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NEGATIVE BINOMIAL FITS TO MULTIPLICITY DISTRIBUTIONS FROM CENTRAL COLLISIONS OF ^{16}O+Cu at 14.6A GeV/c AND INTERMITTENCY

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INTRODUCTION

The concept of "Intermittency" was introduced by Bialas and Peschanski[1] to try to explain the 'large' fluctuations of multiplicity in restricted intervals of rapidity or pseudorapidity.[2,3] A formalism was proposed[1] to to study non-statistical (more precisely, non-Poisson) fluctuations as a function of the size of rapidity interval, and it was further suggested[1] that the "spikes" in the rapidity fluctuations were evidence of fractal or intermittent behavior, in analogy to turbulence in fluid dynamics which is characterized by self-similar fluctuations at all scales—the absence of a well defined scale of length.[4] Bialas and Peschanski proposed that the data be presented as Normalized Factorial Moments of order q :

$$< F_q(\delta\eta) >= \frac{< n(n-1)\ldots(n-q+1) >}{< n >^q} \quad , \qquad (1)$$

where n is the multiplicity in a pseudorapidity interval (bin) of size $\delta\eta$ on a given event and the $< \;\; >$ brackets indicate averaging over all events. Intermittency would be indicated by a power-law increase of multiplicity distribution moments over pseudorapidity bins as the bin size is reduced:

$$F_q(\delta\eta) \propto (\delta\eta)^{-\phi_q} \quad . \qquad (2)$$

The Normalized Factorial Moment with the clearest interpretation is

$$F_2 = \frac{< n(n-1) >}{< n >^2} = \frac{< n^2 > - < n >}{< n >^2} = \frac{\sigma^2 + < n >^2 - < n >}{< n >^2} = 1 + \frac{\sigma^2}{\mu^2} - \frac{1}{\mu} \qquad (3)$$

where $\mu \equiv < n >$ and $\sigma \equiv \sqrt{< n^2 > - < n >^2}$ is the standard deviation. Note that the Normalized Factorial Moments are all equal to unity for a Poisson Distribution.

The formulation of this new concept of "intermittency" in terms of moments was taken by many as an inelegant and confusing development, particularly since the greatest advance in multiplicity distributions in 20 years had recently been made by the UA5 Collaboration[5] who actually determined the functional form of multiplicity distributions. The Negative Binomial Distribution (which had been used sporadically for the total multiplicity[6]) was used by the UA5 collaboration[7] as a "remarkable" description of their measured multiplicity distributions in intervals of rapidity which are not significantly constrained by conservation laws,[8–11] and also for the total multiplicity. Also, a related distribution, the Gamma Distribution, had been used to describe E_T distributions.[12] **One could not help but wonder what "intermittent" behavior would look like in terms of distributions rather than moments—since once the distribution is known, then *ALL* the moments are known.**

An "intermittency" analysis of charged particle multiplicity data from the target multiplicity array (TMA) in central collisions of ^{16}O+Cu at 14.6 A·GeV/c has been published by the AGS-E802 collaboration.[13] The centrality cut was made using the Zero degree Calorimeter (ZCAL) and requiring that the forward energy be less than one projectile nucleon (i.e. $T_{\text{ZCAL}} < 13.6$ GeV). In agreement with previous measurements,[14] an apparent power-law growth of Normalized Factorial moments with decreasing pseudorapidity interval was observed in the range $1.0 \geq \delta\eta \geq 0.1$. In the present work, multiplicity distributions in individual pseudorapidity bins are presented for the same data. These distributions are excellently fit by Negative Binomial Distributions (NBD) in all $\delta\eta$ bins, allowing, for the first time, a systematic formulation of the subject of "intermittency" in terms of distributions, rather than moments.

NEGATIVE BINOMIAL DISTRIBUTION

The Negative Binomial Distribution of an integer m is defined as

$$P(m) = \frac{(m+k-1)!}{m!(k-1)!} \frac{(\frac{\mu}{k})^m}{(1+\frac{\mu}{k})^{m+k}} \tag{4}$$

where $P(m)$ is normalized for $0 \leq m \leq \infty$, $\mu \equiv < m >$, and some higher moments are:

$$\sigma = \sqrt{\mu(1+\frac{\mu}{k})} \qquad \frac{\sigma^2}{\mu^2} = \frac{1}{\mu} + \frac{1}{k} \qquad F_2 = 1 + \frac{1}{k} \tag{5}$$

The Normalized Factorial Moments and Cumulants (K_q)[15,16] of the NBD are particularly simple:

$$F_q = F_{(q-1)}(1 + \frac{q-1}{k}) \qquad\qquad K_q = \frac{(q-1)!}{k^{q-1}} \quad . \tag{6}$$

The NBD, with an additional parameter k compared to a Poisson distribution, becomes Poisson in the limit $k \to \infty$ and Binomial for k equal to a negative integer (hence the name). The extra parameter has made the NBD useful to Mathematical Statisticians in the Likelihood Ratio Test for whether a distribution is Poisson—more precisely as a "test for independence in rare events."[17] The likelihood ratio test for a Poisson distribution consists of determining whether the NBD parameter $1/k$ is consistent with zero to within its error $s_{\frac{1}{k}}$, which is given[17] as:

$$s_{\frac{1}{k}} = \frac{s_k}{k^2} = \frac{1}{\mu}\sqrt{\frac{2}{N}} \tag{7}$$

where N is the total number of events. For statisticians, the NBD represents the first departure from a Poisson Law. Physicists are more likely to describe the NBD as Bose-Einstein ($k = 1$) or Generalized Bose Einstein $k \neq 1$ distributions.[6]

DISCLAIMER

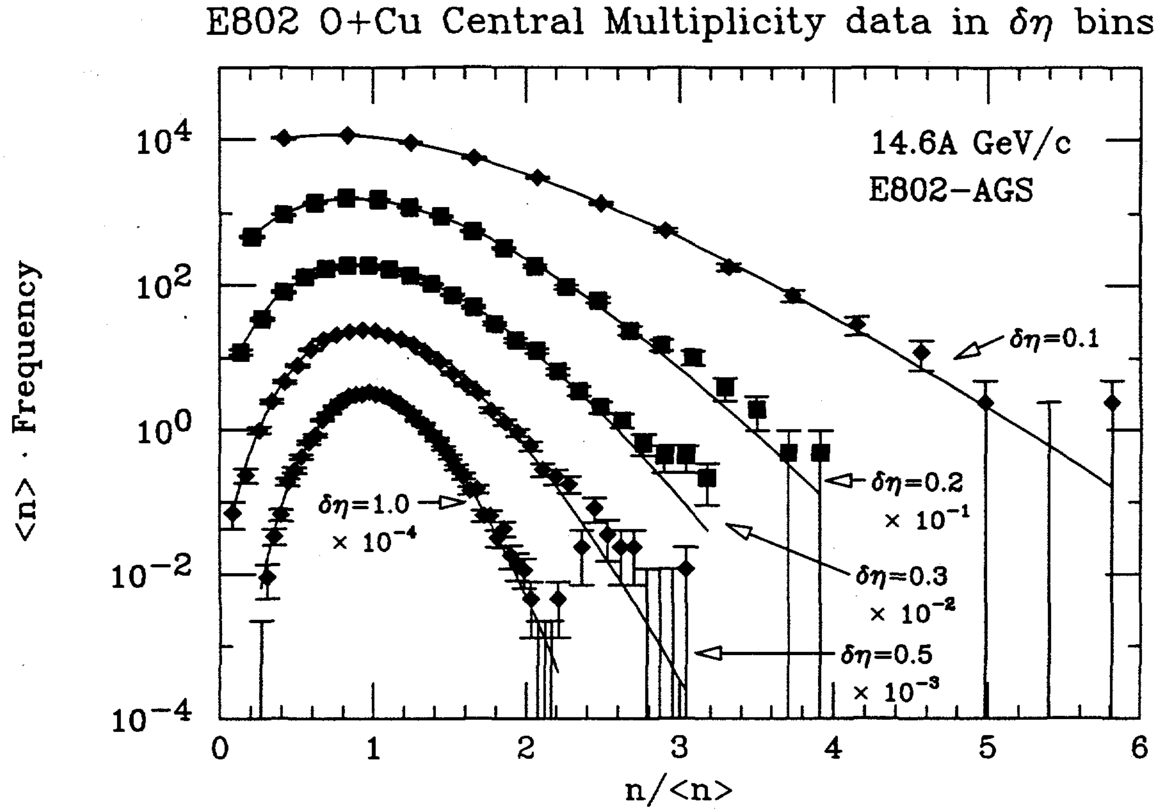

Figure 1: Multiplicity distributions for selected $\delta\eta$ intervals, 0.1,0.2,0.3,0.5,1.0, as indicated. The data for each interval are plotted scaled in multiplicity by $< n >$, the mean multiplicity in the interval. Each successive distribution has been normalized by the factor indicated, for clarity of presentation.

NEGATIVE BINOMIAL FITS TO THE DATA

The AGS-E802 multiplicity distributions for central ^{16}O+Cu in bins of $\delta\eta$= 0.1,0.2,0.3, 0.5,1.0, around a central value of $\eta = 1.7$ in the laboratory, are shown in Fig. 1. Note that $<dn/d\eta>$ is rather constant (2.5% rms variation) for these bins. The distributions for each bin all have approximately the same number of events, 19667. The solid lines on the data are NBD fits. The exact details of the centrality cut are important for Fig. 1, and presumably also for intermittency analyses by moments, since the rising (lower multiplicity) parts of the distributions are determined by the centrality cut. The excellence of the fits of the NBD to the rising as well as the falling parts of all the distributions is attributed to use of the ZCAL centrality cut, which is an indirect cut on multiplicity, rather than the sharp cut on multiplicity which is traditionally used to define centrality. Approximately 3.2% of minimum bias events would be expected to pass the centrality cut, thus the data in Fig. 1 correspond to $\sim$ 0.6 million interactions.

The k parameters of the NBD fits are plotted in Fig. 2 and show a totally unexpected and strikingly steep **linear** dependence on $\delta\eta$—k varies by more than a factor of 3 over 1 unit of $\delta\eta$. This is in sharp contrast to the UA5 results, where k is also linear with $\delta\eta$ but varies by only $\sim$ 10% over the same interval. The linear increase of the NBD parameter $k(\delta\eta)$ with $\delta\eta$ (and thus with $\mu =<n(\delta\eta)>$) indicates that the multiplicity in each $\delta\eta$ bin acts as if it were largely statistically independent of that in the next bin, since the near direct proportionality of the NBD parameter $k(\delta\eta)$ with $<n(\delta\eta)>$ means that the multiplicity distributions convolute as the bin size is extended. The effect of the clear non-zero intercept, $k(0) \neq 0$, is that the ratio $k(\delta\eta)/<n(\delta\eta)>$ does not remain strictly constant with increasing $\delta\eta$, as would be the case for full statistical independence. *The fact that the measured multiplicity distributions are excellently* *fit by distributions (NBD) with well known properties under convolution enables these observations to be made by inspection.*

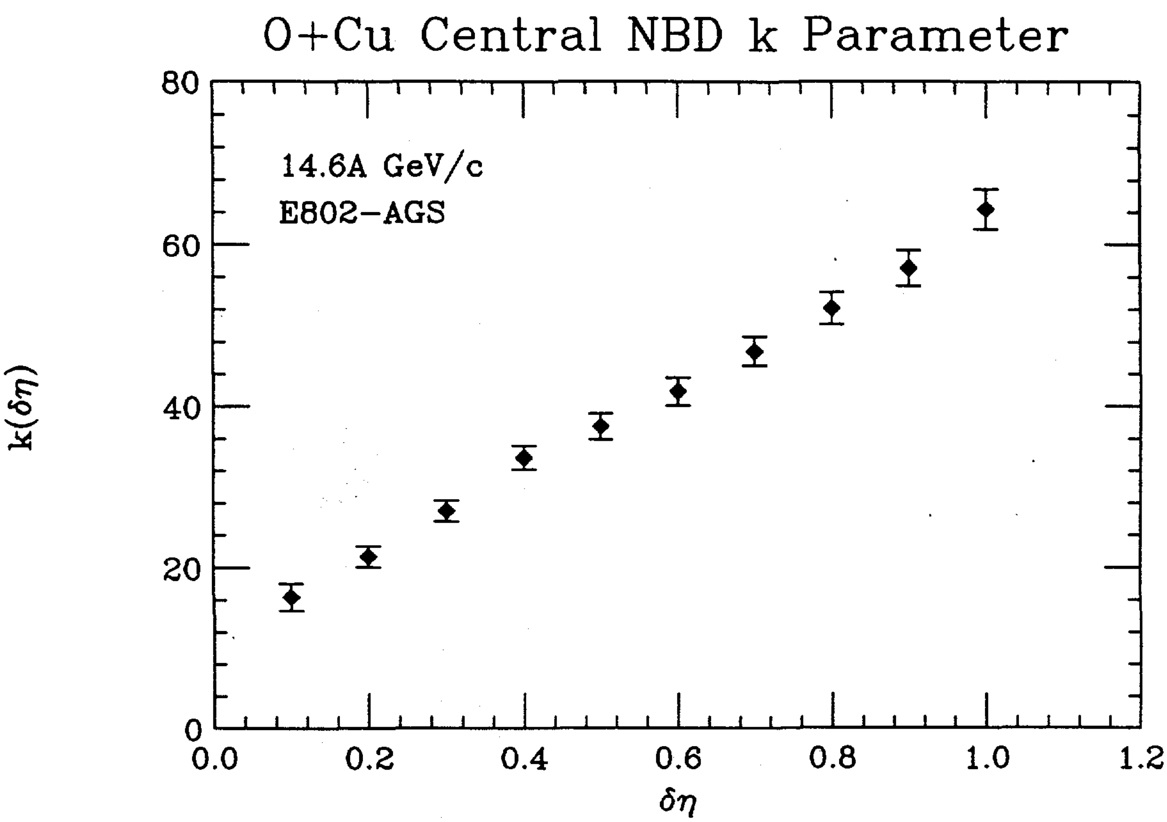

Figure 2: The $k(\delta\eta)$ parameter from NBD fits to the data as a function of the interval $\delta\eta$.

INTERMITTENCY EXPLAINED!

It is now possible to relate the directly measured evolution of the fluctuations of multiplicity with increasing pseudorapidity interval—as described in terms of the Negative Binomial distributions which excellently fit the measurements—to the Normalized Factorial Moment analysis of the same data. The striking linear evolution of the NBD parameter $k(\delta\eta)$ with the width of the interval, explains the observation of fractional power laws based on the "intermittency" formalism in a much more simple, elegant and understandable way. *The apparent power laws with fractional exponent are simply an artifact of using the quantity $F_2 - 1$, which is the inverse of a linear quantity $k(\delta\eta)$.* The "Intermittency" phenomenology, which looks for *self-similar fluctuations at all scales* $\delta\eta$ by a *fractional* power-law increase of bin-averaged Normalized Factorial Moments with decreasing bin size $\delta\eta$, obscures the real physics of multiplicity fluctuations which is given simply and elegantly by the linear evolution of $k(\delta\eta) \equiv 1/(F_2 - 1)$.

Furthermore, for all orders of the Normalized Factorial Moments measured in this experiment[13] the apparent fractional power-law increase with decreasing bin size $\delta\eta$ is entirely given by the Negative Binomial Distribution best fit curves, represented by the single parameter $k(\delta\eta)$—and has nothing to do with the deviations of the measured data points from the best fit curves! The Normalized Factorial moments of all orders can be obtained from the single parameter $k(\delta\eta)$ of the NBD fit (see Eq. 6), and compared point by point with the results of the moment analysis[13] up to 6th order (see Fig. 3). The low order moments agree to well within the statistical errors but there appears to be a small systematic discrepancy between the two methods, which increases slightly with increasing order. Part of the discrepancy may come from the slight difference in the actual data for the two methods and may therefore be real. It is also conceivable[18] that the NBD fits, which give excellent values for the low order moments with the the best statistics, may give smooth values for the high order moments, which miss the fluctuations of the data points at high multiplicity seemingly indicated in Fig. 1.[19] Two comments are relevant on this possibility: the only visible fluctuations of the data from the curves occur for $n \geq 15$ (and therefore are relevant only for the 15th moment or higher); due to the excellence of the χ^2 of the fits, these fluctuations from the NBD best fit curves are constistent with *statistical fluctuations*. In any case, the slight differences in the results of the two method would not affect the fractal interpretation of either set of data points in Fig. 3 by a "true believer" in the factorial moment formalism of "intermittency".

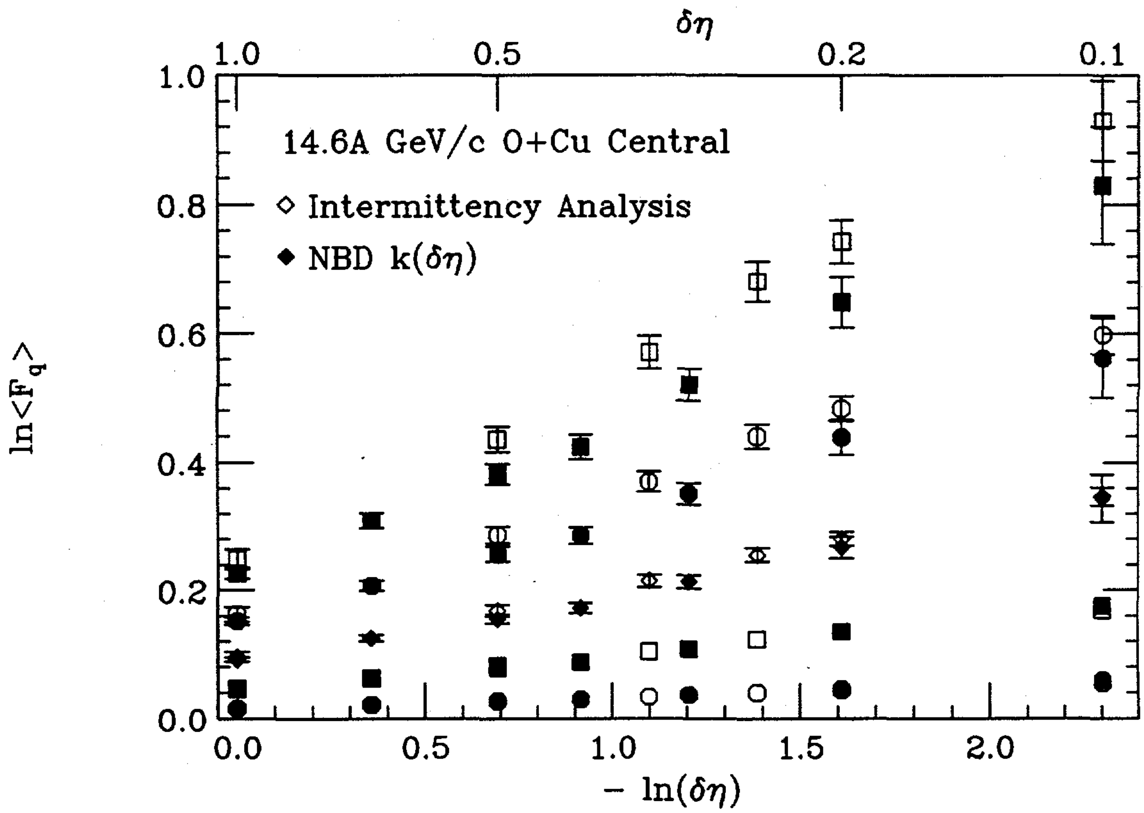

Figure 3: Normalized factorial moments F_q for central collisions of ^{16}O+Cu, for orders q=2,3,4,5,6, from the "intermittency" analysis[13] (open points) compared to the same quantities computed from the NBD parameter $k(\delta\eta)$ of Fig. 2 (solid points).

TWO-PARTICLE CORRELATIONS, THE NBD AND INTERMITTENCY

The importance of two-particle correlations to completely determine the multiplicity distribution was pointed out by Fowler and Weiner,[9] and more recently by Giovannini and Van Hove.[20] The application of two-particle short range correlations to the "intermittency" phenomenology was pioneered by Carruthers, Friedlander, Shih and Weiner,[21] Capella, Fialkowski and Krzywicki,[22] and Carruthers and Sarcevic.[23] The Reduced 2-particle Correlation is parameterized in an exponential form

$$R(y_1, y_2) = \frac{C_2(y_1, y_2)}{\rho_1(y_1)\rho_1(y_2)} = \frac{\rho_2(y_1, y_2)}{\rho_1(y_1)\rho_1(y_2)} - 1 = R(0,0)\, e^{-|y_1 - y_2|/\xi} \tag{8}$$

where $\rho_1(y)$ is the inclusive single particle density (assumed constant), $\rho_2(y_1, y_2)$ is the inclusive two-particle density, $C_2(y_1, y_2)$ is the Mueller[15] 2-particle correlation function, and ξ is the correlation length. Then, the integral can be performed on an interval of full width $\delta\eta$, $0 \leq y_1 \leq \delta\eta$, $0 \leq y_2 \leq \delta\eta$:

$$K_2 \equiv F_2 - 1 = \frac{\int^{\delta\eta} dy_1 dy_2\, C_2(y_1, y_2)}{<n(\delta\eta)>^2} = R(0,0)\, \frac{[1 - \frac{\xi}{\delta\eta}(1 - e^{-\delta\eta/\xi})]}{\delta\eta/2\xi} \quad . \tag{9}$$

For a Negative Binomial Distribution, substitution of the identity $k \equiv 1/(F_2 - 1)$ into Eq. 9 yields the equation for the evolution of the NBD parameter $k(\delta\eta)$:

$$k(\delta\eta) = \frac{1}{F_2 - 1} = \frac{1}{R(0,0)} \frac{\delta\eta/2\xi}{[1 - \frac{\xi}{\delta\eta}(1 - e^{-\delta\eta/\xi})]} \quad . \tag{10}$$

Note that Giovannini and Van Hove[20] were the first give the relationship between the NBD k parameter and the integral of the 2-particle correlation function C_2, and a similar derivation was given by De Wolf.[24] If it is known (eg. from the data) that the multiplicity distribution is Negative Binomial, then the two particle correlation determines the entire distribution. Of course, independently of the distribution, Eq. 10 is valid for the evolution of $1/(F_2 - 1)$ with $\delta\eta$.

This formula, Eq. 10, gives a mathematical explanation of why the linear increase of k with $\delta\eta$ is an indication of the randomness of the multiplicity in adacent $\delta\eta$ bins, while the constancy of k with increasing $\delta\eta$ would be an indication of 100% correlation: in the limit $\delta\eta \ll \xi$, when the $\delta\eta$ interval is well inside the correlation length, $k(\delta\eta) = 1/R(0,0)$, a constant; in the limit $\delta\eta \gg \xi$, k is directly proportional to $\delta\eta$, $k(\delta\eta) \simeq \delta\eta/2\xi$, as expected from convolutions of independent bins.[21] The measured evolution of $k(\delta\eta)$, which appears to be strikingly linear, is equally well described by a fit to Eq. 10 (see Fig. 4) which indicates a weak correlation strength, $R(0,0) = 0.074 \pm 0.005$, and a very short rapidity correlation length, $\xi = 0.12 \pm 0.01$. It is important to note that these results are very sensitive to any short-range two-particle correlation generated by the detector, and in fact, the data of Figs. 1–4 which are uncorrected for instrumental effects, have a known instrumental short range correrelation[13] which constitutes about half the measured effect.

CORRECTION FOR INSTRUMENTAL EFFECTS

A short range correlation was inadvertently built into the Target Multiplicity Array (TMA) used for these measurements, which was constructed of resistive plastic tubes operated in the proportional mode and read out from image signals induced on cathode pads. The detector was composed of individual small panels which were slightly tilted to avoid inefficiency due to the walls of the tubes: the ineffeciency was compensated by a small amount of cross-counting on adjacent pads for particles which cross from one wire to another across a tube wall—a built-in short range correlation. The effect of such cross-counting was studied extensively using Monte Carlo (MC) simulations, test beam data, and finally by comparing the measured rate of two-pad clusters on adjacent wires to that predicted by the MC which included all the physical (conversions, decays, multiple scattering) and geometrical effects. The rate on which pads on adjacent wires fired was $(7.45 \pm 0.11)\%$ in the data, compared to $(3.4 \pm 0.3)\%$ in the MC which is composed of a random effect of 2.3%, with only 1.1% from conversions and Dalitz pairs. (The tracks from conversions and Dalitz pairs generally both land on one pad or else the instrumental background from this effect would be much larger.)

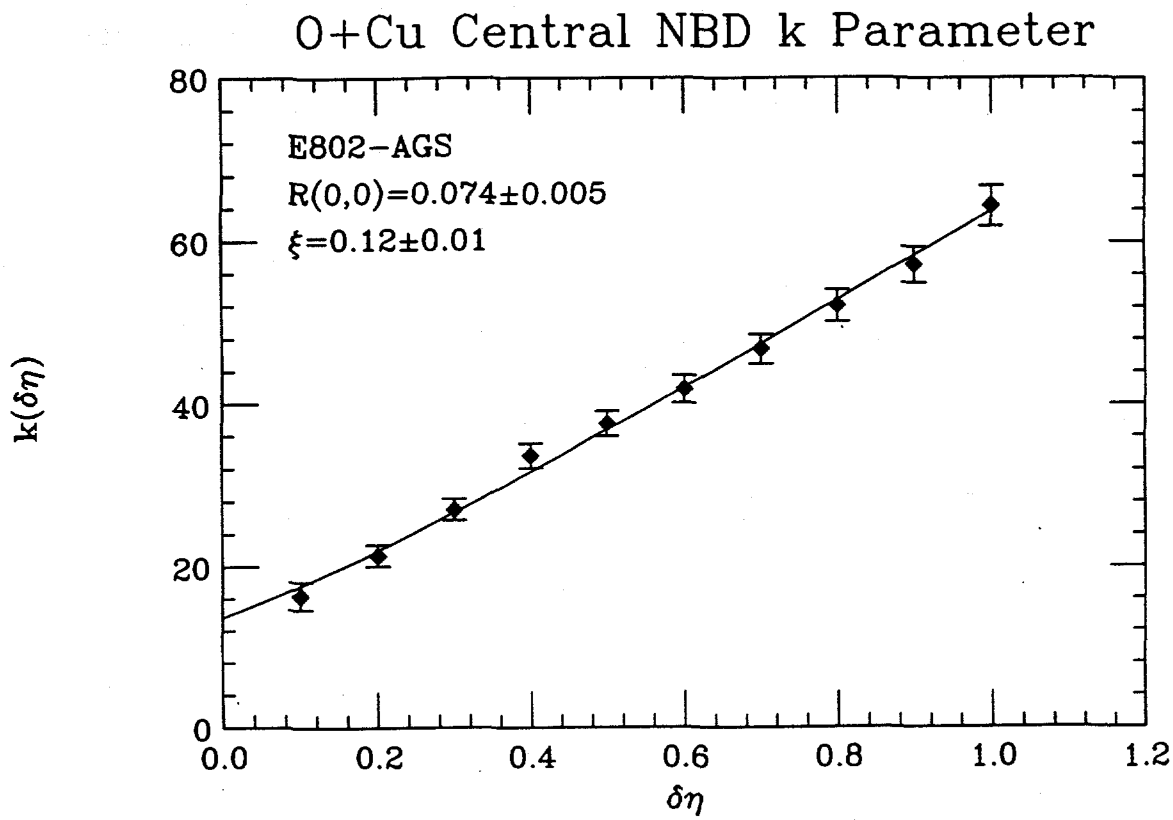

Figure 4: The NBD parameter $k(\delta\eta)$ as a function of the interval $\delta\eta$ (Fig. 2),together with a fit to Eq. 10 with the parameters indicated (solid line).

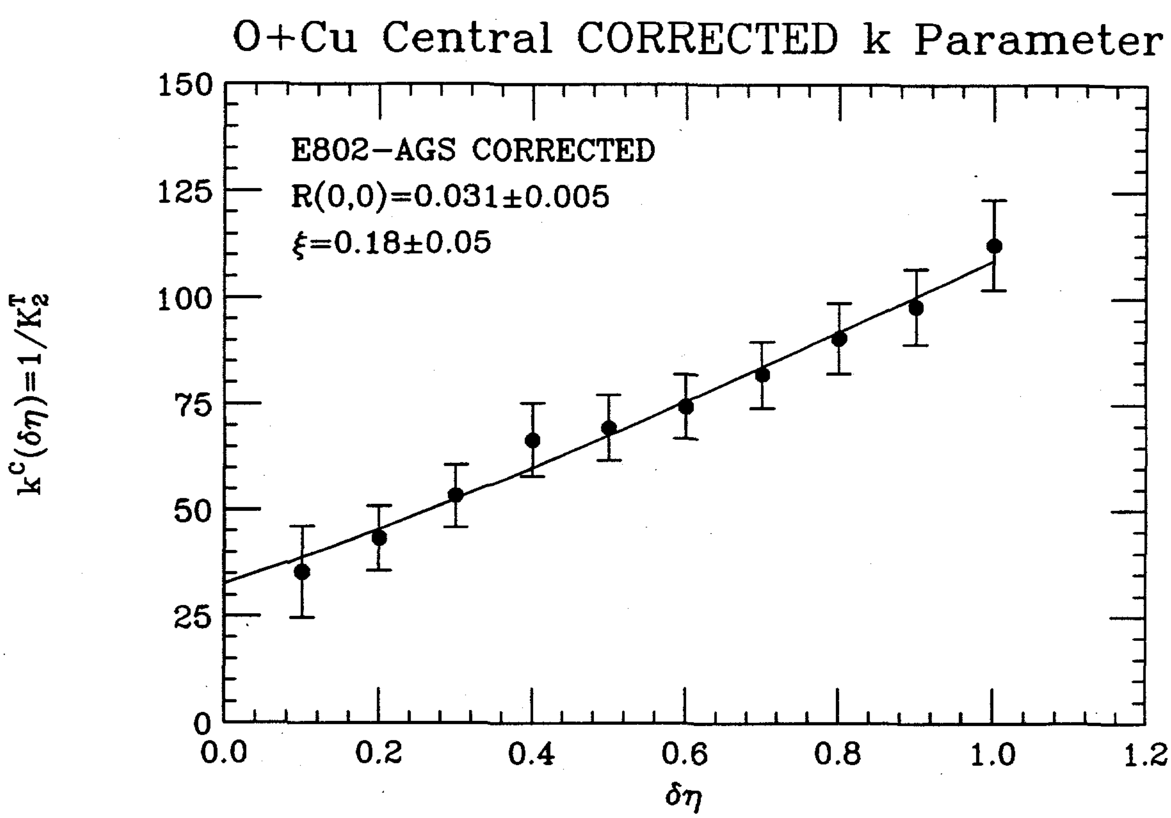

Figure 5: The k parameter from NBD fits to the data, corrected for instrumental effects, as explained in the text, presented as $k^C(\delta\eta) \equiv 1/K_2^T(\delta\eta)$ as a function of the interval $\delta\eta$. The solid line is a fit to Eq. 10 with the parameters indicated.

The difference of 4% was therefore added to the final Monte Carlo used to calculate the instrumental effects. Interestingly, the results of final MC for the instrumental effects, $F_2^{\rm MC} - 1 \equiv K_2^{\rm I}$, can be rather well represented by Eqs. 8, 9, with parameters $R^{\rm I}(0,0) = 0.050 \pm 0.010$ and $\xi = 0.072 \pm 0.020$, which is, in fact, a reasonable mathematical description of the built-in short range correlation of the detector.

The NBD analysis is corrected for the instrumental effect by taking the measured two particle correlation $R(y_1, y_2)$ to be the sum of a true effect plus the instrumental effect:

$$R(y_1,y_2) = R^{\rm T}(y_1,y_2) + R^{\rm I}(y_1,y_2) \quad , \tag{11}$$

with the further assumption that the instrumental effect has minimal influence on the observed $< n(\delta\eta) >$. It then immediately follows from Eqs. 8–10 that the measured $K_2(\delta\eta) = 1/k(\delta\eta)$ is just the sum of the integrals of the true plus the instrumental terms, or

$$K_2(\delta\eta) = \frac{1}{k(\delta\eta)} = K_2^{\rm T}(\delta\eta) + K_2^{\rm I}(\delta\eta) \quad . \tag{12}$$

The true effect $K_2^{\rm T}(\delta\eta)$ is then simply

$$K_2^{\rm T}(\delta\eta) = \frac{1}{k(\delta\eta)} - K_2^{\rm I}(\delta\eta) \quad . \tag{13}$$

FINAL RESULTS FOR $R(0,0)$ AND ξ

In keeping with the notation based on the NBD, the final results are quoted as $1/K_2^{\rm T}(\delta\eta)$, denoted $k^{\rm C}(\delta\eta)$, and are plotted vs $\delta\eta$ in Fig. 5 which clearly illustrates, again, the simple linear evolution and non-zero intercept. The final values of $R(0,0)$ and ξ, corrected for instrumental effects, are derived from a fit of this data to Eq. 10: $R(0,0) = 0.031 \pm 0.005$, $\xi = 0.183^{+0.051}_{-0.042}$ (statistical errors). The systematic error, predominantly from the measured cross-talk uncertainty ($4.0\% \pm 0.4\%$), is ± 0.003 for $R(0,0)$ and ± 0.01 for ξ. The hadron correlation length at low energies is known[25] to be roughly $\xi \sim 2$ units of rapidity, with strength $R(0,0) \sim 0.6$. Thus, for the weak correlation strength and small correlation length derived from the E802 data to make sense, it must be that the standard hadronic short range correlation effect is diluted by the random overlap of the multiple collisions in the ^{16}O+Cu reaction. Similar conclusions in the context of the conventional "intermittency"slope parameters were given in references.[21,22,26,27]

This result further demystifies "intermittency". For ^{16}O+Cu central collisions, *"Intermittency" is nothing more than the apparent statistical independence of the multiplicity in small pseudorapidity bins, $\delta\eta \sim 0.2$, due to the surprisingly short two-particle rapidity correlation length!* The 'large' bin-by-bin fluctuations on individual event rapidity distributions from Si+AgBr interactions in cosmic rays[3,28] and the linear evolution of $k(\delta\eta)$ for the present data are both explained by this effect.

It is interesting that exactly the deduced effects from the E802 data—weakened and very short length rapidity correlations in collisions of relativistic heavy ions—were predicted several years ago.[21,22,29,30] In nucleus-nucleus collisions, the conventional short-range correlations should be washed out by the random superposition of correlated sources,[22,26,27] so that eventually only the Quantum-Statistical Bose-Einstein (B-E) correlations should remain.[21,22,31] Other experiments have reported a relationship of "Intermittency" to B-E correlations.[32,33] If B-E correlations were the entire effect, then direct measurements of B-E correlations in the variable $\delta\eta$, instead of the usual variable[34] ($Q_{\rm inv} = p_1 - p_2$), should reproduce the parameters derived from the evolution of $k^{\rm C}(\delta\eta)$. A preliminary attempt using the E802/E859 spectrometer is shown in

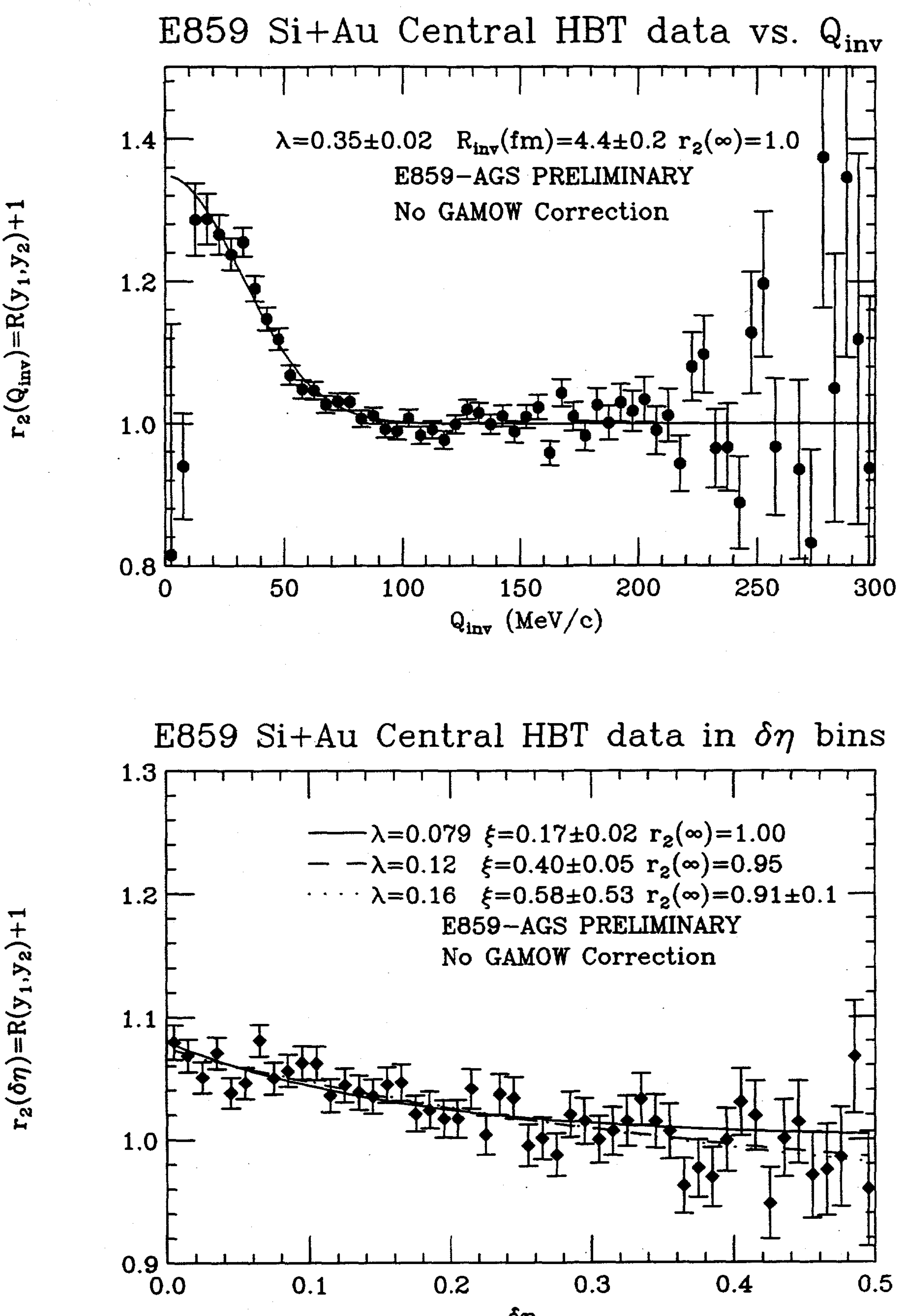

Figure 6: a) E859 two π^- Bose-Einstein Correlation measurements in the variable $Q_{\rm inv}$ from 231K events in Si+Au central collisions. b) the same data as a function of $\delta\eta$. The curves are fits with different normalization, $r(\infty)$. There is no Gamow Correction and the data are **PRELIMINARY**.

Fig. 6. The two pion correlation measurement in the variable Q_{inv}, which is considered to be one of the most demanding in RHI physics, appears to be much easier than the measurement in terms of $\delta\eta$, where the result for ξ is extraordinarily sensitive to the normalization of the correlated and mixed event samples.[35] The results are very preliminary but appear encouraging.

INTERMITTENCY IN TERMS OF DISTRIBUTIONS

Many of the individual components of the present analysis have been noted by previous authors.[9,20–23,31,27,24,32,33,26,29,30,36–38] However, the present data allow for the first time a systematic formulation of the subject of "intermittency" in terms of distributions, rather than moments. Furthermore, the evolution with $\delta\eta$ of the NBD parameters yields a simple, elegant and understandable explanation of the "intermittency" phenomenology. The key to explaining "intermittency", which had not previously been understood, is the dramatic reduction of the two-particle rapidity correlation length for ^{16}O+Cu central collisions from the value in hadron-hadron collisions.[25] Moreover, the correlation length for central ^{16}O+Cu collisions, although smaller than expected, is quite finite and can be measured—which means that a length scale exists in these collisions and therefore there is no intermittency[1,4] in the multiplicity fluctuations.

Since the pioneering work of UA5,[5] many other experiments have shown that the NBD provides excellent fits to charged particle multiplicity distributions in restricted $\delta\eta$ intervals for all reactions studied, for example: p+p (NA22[39]), $e^+ + e^-$ (HRS[40]), μ-p DIS (EMC[41]), S+S central (NA35[42]). All these measurements show the same effect as the present data—linear dependences of the NBD parameter $k(\delta\eta)$ with the pseudo-rapidity interval $\delta\eta$, or equivalently with the mean multiplicity in the interval $\mu(\delta\eta)$, with non-zero intercept, $k(0) \neq 0$ (see Fig. 7a). The present data (and also to a certain extent, the other heavy ion data, NA35 S+S) are quantitatively, rather than qualitatively, different from the others in that $k(\delta\eta)$ is much larger, and the dependence on $\delta\eta$ much steeper. True intermittency, with a zero correlation length $\xi \to 0$, would occur if the intercept $k(0) \to 0$, which is not observed in any experiment!

The Clan Model Appears

Amazingly, the parameters of the clan model of Giovannini and Van Hove[20] can be directly read off Fig. 7a, as shown in Fig. 7b: the mean number of clusters, $<N>= k\ln(1.0+\mu/k)$, and the mean multiplicity in a cluster $<n_c>= \mu/<N>= (\mu/k)/\ln(1.0+\mu/k)$. For the heavy ion data, $k/\mu \gg 1$ for all cases, so that $<n_c>\simeq 1$ and $<N>\simeq \mu$ to an excellent approximation, with the result that the NA35 and both sets of E802 data are nearly indistinguishable on the line $<n_c>\simeq 1$. Only the UA5 data where $k/\mu < 1$, and to a certain extent NA22, show appreciable multiplicity/per cluster and it will be interesting at RHIC or LHC, which are in the UA5 domain, to see how the parameters evolve from p-p to heavy ion collisions.

The Heavy Ion Data

It is instructive to try to understand the precision obtained for the NBD parameter $k(\delta\eta)$ from the two heavy ion experiments, NA35 and the present experiment, E802. The error estimate, $s_{\frac{1}{k}}$, for the NBD parameter $1/k$ was given above (Eq. 7). Thus, to an excellent approximation, the required number of events N, for a fixed percent error

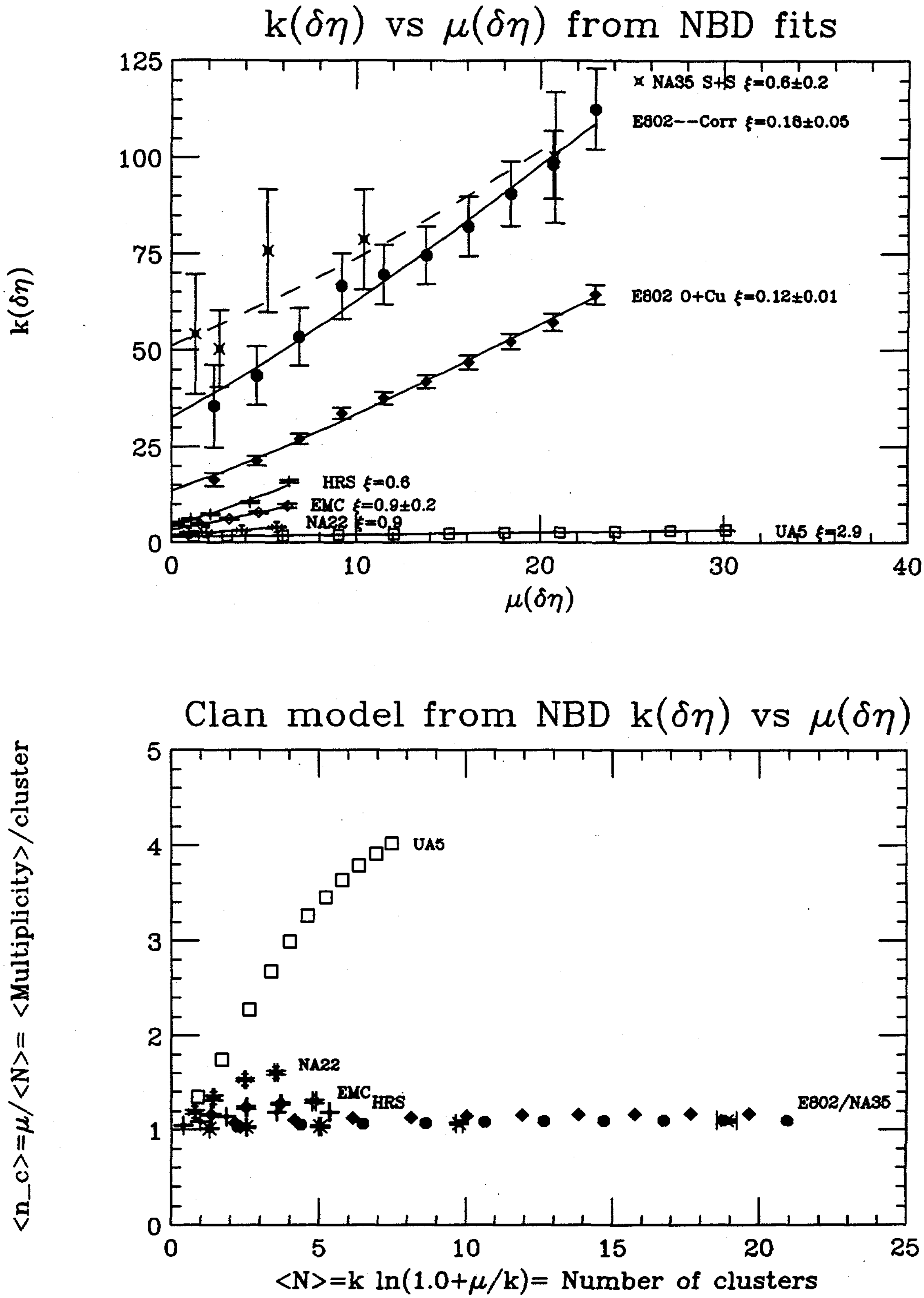

Figure 7: a) The NBD parameter $k(\delta\eta)$ as a function of the mean multiplicity in the pseudo-rapidity interval, $\mu(\delta\eta) = dn/d\eta \times \delta\eta$: UA5 $\bar{\text{p}}$+p $\sqrt{s} = 540$ GeV ($dn/d\eta$=3.01), NA22 p+p $\sqrt{s} = 22$ GeV (1.90), EMC μ-p DIS $W = 18 - 20$ GeV (1.57), HRS $e^+ + e^-$ 2-Jet $\sqrt{s} = 29$ GeV (2.12), E802 O+Cu central $P_{\text{beam}} = 14.6A$ GeV/c (23.0), E802 corrected $k^{\text{C}}(\delta\eta)$, NA35 S+S central $E_{\text{beam}} = 200A$ GeV (10.4). The lines are fits to Eq. 10 with the parameters ξ indicated; b) Parameters of the Clan Model from the same data.

in s_k/k, is

$$N = 2\,\frac{k^2}{\mu^2}\left(\frac{k}{s_k}\right)^2 \quad . \tag{14}$$

This explains why the errors for NA35 S+S with 2856 events are so much larger than E802 O+Cu with 19667 events—(k/μ) is 2 to 3 times larger—even though NA35, in distinction to all the other NBD fit experiments, combined the data from all intervals of a given size (as in the normalized factorial moment analysis) to reduce the errors. Interestingly, the fit of Eq. 10 to this data, shown as the dashed line on Fig. 7 with $\xi = 0.6^{+0.4}_{-0.2}$, indicates that k increases with $\delta\eta$ (i.e. $1/\xi \neq 0$) to 99.4% confidence (2.5σ), which is somewhat in disagreement with the conclusions reached by NA35 from this data,[42] and also by EMU01,[43] that "the NBD parameter (1/k) is (within the errors) independent of the width of the rapidity interval." Of course, the present measurement, corrected for instrumental effects, gives a much clearer (3.5σ) effect for the variation of $k(\delta\eta)$.

Higher Order Correlations

A popular misconception is that[44,45] "there are practically no correlations beyond the second order in the heavy ion data, in contrast to the hadron-hadron and e^+e^- collisions." This is evidently incorrect since all the data, heavy ion included, are fit by the NBD, which exhibits finite K_q for all orders (Eq. 6). Of course, since $k \simeq 50 \cdots 100$ for the heavy ion data, $K_3 = 2/k^2 \simeq 0.0008 \cdots 0.0002$, which is roughly two orders of magnitude lower than the sensitivity of previously published direct searches with 'null' results.[46,47]

SUMMARY AND CONCLUSIONS

E802 ^{16}O+Cu Central(ZCAL) multiplicity distributions in bins of pseudorapidity $\delta\eta = 0.1, 0.2 \ldots 1.0$ show an apparent fractional power-law growth of Normalized Factorial moments with decreasing pseudorapidity interval, in agreement with previous "intermittency" analyses. The same data also exhibit excellent fits to Negative Binomial Distributions. The k parameter of the NBD fits increases steeply and linearly with the $\delta\eta$ interval which is an unexpected and particularly striking result. The linear evolution of the NBD parameter $k(\delta\eta)$ with the width of the interval explains the observation of fractional power laws based on the "intermittency" formalism in a much more simple, elegant and understandable way. *The apparent power laws with fractional exponent of the Normalized Factorial Moments are simply an artifact of using quantities like $F_2 - 1$ which are inversely proportional to a linear quantity*, i.e. $F_2 - 1 = 1/k(\delta\eta)$. Furthermore, the apparent fractional power-law increase of the Normalized Factorial Moments with decreasing bin size $\delta\eta$ for all 6 orders measured in E802 ^{16}O+Cu Central(ZCAL) collisions is entirely given by the Negative Binomial Distribution best fit curves, represented by the single parameter $k(\delta\eta)$—and has nothing to do with the deviations of the measured data points from the best fit curves!

The linear increase of the NBD parameter $k(\delta\eta)$ with $\delta\eta$ can be directly related to the 2-particle short-range rapidity correlation strength and correlation length, conveniently paramterized as an exponential $R(0,0)\, e^{-|y_1 - y_2|/\xi}$, to give the equation

$$k(\delta\eta) = \frac{1}{F_2 - 1} = \frac{<n(\delta\eta)>^2}{\displaystyle\int^{\delta\eta} dy_1 dy_2\, C_2(y_1, y_2)} = \frac{1}{R(0,0)}\frac{\delta\eta/2\xi}{[1 - \frac{\xi}{\delta\eta}(1 - e^{-\delta\eta/\xi})]} \quad ,$$

which describes mathematically why the linear increase of k with $\delta\eta$ is an indication of the randomness of the multiplicity in adacent $\delta\eta$ bins ($\delta\eta \gg \xi$), while the constancy of k with increasing $\delta\eta$ would be an indication of 100% correlation ($\delta\eta \ll \xi$). The evolution of $k(\delta\eta)$, which appears to be strikingly linear, is equally well described by a fit to this equation. After correction for instrumental effects, the best fit parameters indicate a weak correlation strength $R(0,0) = 0.031 \pm 0.005$ and a very short rapidity correlation length $\xi = 0.183^{+0.051}_{-0.042}$, e.g. compared to $p - \bar{p}$ collisions where UA5 measured $R(0,0) \sim 2/3$ and $\xi \sim 3$. The weak and very short-range rapidity correlation in nucleus-nucleus collisions had been predicted—since the conventional nucleon-nucleon short-range correlations should be washed out by the random superposition of correlated sources so that eventually only Quantum Statistical (Bose-Einstein) Correlations should remain. The dramatic reduction of the two-particle rapidity correlation length gives a quantitative demystification of "intermittency". For ^{16}O+Cu central collisions, "intermittency" is nothing more than the apparent statistical independence of the multiplicity in small pseudorapidity bins, $\delta\eta \sim 0.2$, *due to the surprisingly short two-particle rapidity correlation length!*

The present data allow for the first time a systematic formulation of the subject of "intermittency" in terms of distributions to complement the normalized factorial moment formalism. In agreement with all previous measurements of NBD fits to multiplicity distributions in hadron and lepton reactions, the k parameter of the NBD fit for central ^{16}O+Cu collisions is found to exhibit an apparently linear increase with the $\delta\eta$ interval, albeit with a much steeper slope than for the other reactions, and a non-zero intercept, $k(0) \neq 0$. True intermittency, $\xi \to 0$, would occur if the intercept $k(0) \to 0$, which is not observed in any experiment. The correlation length for central ^{16}O+Cu collisions, although smaller than expected, is quite finite and can be measured—which means that a length scale exists in these collisions and therefore there is no intermittency in the multiplicity fluctuations. It is clear that the present E802 data have much more in common with the original UA5 observation—an increase in the width of the multiplicity distributions about the average with decreasing $\delta\eta$ interval—than with any of the classical "intermittency" analyses. The difference is quantitave rather than qualitative: the rapidity correlation length is $\xi \sim 3$ in UA5, $\xi \sim 0.2$ in E802.

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