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DOE/ER/40530-5

STUDIES OF FLUCTUATION PROCESSES IN NUCLEAR COLLISIONS

PROGRESS REPORT

MARCH 1, 1993 - APRIL 30, 1994

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MAY 1994

PREPARED FOR THE U. S. DEPARTMENT OF ENERGY UNDER GRANT

NUMBER DE-FG05-89ER40530

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PREFACE

This report summarizes the progress on grant No. DE-FG05-89ER40530 during the period March 1, 1993 to April 30, 1994.

By extending our previous work on the fluid dynamical treatment of the nuclear collective motion, we deduced from the Boltzmann-Langevin (BL) model a set of transport equations for N collective variables and calculated the associated transport coefficients. Work has also continued on investigating the relation between the BUU model and the BL model for the average evolution. The principle investigator spent a fruitful summer (1993) at LBL, where in collaboration with J. Randrup, we developed a numerical method for simulating the stochastic evolution of the phase-space density near local equilibrium. Two papers have appeared in Phys. Rev. C and Nucl. Phys. A, two papers have been accepted for publication, both in Nucl. Phys. A, and two manuscripts have been submitted to Z. Phys. A for publication. Several seminars/contributed talks were given at various meetings and an invited talk was presented at the NATO Advanced Study Institution Hot and Dense Matter, Bodrum/Turkey.

Most of the work done in collaboration with Y. Abe, M. Belkacem, D. Boilley, Y. B. Ivanov, W. Norenberg, J. Randrup, V. N. Ruskikh and E. Suraud. Their collaboration has been very stimulating and much appreciated.

The secretarial and accounting duties for this grant have been performed by Gloria Julian, CPS, of the TTU Department of Physics, and I wish to express my appreciation. I also wish to thank the theory groups at LBL and at GSI for their hospitality during my visits.

1. SIMPLIFIED SIMULATION OF BOLTZMANN-LANGEVIN EQUATION
(with J. Randrup)

We summarize the recent progress in deriving approximate analytical expression for the transport coefficients associated with the Boltzmann-Langevin (BL) model and describe a numerical method for simulating the stochastic evolution of the phase-space density $f(\mathbf{r},\mathbf{p})$. In the BL model the evolution of the phase-space density in the semi-classical limit is determined by a stochastic transport equation [1,2],

$$\frac{\partial}{\partial t} f - \{h(f), f\} = K(f) + \delta K. \tag{1.1}$$

Here, the l.h.s. describes the Vlasov propagation in terms of the effective one-body Hamiltonian $h(f)$. In the r.h.s. $K(f)$ denotes a binary collision term of the BUU form, and δK is the fluctuating collision term, which is characterized by a correlation function

$$\langle \delta K(\mathbf{r},\mathbf{p},t)\,\delta K(\mathbf{r}',\mathbf{p}',t') \rangle = C(\mathbf{p},\mathbf{p}')\,\delta(\mathbf{r}-\mathbf{r}')\,\delta(t-t') \tag{1.2}$$

assumed to be local in space and time without the memory effects..

It is possible to derive a simple approximate expression for the correlation function $C(\mathbf{p},\mathbf{p}')$ at local equilibrium, where the average phase-space density is of the Fermi-Dirac form, $f_0(\mathbf{r},\mathbf{p}) = 1/[1+\exp(\varepsilon-\mu)/\tau]$. For this, we employ a method developed in connection with the study of Fermi liquids [3], and which is accurate when the temperature is low in comparison with the Fermi energy, $\tau << \varepsilon_F$ We then obtain for the correlation function [4],

$$C(\mathbf{p},\mathbf{p}') \approx W_0\left[f_0(1-f_0)h^3\delta(\mathbf{p}-\mathbf{p}') - f_0(1-f_0)S_{\mathbf{p}\mathbf{p}'}f_0'(1-f_0')\right] \tag{1.3}$$

where W_0 determine the relaxation rate $W_0 = 1/t_0$, and $S_{pp'}$ expresses the correlations induced between the occupancies at the moment **p** and **p′** as a result of the binary collisions. Up to the leading order in τ/ε_F, the relaxation rate is

$$\frac{1}{\tau_0} = W_0 = \pi^2 \rho \sigma V_F \left(\frac{\tau}{\varepsilon_F}\right)^2 \tag{1.4}$$

in agreement with the expression derived for the Fermi Liquids, and the correlation coefficient is given by

$$S_{pp'} = \frac{1}{\rho \phi_0}\left(\frac{1}{\sin\frac{\theta}{2}} - \frac{1}{2\cos\frac{\theta}{2}}\right) \tag{1.5}$$

where $\phi_0 \approx 3\tau/2\varepsilon_F$, ρ is the local density and θ denotes the angle **p** and **p′**. The approximate expression (1.3) provides a very good approximation for the overall behavior in energy and angle of the correlation function near local equilibrium. As an example, figure 1 shows the angular dependence of the covariance part of the correlation function. The exact result obtained by numerical Monte-Carlo evaluation is shown by the solid dots, with the open dots indicating its positive and negative parts. The curves show the corresponding quantity for our analytical approximation which, aside from a normalization, is given by eq. (1.5).

The BL transport problem can be solved by direct numerical simulation. Given the phase-space density $f(\mathbf{r}, \mathbf{p}, t)$ at a time t, the task is to calculate the density $f(\mathbf{r},\mathbf{p},t+\Delta t)$ after a short time interval Δt has elapsed. According to the BL eq. (1.1), this change has two parts, the evolution associated with the collisionless propagation by the one-body Hamiltonian $h(f)$ and a change Δf resulting from the stochastic two-body collisions. The collision-induced changes constitute an ensemble $\{\Delta f\}$ which is constrained by the transport coefficients,

$$\langle \Delta f(\mathbf{r},\mathbf{p}) \rangle = K(f)\Delta t\,, \qquad \langle \Delta f(\mathbf{r},\mathbf{p})\,\Delta f(\mathbf{r}',\mathbf{p}') \rangle = C(\mathbf{p},\mathbf{p}')\,\delta(\mathbf{r}-\mathbf{r}')\,\Delta t\,. \tag{1.6}$$

We can obtain one of the possible new densities, $f(\mathbf{r},\mathbf{p},t+\Delta t) = f(\mathbf{r},\mathbf{p},t) + \Delta f(\mathbf{r},\mathbf{p})$, by picking the random increment $\Delta f(\mathbf{r},\mathbf{p})$ according to the constraints (1.6). This procedure is then repeated on the basis of new density, and, in this manner one entire history $f(\mathbf{r},\mathbf{p},t)$ is generated. An ensemble of N independent histories $\{f^{(n)}(\mathbf{r},\mathbf{p},t)\}$ can be obtained by performing N such "runs," each time starting with the same specified initial density $f(\mathbf{r},\mathbf{p},t=0)$. The simple approximate forms derived for the transport coefficients make it possible to calculate these extremely fast. Consequently, the numerical simulation of realistic scenarios has been brought within the realm of practicality, provided that a suitable fast method can be devised for picking the random charges $\Delta f(\mathbf{r},\mathbf{p})$ according to eq. (1.6).

A paper on this topic will appear in Nucl. Phys. A (1994).

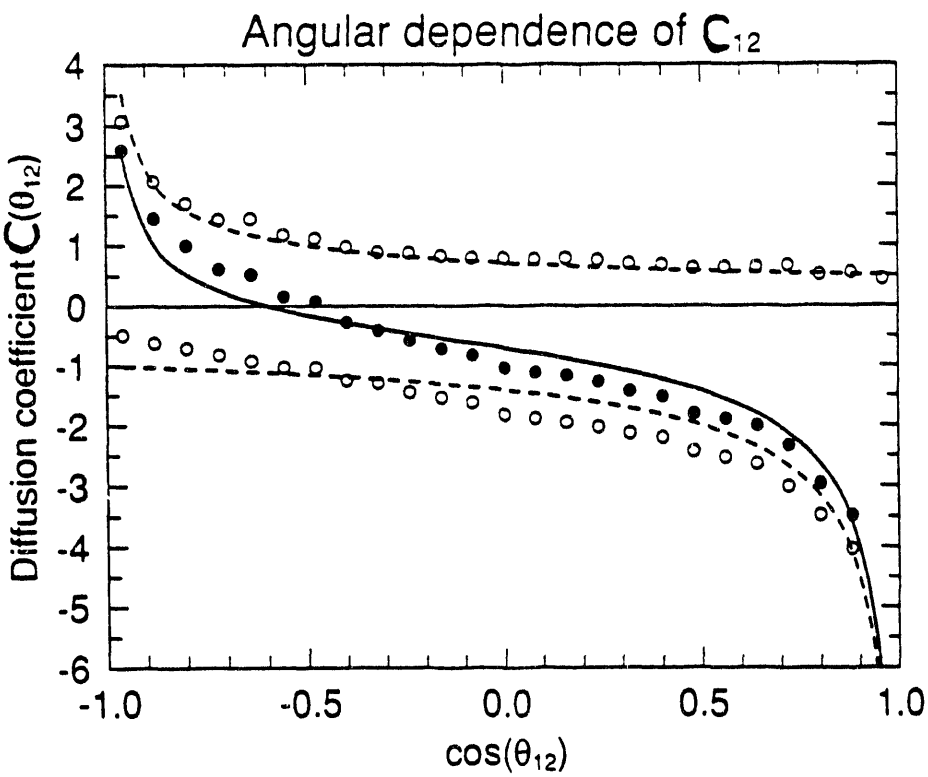

FIGURE 1

2. LONG-RANGE CORRELATIONS IN BOLTZMANN-LANGEVIN MODEL

The BL model provides a useful basis for studying processes involving large density fluctuations such as phase-transitions and nuclear multifragmentations. It is also interesting to investigate the evolution of the average phase-space density on the basis of the BL model. As long as the density fluctuations remain small, the average description of the BL model is equivalent to the one provided by the BUU model. However, when the density fluctuations become large, as it happens for example near the region of instabilities, as a result of non-linear evolution of the fluctuations, even the average description of the BL model can be very different from the one described by the BUU model.

We consider the quantal version of the BL model for the fluctuating s.p. density $\hat{\rho}(t)$ matrix,

$$\frac{\partial}{\partial t}\hat{\rho}+i\left[h(\hat{\rho}),\hat{\rho}\right]=K(\hat{\rho})+\delta K(t) \tag{2.1}$$

where $h(\hat{\rho})$ is the mean-field Hamiltonian, $K(\hat{\rho})$ is the binary collision term and δK the fluctuating collision term which is characterized by a correlation function similar to the semi-classical form of eq. (1.2). A transport equation for the average s.p. density $\rho(t)=\langle\hat{\rho}(t)\rangle$ is obtained by statistical averaging of the BL eq. (2.1),

$$\frac{\partial}{\partial t}\rho+i\left[h(\rho),\rho\right]=K(\rho)+K_c(\rho). \tag{2.2}$$

Here, $K_c(\rho)$ denotes an additional collision term arising from the long-range correlations induced by collective density fluctuations [5,6],

$$K_c(\rho)=-i\langle[\delta h,\delta\rho]\rangle \tag{2.3}$$

with $\delta h = h(\hat{\rho}) - h(\rho)$ and $\delta\rho = \hat{\rho} - \rho$ are deviations of the mean-field and the s.p. density from their averages, respectively. The BUU model corresponds to the semi-classical limit of eq. (2.2), when the density fluctuations are small so that the collision term $K_c(\rho)$ can be neglected. In low-energy nuclear processes $K_c(\rho)$ describes a surface dissipation mechanism, which dominates the damping of collective motion [7]. Also, the magnitude of $K_c(\rho)$ becomes larger as the system moves toward the region of instabilities due to the growing of density fluctuations. Hence, it can strongly modify the average evaluation of the s.p. density near the regions of instabilities, where the effects of the BUU collision term become small. It is, therefore, important to derive an explicit expression for $K_c(\rho)$, which eventually may be incorporated into numerical simulations.

For small amplitude density fluctuations, it is possible to derive an explicit expression for $K_c(\rho)$ by employing the linearized BL equation around the average. For simplicity, we limit our treatment here to small-amplitude, long-wavelength density fluctuations around a spatially uniform average density. In this case, the average density is diagonal in the momentum representation $\langle \mathbf{p}|\rho|\mathbf{p}'\rangle = \delta(\mathbf{p}-\mathbf{p}')f_{\mathbf{p}}(t)$, with $f_{\mathbf{p}}(t)$ as the average occupation factor, and eq. (2.2) becomes a transport equation for the average occupation factors,

$$\frac{\partial}{\partial t} f(t) = K(f) + K_c(f) \tag{2.4}$$

where $K(f)$ is a binary collision term of the BUU form, and $K_c(f)$ is given by

$$K_c(f) = -\int \frac{d\mathbf{p}'}{(2\pi)^3} M(\mathbf{p},\mathbf{p}')\left[f_{\mathbf{p}} - f_{\mathbf{p}'}\right]. \tag{2.5}$$

Here the transition rate $M(\mathbf{p},\mathbf{p}')$ is determined in terms of the density correlation function, which can be calculated using the linearized BL eq. (2.1). For a weak collisional damping, we find

$$M(\mathbf{p},\mathbf{p}') \approx \left|\frac{\partial}{\partial n}U_k\right|^2 \frac{nk^2}{m\omega_k} N_k \frac{\pi}{2}\left[\delta(\omega_k-\omega)+\delta(\omega_k+\omega)\right] \tag{2.6}$$

with $\mathbf{k}=\mathbf{p}-\mathbf{p}'$ and $\omega=\varepsilon_{\mathbf{p}}-\varepsilon_{\mathbf{p}'}$. Here, n denotes the average density and the frequencies of the collective vibrations are found from the dispersion relation, $1+\frac{\partial}{\partial n}U_k\ \chi(\mathbf{k},\omega_k)=0$, with χ as the free response function. The quantity N_k can be regarded as the phonon occupation factor, and it goes over the expected result at high temperature T, $N_k \rightarrow 2T/\omega_k$ [8].

A paper on this topic has been submitted to Z. Phys. A (1994) for publication and another one is in preparation in collaboration with Y. B. Ivanov.

3. A BOHR-MOTTELSON MODEL OF NUCLEI AT FINITE TEMPERATURE (with D.Boilley, Y. Abe and E. Suraud)

The BL approach can be applied to study the gross properties of nuclear collective motion at low energies, including inelastic collisions and induced fission. In particular, it can be used to deduce collective transport models and calculate transport coefficients associated with collective variables. In a previous work, considering a particular situation in which the collective motion is described by a single collective variable, we derived a transport equation within a fluid dynamical treatment by retaining only the two-body dissipation mechanism [9,10]. Here, we generalize this treatment for N collective variables, however, still keeping only the two-body dissipation mechanism.

We consider a large amplitude collective motion characterized by N collective variables, $\mathbf{q}\equiv(q_1,q_2,..q_N)$. In the spirit of ref. [11], we study the collective motion in a diabatic approximation and assume that the collective dynamics is characterized by a set of N linearly independent, irrotational velocity fields,

$$\mathbf{u}(\mathbf{r},t)=\sum \dot{q}_j(t)\ \nabla\phi_j(r). \tag{3.1}$$

Furthermore, we assume that the density is determined in a quasi-static approximation $\rho(\mathbf{r},t) \approx \rho_0[\mathbf{r},\mathbf{q}(t)]$, and it is not affected by the relaxation process. As a result, the velocity fields are related to the density variations by scaling conditions in each independent direction. We can deduce equations of motion for the collective variables, by multiplying eq. (1.1) with $\mathbf{p}\cdot\nabla\phi_J$ and by integrating over the phase-space. We obtain,

$$\sum M_{JJ'}\ddot{q}_{J'}+\frac{\partial}{\partial q_J}E = F_J. \tag{3.2}$$

Here, $M_{JJ'}$ is the irrotational inertial tensor and E denotes the mechanical energy associated with the collective motion,

$$E(\mathbf{q},\dot{\mathbf{q}}) = \frac{1}{2}\sum \dot{q}_J\dot{q}_J M_{JJ'}+V(\mathbf{q}) \tag{3.3}$$

in which $V(\mathbf{q})$ may be identified with the liquid-drop potential energy. The coherent coupling between the intrinsic degrees of freedom and the collective motion induced by the velocity fields gives rise to an additional dynamical force, $F_J = C_J + Q_J$, which has a part C_J due to the compressibility and another part Q_J arising from the distortions of the Fermi surface of the local momentum distribution. The dynamical force due to the deformation of the Fermi surface can be calculated explicitly with the help o the linearized BL eq. (1.1). This yields a Langevin equation for Q_J

$$\frac{d}{dt}Q_J + \sum \Gamma_{JJ'}\dot{q}_{J'} = -\sum \tau^{-1}_{JJ'}Q_{J'} + \delta K_J \tag{3.4}$$

where $\Gamma_{JJ'}$ and $\tau_{JJ'}$ denote the stiffness and the relaxation matrices, respectively, and the correlation function of the random force Q_J is given by

$$\langle \delta K_J(t)\,\delta K_{J'}(t')\rangle = 2D_{JJ'}\delta(t-t'). \tag{3.5}$$

Here the diffusion tensor $D_{JJ'}$ can be expressed in terms of the temperature and the relaxation matrix $\tau_{JJ'}$. The Langevin eq. (3.4) describes a relaxation process. Due to the coherent coupling between s.p. and collective motion, the collective kinetic energy is initially stored as a dynamical potential energy and subsequently it is dissipated into incoherent excitations by two-body collisions. As a result, the dynamical force Q_J appears as a retarded friction on the collective variables.

A paper on this topic has been submitted to Z. Phys. A (1994) for publication.

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APPENDIX I

Papers

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"Nuclear Fission with a Langevin Equation," D. Boilley, E. Suraud, Y. Abe and S. Ayik, Nucl. Phys. A556 (1993) 67.

"Stochastic Multi-Fluid Models for Intermediate Energy Heavy-Ion Collisions," S. Ayik, Y. B. Ivanov, V. N. Ruskikh and W. Norenberg, Nucl. Phys. A (1994), in press.

"Simplified Nuclear Boltzmann-Langevin Simulation," J. Randrup and S. Ayik, Nucl. Phys. A (1994), in press.

Seminars

"Damping of Collective Vibrations in a Memory Dependent Transport Model," LBL, Berkeley, CA (1993), and Workshop on Large Amplitude Nuclear Collective Motion, INT, Seattle, WA (1993).

"The Boltzmann-Langevin Model," Univ. of P. Sabatier, Toulouse, France (1993), and Workshop on Fluctuation Dynamics in Nuclear Collisions, Trento, Italy (1994).

Conference Contributions

"Boltzmann-Langevin Transport Model for Heavy-Ion Collisions," S. Ayik, NATO Advanced Study Institute on Hot and Dense Matter, Bodrum, Turkey (1993).

"Fluctuations in Microscopic Nuclear Simulations," J. Randrup and S. Ayik, International Workshop on the Dynamical Features of Nuclei and Finite Fermi Systems, Stiges, Spain (1993).

"Simulation of Boltzmann-Langevin Equation," S. Ayik and J. Randrup, Workshop on Gross Properties of Nuclei and Nuclear Excitations XXII, Hirschegg, Austria (1994).

APPENDIX II
PERSONNEL

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D. Boilley, Postdoc. Fellow	Yukawa Institute for Theoretical Physics, Kyoto, Japan
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W. Norenberg, Professor	GSI, Darmstadt, Germany
J. Randrup, Senior Scientist	Lawrence Berkeley Laboratory, Berkeley, CA
V. N. Ruskikh, Research Staff	Kurchatov Institute, Moscow, Russia
E. Suraud, Professor	University of P. Sabatier, Toulouse, France

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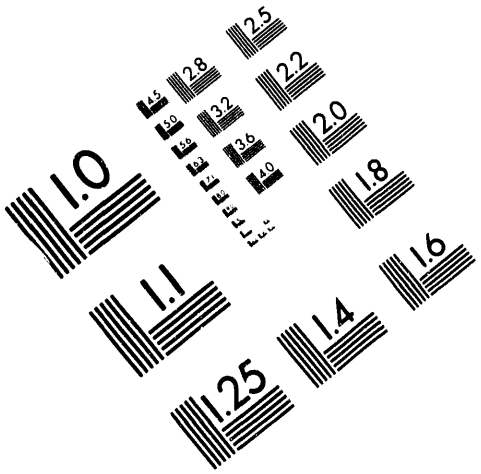

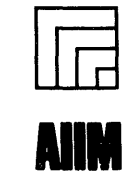

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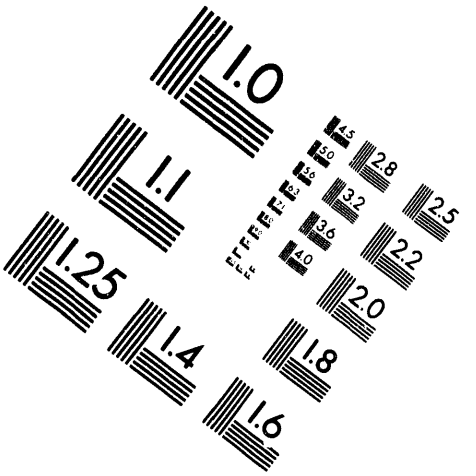

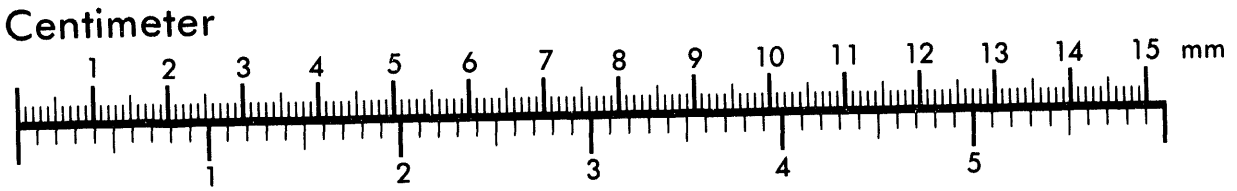

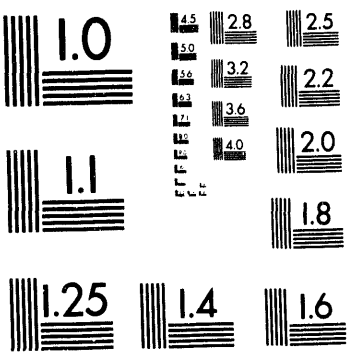

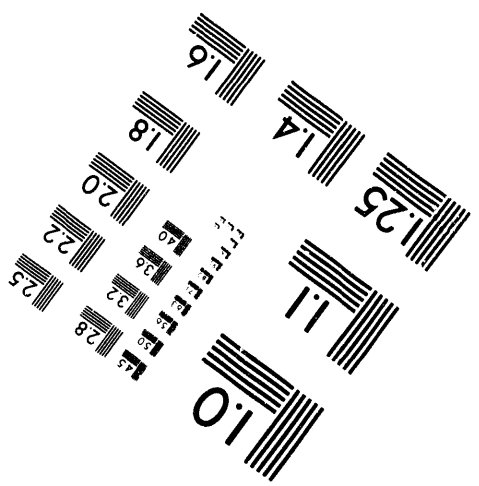

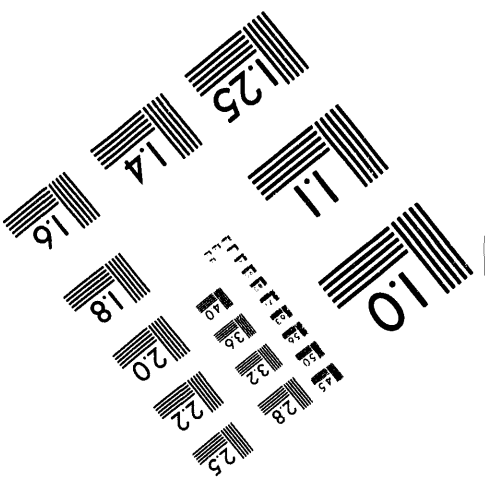

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