thus the force (27) counteracts the displacement.

To illustrate this result we now imagine an unbalance to take place, by which a total current $\tilde{J} = (-\tilde{J}, 0, 0)$ with $\tilde{J} > 0$ flows across the plasma and is closed through the surrounding vessel wall. The destabilizing force $\tilde{J}B_{ta}$ for $B_{ta} > 0$ can then be counteracted by the induced surface force as follows. The current due to the unbalance is assumed to have the relative magnitude $\tilde{U} = \tilde{J} / J_0$ as compared to the total toroidal plasma current. Combination of Eqs. (21) and (24)-(26) yields

$$\tilde{U} = 3(g_0^2\tilde{\xi}_c / Rq_a) \qquad (28)$$

Two examples can be given for a relative vertical displacement $\tilde{\xi}_c / R = 0.02$, say, in a configuration having the toroidal current $J_0$ = 6 MA, a major radius $R$ = 3 m, and being subject to a displacement $\tilde{\xi}_0$ = 0.06 m:

- In an extreme case where $q_a = 1$ and $g_0 = 1$, a disturbance current $\tilde{J}$= 360 kA can be outbalanced according to Eq. (28).

- In a more moderate case where $q_a = 2$ and $g_0 = 1/3$, the same maximum disturbance current becomes $\tilde{J} \approx 20$ kA.

The additional technological arrangements being necessary for stabilization by a superimposed multipole field have, of course, to be analysed with respect to technological complexity and costs. Thereby the possibility for such fields to be used at the same time in divertor geometry should be considered.

## VI. CONCLUSIONS

In this paper free-boundary, incompressible, nearly rigid displacements have been studied in a magnetically confined plasma. The magnetic field is partly generated by plasma volume currents, partly by steady currents in a set of fixed external conductors. The inhomogeneity of the externally applied magnetic field part gives rise to an induced surface current effect and a related force on the plasma.

A concrete illustration of this effect is given by rigid vertical plasma displacements in tokamaks:

— The induced surface current effect produces an additional force and change in potential energy which have not been taken into account in conventional MHD theory.

— The interaction between the induced surface current and the externally imposed magnetic field gives rise to a restoring force and a positive contribution to the change in potential energy. A special design of poloidal field coils can therefore provide additional means for vertical position control in tokamaks, also in the case of strongly elongated cross-sections.

— The earlier developed simplified model in which a toroidal plasma is represented by a small current-carrying conductor has to be modified. This model does not take into account that the plasma is a highly conducting body of finite size.

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Abstract

Free-boundary nearly rigid displacement are considered in a plasma confined by a magnetic field consisting of one part generated by the plasma current density, and one part being due to steady currents in fixed external conductors. An induced surface current effect and a related force on the plasma arise when the externally applied field is inhomogeneous in the direction of displacement. This additional force has not been taken into account in conventional MHD theory.

In the particular case of tokamaks, the induced surface current effect has two impacts on vertical nearly rigid displacements. First, there arises an additional restoring force and a positive contribution to the change in potential energy when the externally applied field is inhomogeneous in the vertical direction. A special design of poloidal field coils can thus provide new means for vertical position control in tokamaks, also in the case of strongly elongated cross-sections. Second, an earlier simplified model, in which the plasma is represented by a line current, has to be modified since the plasma is a highly conducting body of finite size.

*Key words*: Magnetic confinement, energy principle, position control, tokamaks.