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TESTS FOR LEPTONIC CP VIOLATION IN TAU DECAYS

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Abstract

At the Z^o or a B factory, there are two tests for non-CKM-type leptonic CP violation in the $\tau \rightarrow \rho\nu(a_1\nu)$ decay channel by inclusion of $\rho(a_1)$ polarimetry. By CP invariance, the moduli ratio of, and the phase difference between, the two helicity amplitudes for $\tau^- \rightarrow \rho^-\nu(a_1^-\nu)$ decay should equal those for $\tau^+ \rightarrow \rho^+\bar{\nu}(a_1^+\bar{\nu})$ decay. Formulas are given for a L-handed ν_τ, and also for an arbitrary mixture of ν_L and ν_R neutrino helicities. Statistical errors are listed for both the case that the τ^- momentum direction is not known, and when it is known via a silicon vertex detector.

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Introduction

After almost 30 years, the fundamental origin and significance of the observed CP and T violations in kaon decays is still a mystery. On the other hand, it is possible to use new collider data to rigorously search for CP and T violations in the tau lepton decays.[1] While such an observation would be surprising, nevertheless we won't be sure of its absence unless we search for it. Spin-correlation effects plus decay polarimetry enable[2,3] such a search in $e^-e^+ \rightarrow Z^o$, or $\gamma^* \rightarrow \tau^-\tau^+$.

As shown in Ref. 1, by use of ρ polarimetry, there are two tests for "non-CKM-type" leptonic CP violation in $\tau \rightarrow \rho\nu$ decay.

This is easily seen because by rotational invariance there are two independent helicity amplitudes for $\tau^- \rightarrow \rho^-\nu_\tau$ decay

$$A(-1, -1/2) = |A(-1, -1/2)|\, e^{i\phi^a_{-1}}, \quad A(0, -1/2) = |A(0, -1/2)|\, e^{i\phi^a_o} \tag{1}$$

assuming a L-handed ν_τ. The CP-conjugate decay $\tau^+ \rightarrow \rho^+\bar{\nu}_\tau$ depends on

$$B(1, 1/2) = |B(1, 1/2)|\, e^{i\phi^b_1}, \quad B(0, 1/2) = |B(0, 1/2)|\, e^{i\phi^b_o} \tag{2}$$

assuming a R-handed $\bar{\nu}_\tau$. By CP invariance $B(\lambda_{\bar{\rho}}, \lambda_{\bar{\nu}}) = A(-\lambda_{\bar{\rho}}, -\lambda_{\bar{\nu}})$. The two tests are that the phase difference and moduli ratio for the two amplitudes for $\tau^- \rightarrow \rho^-\nu_\tau$ decay must equal those for the CP-conjugate decay. That is

$$\beta_a = \beta_b \text{ (1st test)} \tag{3}$$

where $\beta_a \equiv \phi^a_{-1} - \phi_o^a$, $\beta_b \equiv \phi_1^b - \phi_o^b$; and

$$r_a = r_b \text{ (2nd test)} \tag{4}$$

in terms of the moduli ratios

$$r_a \equiv \frac{|A(-1, -1/2)|}{|A(0, -1/2)|}, \quad r_b \equiv \frac{|B(1, 1/2)|}{|B(0, 1/2)|} \tag{5}$$

It is important to realize that any leptonic-CKM t-type phases will equally affect the $A(-1, -1/2)$ and $A(0, -1/2)$ amplitudes. Therefore, they will cancel out in β_a and in r_a. Hence, $\beta_a = \beta_b$ and $r_a = r_b$ test for a non-CKM-type leptonic CP violation.

In the standard lepton model (pure V-A and no CP violation), $\beta_a = \beta_b = 0$ and the moduli ratio $r_a = r_b \approx \sqrt{2}\, m_\rho/m_\tau = 0.613$. These two tests, (3) and (4), should be compared with the classic CP test for partial width asymmetry of CP-conjugate reactions:

$$A_\Gamma \equiv \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}} \tag{6}$$

where, e.g. $\Gamma = \Gamma(\tau^- \to \rho^- \nu_\tau)$ and $\bar{\Gamma} = \bar{\Gamma}(\tau^+ \to \rho^+ \bar{\nu}_\tau)$. For τ two-body decay modes, the denominator of (6) is known to (1 to 4)%, so (at best) we know $A_\Gamma \sim$ (1 to 4)% whereas we find (see below) that the fractional uncertianty of the moduli can be measured to the $(\delta r_a)/r_a \sim$ (0.1 to 1)% level from data, respectively, at γ^* energies (at the Z^o).

Contents of this Paper

This conference contribution extends the analysis of Ref. 1 in three ways:

(a) Ref. 1 considered the $\tau \to \rho\nu$ decay mode. Here the two tests for non-CKM-type leptonic CP violation are extended to the $\tau \to a_1 \nu$ mode.

(b) Ref. 1 assumed a L-handed ν_τ for the $\tau \to \rho\nu$ mode. Here formulas are given for a mixture of V-A and V+A couplings, and for both left-handed and right-handed neutrinos in $\tau^- \to \rho^- \nu$ $(a_1^- \nu)$ decay.

(c) Ref. 1 assumed that the τ^- momentum direction was only known kinematically up to two possible directions. However, by a silicon vertex detector, the τ^- momentum direction may be known at a B factory. Here we obtain and discuss the improvement in the statistical errors for the two tests for the $\tau \to \rho\nu$ mode when the τ^- momentum direction is measured.

Formulas for $\tau^- \to \rho^- \nu$ including both V±A, and both ν helicities.

Including both ν_L and ν_R helicities and using a "compact boldface formalism," we find the composite decay density matrix for $\tau^- \to \rho^- \nu \to (\pi^- \pi^o) \nu$ is

$$\mathbf{R} = \begin{pmatrix} \mathbf{R}_{++} & e^{i\phi_1^\tau} \mathbf{r}_{+-} \\ e^{-i\phi_1^\tau} \mathbf{r}_{-+} & \mathbf{R}_{--} \end{pmatrix} \tag{7}$$

The diagonal elements are

$$\begin{aligned} \mathbf{R}_{\pm\pm} = \mathbf{n}_a [1 \pm \mathbf{f}_a \cos\theta_1^\tau] \\ \mp (1/\sqrt{2}) \sin\theta_1^\tau \sin 2\tilde{\theta}_a \, [\cos(\tilde{\phi}_a - \beta_a) \, |A(0,-1/2)| \, |A(-1,-1/2)| \\ - \cos(\tilde{\phi}_a + \beta_a^R) \, |A(0,1/2)| \, |A(1,1/2)|] \end{aligned} \tag{8}$$

and

$$\begin{aligned} \mathbf{r}_{+-} &= (\mathbf{r}_{-+})^* \\ &= \mathbf{n}_a \; \mathbf{f}_a \sin\theta_1^\tau \\ &+ (1/\sqrt{2}) \sin 2\tilde{\theta}_a \, \{[\cos\theta_1^\tau \cos(\tilde{\phi}_a - \beta_a) + i \sin(\tilde{\phi}_a - \beta_a)] \, |A(0,-1/2)| \, |A(-1,-1/2)| \\ &- [\cos\theta_1^\tau \cos(\tilde{\phi}_a + \beta_a^R) + i \sin(\tilde{\phi}_a + \beta_a^R)] \, |A(0,1/2)| \, |A(1,1/2)|\} \end{aligned} \tag{9}$$

Note that the two observable phase differences are

$$\beta_a \equiv \phi^a_{-1} - \phi^a_o \tag{10a}$$

$$\beta_a^R \equiv \phi^a_1 - \phi^{aR}_o \tag{10b}$$

In Eqs. (8-9),

$$\begin{pmatrix} \mathbf{n}_a \\ \mathbf{n}_a \mathbf{f}_a \end{pmatrix} = \cos^2\tilde{\theta}_a \, (|A(0,-1/2)|^2 \pm |A(0,1/2)|^2) \pm \frac{1}{2} \sin^2\tilde{\theta}_a \, (|A(-1,-1/2)|^2 \pm |A(1,1/2)|^2) \tag{11}$$

Similarly, for the conjugate process $\tau^+ \to \rho^+ \bar{\nu} \to (\pi^+ \pi^o) \bar{\nu}$, including both $\bar{\nu}_R$ and $\bar{\nu}_L$ helicities,

$$\bar{\mathbf{R}} = \begin{pmatrix} \bar{\mathbf{R}}_{++} & e^{i\phi_2^\tau} \bar{\mathbf{r}}_{+-} \\ e^{-i\phi_2^\tau} \bar{\mathbf{r}}_{-+} & \bar{\mathbf{R}}_{--} \end{pmatrix} \tag{12}$$

where

$$\begin{aligned} \mathbf{R}_{\pm\pm} = n_b (1 \mp f_b \cos\theta_2^\tau) \\ \pm (1/\sqrt{2}) \sin\theta_2^\tau \sin 2\tilde{\theta}_b \, [\cos(\tilde{\phi}_b + \beta_b) \, |B(0,1/2)| \, |B(1,1/2)| \\ - \cos(\tilde{\phi}_b - \beta_b^L) \, |B(0,-1/2)| \, |B(-1,-1/2)|] \end{aligned} \tag{13}$$

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$$\bar{r}_{+-} = (\bar{r}_{-+})^*$$
$$= -n_b \; f_b \sin\theta_2^\tau$$
$$- (1/\sqrt{2}) \; \sin 2\tilde{\theta}_b \; \{[\cos\theta_2^\tau \cos(\tilde{\phi}_b + \beta_b) + \iota \sin(\tilde{\phi}_b + \beta_b)] \; |B(0, 1/2)| \, |B(1, 1/2)|$$
$$- [\cos\theta_2^\tau \cos(\tilde{\phi}_b - \beta_b^L) + \iota \sin(\tilde{\phi}_b - \beta_b^L)] \; |B(0,-1/2)| \, |B(-1,-1/2)|\} \qquad (14)$$

In Eqs. (13-14),

$$\beta_b \equiv \phi_1^b - \phi_o^b \qquad (15a)$$

$$\beta_b^L \equiv \phi_{-1}^b - \phi_o^{bL} \qquad (15b)$$

and

$$\begin{pmatrix} n_b \\ n_b f_b \end{pmatrix} = \cos^2\tilde{\theta}_b \; (|B(0, 1/2)|^2 \pm |B(0,-1/2)|^2) \pm \frac{1}{2} \sin^2\tilde{\theta}_b \; (|B(1, 1/2)|^2 \pm |B(-1,-1/2)|^2) \qquad (16)$$

The full "Stage 2 Spin-Correlation" function (S2SC) is given by

$$I_7 = I_7 (R \rightarrow R, \; \bar{R} \rightarrow \bar{R}) \qquad (17)$$

where the I_7 function on the right-hand side is given in the next equation from Ref. 1. The simpler I_7 of Ref. 1 assumed a L-handed ν in $\tau^- \rightarrow \rho^- \nu$, and a R-handed $\bar{\nu}$ in $\tau^+ \rightarrow \rho^+ \bar{\nu}$.

$$I_7 = I(E_1, E_2, \phi; \tilde{\theta}_1, \tilde{\phi}_1; \tilde{\theta}_2, \tilde{\phi}_2)$$
$$= \sum_{h_1, h_2} |T(h_1, h_2)|^2 \; R_{h_1, h_1} \; \bar{R}_{h_2, h_2}$$
$$+ \; e^{\iota\phi} \; T(++) \; T^*(--) \; r_{+-} \; \bar{r}_{+-} \; + \; e^{-\iota\phi} \; T(--) \; T^*(++) \; r_{-+} \; \bar{r}_{-+} \qquad (18)$$

where $T(\lambda_1, \lambda_2)$ are the helicity amplitudes[4] describing $Z^o, \gamma^* \rightarrow \tau^- \tau^+$.

Similarly, the simpler 4 (5) variable S2SC functions are

$$I_{4,5} = I_{4,5} + (\lambda_R)^2 \, I_{4,5} \, (\rho \rightarrow \rho^R) + (\bar{\lambda}_L)^2 \, I_{4,5} \, (\bar{\rho} \rightarrow \bar{\rho}^L)$$
$$+ \; (\lambda_R \bar{\lambda}_L)^2 \, I_{4,5} \, (\rho \rightarrow \rho^R, \; \bar{\rho} \rightarrow \bar{\rho}^L) \qquad (19)$$

where the ratios of the R-handed to L-handed $\tau^- \rightarrow \rho^- \nu$ moduli (and vice versa for $\tau^+ \rightarrow \rho^+ \bar{\nu}$) are

$$\lambda_R \equiv \frac{|A(0, 1/2)|}{|A(0, -1/2)|} \qquad (20a)$$

$$\bar{\lambda}_L \equiv \frac{|B(0, -1/2)|}{|B(0, 1/2)|} \qquad (20b)$$

In Eq. (19), the 4-variable S2SC of Ref. 1 is

$$I(E_{\rho^-}, E_{\rho^+}; \tilde{\theta}_1, \tilde{\theta}_2) = |T(+-)|^2 \; \rho_{++} \; \bar{\rho}_{--}$$
$$+ \; |T(-+)|^2 \; \rho_{--} \; \bar{\rho}_{++} \; + \; |T(++)|^2 \; \rho_{++} \; \bar{\rho}_{++} \; + \; |T(--)|^2 \; \rho_{--} \; \bar{\rho}_{--} \qquad (21)$$

with the <u>integrated</u>, composite decay density matrix for $\tau^- \rightarrow \rho^- \nu_\tau \rightarrow (\pi^- \pi^o) \nu_\tau$ with τ^- helicity $\lambda_1 = h/2$

$$\rho_{hh} = (1 + h\cos\theta_1^\tau) \, [\cos^2\omega_1 \cos^2\tilde{\theta}_1 \; + \; 1/2 \sin^2 \omega_1 \; \sin^2\tilde{\theta}_1]$$
$$+ \; (r_a^2/2) \; (1 - h\cos\theta_1^\tau) \; [\sin^2\omega_1 \; \cos^2\tilde{\theta}_1 \; + \; 1/2 \, (1 + \cos^2\omega_1) \cdot \sin^2\tilde{\theta}_1]$$
$$+ \; h \, (r_a/\sqrt{2}) \; \cos\beta_a \; \sin\theta_1^\tau \sin 2\omega_1 \; [\cos^2\tilde{\theta}_1 - 1/2 \sin^2\tilde{\theta}_1] \qquad (22)$$

For the CP conjugate process with τ^+ with helicity $\lambda_2 = h/2$,

$$\bar{\rho}_{h,h} = \rho_{-h,-h} \; \text{(subscripts } 1 \rightarrow 2, \; a \rightarrow b) \qquad (23)$$

The additional ρ^R and $\bar{\rho}^L$ needed for Eq. (19) are defined (and given) by

$$\rho^R_{\pm\pm} \equiv \frac{1}{2\pi} \int_o^{2\pi} d\tilde{\phi}_1^{(\iota)} \; \frac{R^R_{\pm\pm}}{|A(0, 1/2)|^2}$$
$$= \rho_{-h-h} \; (r_a \rightarrow r_a^R, \; \beta_a \rightarrow \beta_a^R) \qquad (24)$$

with β_a^R given in Eq. (10b), and

$$r_a^R \equiv \frac{|A(1, 1/2)|}{|A(0, 1/2)|} \, . \tag{25}$$

Also

$$\bar{\rho}^L_{\pm\pm} \equiv \frac{1}{2\pi}\int_0^{2\pi} d\tilde{\phi}_2^{(c)} \frac{\bar{R}^L_{\pm\pm}}{|B(0, -1/2)|^2}$$

$$= \bar{\rho}_{-h\,-h} \; (r_b \rightarrow r_b^L, \; \beta_b \rightarrow \beta_b^L) \tag{26}$$

with β_b^L of Eq. (15b) and

$$r_b^L \equiv \frac{|B(-1, -1/2)|}{|B(0, -1/2)|} \, . \tag{27}$$

For the 5-variable S2SC, the additional formulas are

$$\rho^R_{\pm\mp} \equiv \frac{1}{2\pi}\int_0^{2\pi} d\tilde{\phi}_1^{(c)} \frac{r^R_{\pm\mp}}{|A(0, 1/2)|^2}$$

$$= (\rho^R_{\mp\pm})^* \tag{28}$$

$$\rho^R_{+-} = -\rho_{+-} \; (r_a \rightarrow r_a^R, \; \beta_a \rightarrow -\beta_a^R) \tag{29}$$

and

$$\bar{\rho}^L_{\pm\mp} \equiv \frac{1}{2\pi}\int_0^{2\pi} d\tilde{\phi}_2^{(c)} \frac{\bar{r}^L_{\pm\mp}}{|B(0, -1/2)|^2}$$

$$= (\rho^L_{\mp\pm})^* \tag{30}$$

$$\bar{\rho}^L_{+-} = -\bar{\rho}_{+-} \; (r_b \rightarrow r_b^L, \; \beta_b \rightarrow -\beta_b^L) \, . \tag{31}$$

See Ref. 1 for the definitions of ρ_{+-} and $\bar{\rho}_{+-}$.

Additional $\nu_R / \bar{\nu}_L$ Tests for CP Violation

There are two tests of "non-CKM" type leptonic CP violation if R-handed ν (and L-handed $\bar{\nu}$) exist:

$$\beta_a^R = \beta_b^L \quad (\text{1st } \nu_R / \bar{\nu}_L \text{ test})$$

$$r_a^R = r_b^L \quad (\text{2nd } \nu_R / \bar{\nu}_L \text{ test})$$

where the phase differences are defined by Eqs. (10b, 15b) and the moduli ratios by Eqs. (25, 27).

In the case of both $(V \mp A)$ couplings and possibly $m_\nu \neq 0$, the $\tau^- \rightarrow \rho^- \nu$ amplitudes for $\lambda_\nu = -1/2$ are

$$A(0, -1/2) = g_L \left(\frac{E_\rho + q_\rho}{m_\rho}\right) \sqrt{m_\tau (E_\nu + q_\rho)} \quad - g_R \left(\frac{E_\rho - q_\rho}{m_\rho}\right) \sqrt{m_\tau (E_\nu - q_\rho)} \tag{32a}$$

$$A(-1, -1/2) = g_L \sqrt{2 m_\tau (E_\nu + q_\rho)} \quad - g_R \sqrt{2 m_\tau (E_\nu - q_\rho)} \tag{32b}$$

For $\lambda_\nu = 1/2$ they are

$$A(-1, 1/2) = 0$$

$$A(0, 1/2) = -g_L \left(\frac{E_\rho - q_\rho}{m_\rho}\right) \sqrt{m_\tau (E_\nu - q_\rho)} \quad - g_R \left(\frac{E_\rho + q_\rho}{m_\rho}\right) \sqrt{m_\tau (E_\nu + q_\rho)} \tag{33a}$$

$$A(1, 1/2) = -g_L \sqrt{2 m_\tau (E_\nu - q_\rho)} \quad - g_R \sqrt{2 m_\tau (E_\nu + q_\rho)} \, . \tag{33b}$$

Note that $g_{L,R}$ respectively denote the chirality $(V \mp A)$ of the $\tau^- \rightarrow \rho^- \nu$ coupling whereas $\lambda_\nu = \mp 1/2$ denotes the handedness of the (massive) tau neutrino.

Formulas for $\tau^- \rightarrow a_1^- \nu$ including both $V \pm A$, and both ν helicities.

First, note that in kinematically describing the $\tau^- \rightarrow a_1^- \nu \rightarrow (\pi_1^- \pi_2^- \pi_3^+)$ mode, one can use the normal to the $(\pi_1^- \pi_2^- \pi_3^+)$ decay triangle in place of the π^- momentum direction of $\rho^- \rightarrow \pi_1^- \pi_2^o$ of the $\tau^- \rightarrow \rho^- \nu$ decay mode. Then, the various S2SC functions given above still hold, Eqs. (17) and (19).

Including both ν_L and ν_R helicities, we find composite decay density matrices for the $\tau^- \rightarrow a_1^- \nu \rightarrow (\pi_1^- \pi_2^- \pi_3^+) \nu$ decay sequence[5]

$$R^\nu = S_1^+ R^+ + S_1^- R^- \tag{34}$$

where $R^\pm$ have the same form as Eq. (12) except the elements have "±" superscripts (see below). $S_1^\pm$ describe $a_1^- \rightarrow \pi_1^- \pi_2^- \pi_3^o$. When the 3-body Dalitz plot is integrated over, only the S_1^+ term remains. In Eq. (34), the R^+ matrix elements are

$$R^{+}_{\pm\pm} = \{\text{Eq. (8) except } (\tfrac{1}{\sqrt{2}}) \rightarrow (-\tfrac{1}{\sqrt{2}})\} \tag{35a}$$

$$\begin{aligned} r^{+}_{\pm\pm} &= (r^{+}_{-+})^{*} \\ &= \{\text{Eq. (9) except } (\tfrac{1}{\sqrt{2}}) \rightarrow (-\tfrac{1}{\sqrt{2}})\} \end{aligned} \tag{35b}$$

with

$$\begin{pmatrix} n_a \\ n_a f_a \end{pmatrix} = \sin^2 \tilde{\theta}_a \, (|A(0,-1/2)|^2 \pm |A(0,1/2)|^2) \pm \left(1 - \tfrac{1}{2}\sin^2\tilde{\theta}_a\right) (|A(-1,-1/2)|^2 \pm |A(1,1/2)|^2) \tag{36}$$

Similarly, the R^- matrix elements are

$$\begin{aligned} R^{-}_{\pm\pm} = & -n_a^- \,(1 \mp f_a^- \cos\theta_q^{\tau}) \\ & \mp \sqrt{2}\,\sin\theta_1^{\tau}\,\sin\tilde{\theta}_a\,[\cos(\tilde{\phi}_a - \beta_a)\; |A(0,-1/2)|\,|A(-1,-1/2)| \\ & + \cos(\tilde{\phi}_a + \beta_a^{R})\; |A(0,1/2)|\,|A(1,1/2)|] \end{aligned} \tag{37a}$$

with

$$\begin{pmatrix} n_a^- \\ n_a^- f_a^- \end{pmatrix} = \cos\tilde{\theta}_a\,(|A(-1,-1/2)|^2 \mp |A(1,1/2)|^2), \tag{37b}$$

and

$$\begin{aligned} r^{-}_{+-} &= (r^{-}_{-+})^{*} \\ &= \sin\theta_1^{\tau}\cos\tilde{\theta}_a\,(|A(-1,-1/2)|^2 + |A(1,1/2)|^2) \\ &+ \sqrt{2}\,\sin\tilde{\theta}_a\,\{[\cos\theta_1^{\tau}\cos(\tilde{\phi}_a - \beta_a) + \iota\sin(\tilde{\phi}_a - \beta_a)]\; |A(0,-1/2)|\,|A(-1,-1/2)| \\ &+ [\cos\theta_1^{\tau}\cos(\tilde{\phi}_a + \beta_a^{R}) + \iota\sin(\tilde{\phi}_a + \beta_a^{R})]\; |A(0,1/2)|\,|A(1,1/2)|\} \end{aligned} \tag{38}$$

For the conjugate decay sequence, $\tau^+ \rightarrow a_1^+ \bar{\nu} \rightarrow (\pi_1^+ \pi_2^+ \pi_3^-)\,\bar{\nu}$,

$$\bar{R}^{\bar{\nu}} = \bar{S}_1^{+}\,\bar{R}^{+} + \bar{S}_2^{-}\,\bar{R}^{-} \tag{39}$$

The $\bar{R}^+$ matrix elements (see Eq. (12)) are

$$\bar{R}^{+}_{\pm\pm} = \{\text{Eq. (13) except } (\tfrac{1}{\sqrt{2}}) \rightarrow (-\tfrac{1}{\sqrt{2}})\} \tag{40a}$$

$$\begin{aligned} \bar{r}^{+}_{+-} &= (\bar{r}^{+}_{-+})^{*} \\ &= \{\text{Eq. (14) except } (\tfrac{1}{\sqrt{2}}) \rightarrow (-\tfrac{1}{\sqrt{2}})\} \end{aligned} \tag{40b}$$

with

$$\begin{pmatrix} n_b \\ n_b f_b \end{pmatrix} = \sin^2 \tilde{\theta}_b \, (|B(0,1/2)|^2 \pm |B(0,-1/2)|^2) \pm \left(1 - \tfrac{1}{2}\sin^2\tilde{\theta}_b\right) (|B(1,1/2)|^2 \pm |B(-1,-1/2)|^2) \tag{41}$$

The $\bar{R}^-$ matrix elements are

$$\begin{aligned} \bar{R}^{-}_{\pm\pm} = & \; n_b^- \,(1 \pm f_b^- \cos\theta_2^{\tau}) \\ & \mp \sqrt{2}\,\sin\theta_2^{\tau}\,\sin\tilde{\theta}_b\,[\cos(\tilde{\phi}_b + \beta_b)\; |B(0,1/2)|\,|B(1,1/2)| \\ & + \cos(\tilde{\phi}_b - \beta_b^{L})\; |B(0,-1/2)|\,|B(-1,-1/2)|\} \end{aligned} \tag{42a}$$

with

$$\begin{pmatrix} n_b^- \\ n_b^- f_b^- \end{pmatrix} = \cos\tilde{\theta}_b\,(|B(1,1/2)|^2 \mp |B(-1,-1/2)|^2), \tag{42b}$$

$$\begin{aligned} \bar{r}^{-}_{+-} &= (\bar{r}^{-}_{-+})^{*} \\ &= \sin\theta_2^{\tau}\cos\tilde{\theta}_b\,(|B(1,1/2)|^2 + |B(-1,-1/2)|^2) \\ &+ \sqrt{2}\,\sin\tilde{\theta}_b\,\{[\cos\theta_2^{\tau}\cos(\tilde{\phi}_b + \beta_b) + \iota\sin(\tilde{\phi}_b + \beta_b)]\; |B(0,1/2)|\,|B(1,1/2)| \\ &+ [\cos\theta_2^{\tau}\cos(\tilde{\phi}_b - \beta_b^{L}) + \iota\sin(\tilde{\phi}_b - \beta_b^{L})]\; |B(0,-1/2)|\,|B(-1,-1/2)|\} \end{aligned} \tag{43}$$

Ideal Statistical Errors

$\tau^- \rightarrow \rho^- \nu$ mode with L-handed ν:

Tables 1, 2 and 3 list the ideal statistical errors[6] of Ref. 1 for the CP and $\tilde{T}_{FS}$ discrete symmetry tests. Here $\tilde{T}_{FS}$ is the approximate time-reversal-operation which holds only if possible final-state-interactions are neglected. Such effects are indeed negligible in the usual V-A, $m_{\nu\tau} = 0$ lepton model. By $\tilde{T}_{FS}$ the decay amplitude (A or B above) is purely real.

See conference contribution ICHEP-0099 for further discussion[1,7] these statistical errors.

$\tau^- \to a^- \nu$ mode with L-handed ν:

Tables 4, 5, 6 list the analogous ideal statistical errors for the CP and $\tilde{T}_{FS}$ discrete symmetry tests in the case of the $\tau \to a_1 \nu$ decay mode.

Improvement from measurement of τ^- momentum direction:

As discussed above, by use of a silicon vertex detector it may be possible to uniquely determine the τ^- momentum direction. Table 7 shows the improvement for the 2 tests for "non-CKM" type CP-violation in $\tau \to \rho\nu$ decay.

Conclusions About Ideal Statistical Errors

At the Z^o, 10^7 Z^o's are assumed, and at each γ^* energy we assumed 10^7 $\tau^- \tau^+$ pairs. Notice that in the measurement of the phase differences at γ^* energies, versus at the Z^o, there is not as much improvement as would be expected due to the increase in the number of events. This is because in using ρ-polarimetry (or a_1-polarimetry) a Wigner-rotation is involved in going from the center of mass frame's ρ-observables (or a_1-obserables) to the respective τ rest frame's ρ-observables (or a_1-observables). For instance, see Tables 3 and 6.

τ spin correlations are necessary to measure β_a at γ^* energies; at the Z^o, without using spin-correlations there would be an extra suppression factor of $< P_\tau > = -0.138$.

Since the direction of the initial e^- beam has been integrated out, there is no obvious source for a violation of $\tilde{T}_{FS}$ invariance for the S2SC processes considered here. For instance, unlike in $K_{\ell 3}$ decays, since ν_τ is only weakly interacting there is no "old physics" source for electromagnetic rescattering of the ν_τ and the ρ^- (or a_1).

The tables for the a_1 decay modes show approximately the same patterns as those for the ρ decay modes obtained earlier in Ref. 1 and shown here in the first three tables. However, the net sensitives differ---the sensitivity for the $\beta_a = \beta_b$ test is about 10 times worse in the a_1 mode, but the **normalized** sensitivity is about the same for the $r_a = r_b$ test for both the ρ mode and for the a_1 mode. "**Normalized** sensitivity" refers to the value of the fractional error { $\sigma(r_a) / r_a$ }.

For measurement of β_a at γ^* energies, knowledge of the τ momentum direction improves the sensitivity by about a factor of ($1/\sqrt{2} = 0.707$) which is what would be expected by statistics. However, there is a small (about 10 %) improvement in the measurement of r_a by measurement of the τ momentum direction.

In conclusion, at γ^* energies one can perform the 1st test , $\beta_a = \beta_b$, to about the 0.5^o level, and the 2nd test, $r_a = r_b$, to about the 0.1% level by the ρ decay mode. For the a_1 , the sensitivity for the lst test is about 10 times worse, but is about the same for the 2nd test.

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6. We calculate ideal statistical errors as in C.A. Nelson, Phys. Rev. D40, 123 (1989). For instance, for a 2-variable correlation we would distribute N events ideally over a 2-dimensional ℓj-grid according to the theoretical result, $I(x, y) = Z_o(x, y) + a\, Z_1(x, y)$; The ideal error in bin ℓj is $\sigma_{\ell j} = \sqrt{I(x_\ell, y_j)}$. By χ^2 minimization, the "ideal statistical error" in the measurement of "a" is $\sigma_a = \{ \sum_{\ell j} [Z_1(x_\ell, y_j) / \sigma_{\ell j}]^2 \}^{1/2}$.

7. C.A. Nelson, SUNY BING 4/30/94 and ICHEP-0099.

Table Captions

Table 1: At $E_{cm} = M_Z$, ideal statistical errors for two tests for CP violation in $\tau \to \rho\nu$ by the simpler S2SC function $I(E_1, E_2\, \tilde{\theta}_1, \tilde{\theta}_2)$, see Eq. (21), for the sequential decay $Z^o \to \tau^-\tau^+$ with $\tau^- \to \rho^-\nu \to (\pi^-\pi^o)\nu$ and $\tau^+ \to \rho^+\bar{\nu}$, $\pi^+\bar{\nu}$, or $\ell^+ \nu_\ell \bar{\nu}_\tau$. We use 10^7 Z^o events.

Table 2: At E_{cm} = 10 GeV and 4 GeV respectively, ideal statistical errors for two tests for CP violation in $\tau \to \rho\nu$ by the simpler S2SC function, Eq. (21), for the decay of an off-mass-shell photon $\gamma^* \to \tau^-\tau^+$ with $\tau^- \to \rho^-\nu \to (\pi^-\pi^o)\nu$, and $\tau^+ \to \rho^+\bar{\nu}$, $\pi^+\bar{\nu}$, or $\ell^+ \nu_\ell \bar{\nu}_\tau$. We use 10^7 $\gamma^* \to \tau^-\tau^+$ events.

Table 3: Ideal statistical errors for CP/T violation tests based on the full S2SC function of Eq. (18) for the $\{\rho^-\rho^+\}$ sequential decay mode. Note that $\tilde{\beta} \equiv \beta_a - \beta_b$ and $\beta' \equiv \beta_a + \beta_b$.

Table 4: Ideal statistical errors for CP tests for $\tau^- \to a_1^-\nu \to (\pi_1^-\pi_2^-\pi_o^+)\nu$ at Z^o from the simpler $I(E_1, E_2\, \tilde{\theta}_1, \tilde{\theta}_2)$.

Table 5: Same as Table 4 except at E_{cm} = 10 GeV and 4 GeV.

Table 6: Ideal statistical errors for CP/T violation tests based on full S2SC for $\{a_1^- a_1^+\}$ sequential decay mode.

Table 7: Percentage improvement for tests for $\tau \to \rho\nu$ mode (compare Table 3) when τ^- direction is known, e.g. via silicon vertex detector.

TABLE 1

$E_{cm} = M_Z$ Mode	Number of events.	Ideal statistical errors $\sigma(r_a)$	$\sigma(\beta_a^2)$
$\{\rho^- \rho^+\}$	20,302	0.0065	$(12°)^2$
$\{\rho^- \pi^+\}$	9,847	0.0091	$(12°)^2$
$\{\rho^- \ell^+\}$	29,074	0.0056	$(15°)^2$
Sum of above modes	59,223	0.0039 [0.6%]	$(10°)^2$

TABLE 2

Mode	Number of events.	$E_{cm} = 10$ GeV $\sigma(r_a)$	$\sigma(\beta_a^2)$	$E_{cm} = 4$ GeV $\sigma(r_a)$	$\sigma(\beta_a^2)$
$\{\rho^- \rho^+\}$	605,127	.0012	$(5.5°)^2$	.0011	$(8.8°)^2$
$\{\rho^- \pi^+\}$	293,527	.0017	$(5.9°)^2$	.0016	$(9.1°)^2$
$\{\rho^- \ell^+\}$	866, 658	.0010	$(7.5°)^2$	.0010	$(11.5°)^2$
Sum of above modes	1,765,312	.0007 [0.1%]	$(4.7°)^2$	.0007 [0.1%]	$(7.3°)^2$

TABLE 3

E_{cm}	Number of $\{\rho^- \rho^+\}$ events	Ideal Statistical Errors $\sigma(\tilde{\beta})$	$\sigma(\beta')$	$\sigma(\beta_a)$
M_Z	20,302	1.88°	3.15°	1.84°
10 GeV	605, 127	0.43°	0.74°	0.42°
4 GeV	605,127	0.86°	1.13°	0.71°

TABLE 4

$E_{cm} = M_Z$ Mode	Number of events.	Ideal statistical errors $\sigma(r_a)$	$\sigma(\beta_a^2)$
$\{a_1^- a_1^+\}$	2,718	0.019	$(41°)^2$
$\{a_1^- \rho^+\}$	7,428	0.011	$(22°)^2$
$\{a_1^- \pi^+\}$	3,603	0.016	$(24°)^2$
$\{a_1^- \ell^+\}$	10,638	0.009	$(29°)^2$
Sum of above modes	24,387	0.0062 [0.6%]	$(18°)^2$

TABLE 5

Mode	Number of events.	$E_{cm} = 10$ GeV $\sigma(r_a)$	$\sigma(\beta_a^2)$	$E_{cm} = 4$ GeV $\sigma(r_a)$	$\sigma(\beta_a^2)$
$\{a_1^- a_1^+\}$	81,000	.0035	$(21°)^2$	.0035	$(26°)^2$
$\{a_1^- \rho^+\}$	221,400	.0021	$(10°)^2$	.0021	$(15°)^2$
$\{a_1^- \pi^+\}$	107,388	.0030	$(11°)^2$	.0030	$(15°)^2$
$\{a_1^- \ell^+\}$	317,070	.0018	$(14°)^2$	.0018	$(19°)^2$
Sum of above modes	726,858	.0011 [0.1%]	$(5°)^2$	.0012 [0.1%]	$(12°)^2$

TABLE 6

E_{cm}	Number of $\{a_1^- a_2^+\}$ events	Ideal Statistical Errors $\sigma(\tilde{\beta})$	$\sigma(\beta')$	$\sigma(\beta_a)$
M_Z	2,718	20°	32°	17°
10 GeV	81,000	4°	6°	5°
4 GeV	81,000	8°	10°	8°

TABLE 7

E_{cm}	Number of $\{\rho^- \rho^+\}$ events	Percentage Improvement by τ Direction	
		$\sigma(r_a)$	$\sigma(\beta_a)$
10 GeV	605,127	7%	27%
4 GeV	605,127	12%	26%