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ABSTRACT

Expressions for SO(4) invariant euclidean QCD generating functionals are introduced which should produce non-vanishing gluon condensates . Their investigation is started here by initially considering the loop expansion of the corresponding effective action searching for a description differing from the usual perturbation theory. At this level, we consider special free propagators showing a sort or off-diagonal long range order. The calculation of the polarization tensor leads to a gluon mass term which is proportional to the squared root of the also finite value for $< G^2 >$. The summation of all the one-loop contributions to the energy having only mass insertions, indicates the spontaneous generation of the condensate from the perturbative ground state in a way resembling the similar effect in the case of the chromomagnetic field models. This initial inspection suggests the need for a closer investigation which will be considered elsewhere.

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§1 Introduction

The structure of the vacuum state is yet a fundamental problem in Quantum Chromodynamics (QCD) [1],[2]. Correspondingly with the success of the theory at high energies, in which the asymptotic freedom property simplifies the treatment, numerous approaches have been devised in order to understand the basic properties at low energies [1]. The bag and strings models [3], chromomagnetic vacuum field pictures [4],[5], the instanton gas [6] and Dual Chromodynamics [7] are just some of the most relevant developments.

In this work we intend to present some remarks about the Green's functions generating functionals of the QCD vacuum, which were motivated in a previous work [8], done in the framework of the chromomagnetic field approach . In [8] a constant chromomagnetic field, which is classically associated to an also constant current fully breaking the gauge invariance, was quantized using the Dirac's brackets procedure.

The selected magnetic like field had translational as well as a combined rotational and gauge symmetry, which led us to consider it as a possible improved background field over the spatially and rotationally asymmetric Savvidi field. The main motivating property of these fields is that the spectrum of small gluonic waves gives six propagating modes from which only one was massive. The other waves were massless and also unstable. We interpreted that mode as a possible signal of a corresponding massive gluon excitation within a close connected but relativistic treatment.

A factor which could open the possibility for the developing of the above mentioned covariant approach, is to notice that the Schwinger functional differential equation for the Green's functions generating functional is a linear one as also it is the relation specifying the mean field as its first derivate over the sources. Therefore, if the Schwinger equations and its boundary conditions in the space of the sources are taken as basic dynamical principles determining the ground states, the mentioned linearity could allow the generation of new ground states functionals from other known ones.

The above remark and the assumption of the existence of fields being extrema (possibly unstable) of the effective action (Savvidi fields) out of the zero field configuration, motivated the proposition given in this work of an ansatz for the generating functional of the QCD vacuum. It is defined as a special mean value over the SU(3) and SO(4) transformations (the counterpart of the Lorentz group in euclidean space) of the generating functionals associated to the correspondingly transformed chromomagnetic vacuum fields.

The discussions is done here for the kind of field treated in [8] but quantized in the standard fashion by fixing the gauge. The gauge parameter α will be not introduced because in external field problems the effective action may become α dependent in perturbation theory as remarked in [9],[10]. Those fields were recently considered in [11] in searching possibilities for relativistic background fields. In spite of the fact that in [11] they were found as non satisfying the euclidean invariance requirement by themselves, the mean value procedure introduced here may help to overcome this limitation for their application .

It seems worth to remark that the problem of the gauge dependence of the results for such fields having sources at classical level is only limited to an indefinition in the perturbative calculation. This should be the case because it has been argued in general, [12] [13], that if the field exactly satisfies the effective action lagrange equation, then, the results are gauge independent.

The further investigation of the introduced functionals begins in the work not by entering in the analysis of the promediated perturbation theory. We defer that study to following discussions. In place of that, the related effective action functional is introduced and the possibilities for producing main properties like mass gap and gluon condensates parameters at one loop are examined.

It is found that a free gluon propagator consisting in the usual one plus a term seemingly describing a condensate of zero momentum gluons, leads in the first order to both: a gluon mass contribution to the polarization tensor and a field intensity condensate $< G^2 >$. The two quantities are simply related and the assumption of a current estimate for the gluon condensate parameter leads to a gluon mass term having a value near 0.4 GeV/c^2. This is a value, as taken as an indicator of the QCD mass gap, somewhat smaller than the expected one. However,it should be considered that this result was obtained on assuming a simple perturbative quantization. We expect that a direct calculation within the promediated functional could produce a better evaluation because the one-loop result seem to be related merely to the mean value of the classical action evaluated at the mean fields. But, the promediated one loop contributions, clearly, could be expected to be appreciable. The summation of all one loop mass insertions to the effective potential also indicates the spontaneous generation of the condensate from the perturbative vacuum.

It is interesting to remark that the propagator in the tree and one loop approximations and a zero mean field seem to be obtainable in the framework of the effective action for composites, [14][15], in the first approximations. The detailed checking of this possibility will be considered elsewhere. Under the assumption that the effective action discussion in the first loop performed here matches with the mean value approach , both in common, seem to indicate a way of clarifying the connections between the properties arising within the chromomagnetic and leading logarithm models [16],[17] and the behaviour of the real relativistic invariant vacuum state.

The gluon mass term appears to be generated by a sort of spontaneous breaking of the gauge symmetry created by a condensate. A zero mass mode is present since the scalar polarization remains massless because the calculated one loop polarization tensor is transverse. The situation appear to be similar to what happens for longitudinal and transverse photons in a zero temperature plasma, both take the same plasmon mass.

Possibilities of connections with the Dual QCD approach and the treatments in [18] and [11] are planned to be examined in a further extension of the work.

In Section 2 the generating functional ansatz for the QCD is defined. In the third section after passing to the effective action functional the particular free propagator for

the gluon is introduced and the one loop polarization tensor calculated. The condensate contribution to the field intensity condensate is also given and the summation of one loop mass insertions to the effective potential is evaluated. Finally the results are briefly reviewed in the conclusions.

§2 Generating Functional ansatz for QCD.

The discussion below will be referred to the Euclidean space and the following conventions used:

$$\begin{aligned} \nabla_\mu^{ab} &= \partial_\mu \delta^{ab} + g f^{abc} A_\mu^c, \\ F_{\mu\nu}^a &= \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^c A_\nu^b, \end{aligned} \qquad (\ )$$

in which g is the coupling constant and f^{abc} are structure constants for SU(3).

The gluon and ghost dependent action including their corresponding sources will be taken in the form

$$\begin{aligned} S_T[A,\tilde{C},C] &= S[A,\tilde{C},C] + S_s[A,\tilde{C},C] \\ &= \int d^4x\{-\frac{1}{4}F_{\mu\nu}^a F_{\mu\nu}^a + \frac{1}{2\alpha}\partial_\mu A_\mu^a \partial_\mu A_\mu^a + \tilde{C}^a \nabla_\mu^{ab} \partial_\mu C^b\} + \\ & \quad \int d^4x\{j_\mu^a A_\mu^a + \tilde{\xi}^a C^a + \xi^a \tilde{C}^a\}, \end{aligned} \qquad (2)$$

where T^a are the generators of SU(3) in the fundamental representation for quarks. S is the source free part of the action and α is the lorentz gauge parameter. The α parameter will be taken in the limit $\alpha \to 0$ as suggested by the lack of α independence in the external field problems [9],[10].

Let us consider that the exact QCD effective action lagrange equation related with (2) has some non-vanishing x_4 independent field extremals $\mathcal{A}_\mu^a$ for which $\mathcal{A}_4^a = 0$. In other words, that the system quantum mean fields in the absence of auxiliary sources satisfy

$$\frac{\delta\Gamma}{\delta A_\mu^a(x)}[\mathcal{A}_\mu^a, 0, 0, 0] = 0, \qquad (3)$$

$$\mathcal{A}_\mu^a(x) = \left.\frac{\delta \ln Z}{\delta j_\mu^a(x)}\right|_{s=0} \neq 0, \quad \mathcal{A}_4^a = 0, \qquad (4)$$

where $s = (j, \xi, \tilde{\xi})$ indicates the set of all the auxiliary sources inducing the mean gluonic $\mathcal{A}_\mu^a$ and ghost fields $\tilde{C}, C$.

It is possible now to examine the set of fields $\mathcal{F}$ having elements defined through

$$\mathcal{A}_\mu^a(\Omega) = U^{ab}\Lambda_{\mu\nu}^{\nu}\mathcal{A}_\nu^b = \Omega_{\mu\nu}^{ab}\mathcal{A}_\nu^b, \qquad (5)$$

where Ω is an arbitrary element of the direct product of the SU(3) group and the full SO(4) group representing the Lorentz invariance in Euclidean space.

It is clear that, due to the SO(4) and SU(3) global invariances of the theory, the fields $\mathcal{A}(\Omega)$ should be also extremals of the effective action.

The existence of these kinds of chromomagnetic non-vanishing vacuum fields have been the basis of a wide research activity on lower energy QCD starting with the works of Savvidi [4]. The first one loop correction to the effective action has been argued, at least for constant field intensity, to only depend on the field intensity squared $G^a_{\mu\nu}G^a_{\mu\nu}$ scalar. As the Savvidi's original field is one breaking lorentz and rotational invariances in the discussion below we will use an alternative chromomagnetic field having the form ([19],[20],[8],[11]):

$$\begin{aligned} \mathcal{A}^a_i &= \lambda\delta^a_i, \ i=1,2,3; \ a=1,2,3, \\ \mathcal{A}^a_i &= 0, \ i=1,2,3; \ a=4,...,8, \qquad (6) \\ \mathcal{A}^a_4 &= 0, \ a=1,...,8. \end{aligned}$$

Note that the a=1,2,3 Gell'man matrices form a SU(2) subalgebra of SU(3). Then, the field (6) becomes the embedding in SU(3) theory of the field which is most commonly used in the literature in the framework of SU(2) gauge theory.

The potentials (6) need for external sources to be solution of the classical equation of motion. The quantization of such non vanishing external sources problems show indefinitions with the selection of the gauge. However, if at quantum level those fields become solutions without external sources, then, the indefinitions disappear at the exact results on mass shell [12],[13].

Of course, at each order a dependence of the approximate physical magnitudes could appear. However, the full independence should allow to consider the gauge constant as a sort of variational parameter to be fixed by some appropriate criteria.

The fields (6) have the advantage of being invariant under translations and also upon combined rotational and SU(3) global transformations [8]. Recently in [11] the possible role of the fields (6) in the description of the QCD ground state was investigated.

In order to start the construction of the ansatz for Z, let us consider the following auxiliary generating functionals

$$Z^\Omega\left[j,\eta,\bar{\eta},\bar{\xi},\xi\right] = \frac{1}{N}\int \mathcal{D}A\mathcal{D}\bar{C}\mathcal{D}C\mathcal{D}\bar{\psi}\mathcal{D}\psi exp\{S_T\left[A+\mathcal{A}(\Omega),\bar{C},C,\bar{\psi},\psi\right]\}. \qquad (7)$$

That is, the field in the total action S_T have been shifted in the value of the mean field $\mathcal{A}(\Omega)\in\mathcal{F}$. N normalizes $Z^\Omega[0]$ to one.

Differentiating the exponential in the integrand of (7) over the fields, the following Schwinger functional equations for Z^Ω follows

$$\{\frac{\delta S_T}{\delta\phi^i}[\frac{\delta}{\delta j}]+j_i\}Z^\Omega[j] = 0, \qquad (8)$$

$$Z^\Omega[0] = 1, \qquad (9)$$

$$\frac{\delta Z^\Omega}{\delta j}[0] = \phi^i(\Omega),\ldots, \qquad (10)$$

where the DeWitt compact notation was used as defined below

$$\begin{aligned} \phi^i &\equiv (A,C,\bar{C},\psi,\bar{\psi}), \\ j_i &\equiv (j,\bar{\xi},\xi,\bar{\eta},\eta), \qquad (11) \\ \phi^i(\Omega) &= (\mathcal{A}(\Omega),0,0,0,0) \end{aligned}$$

and the repeated dots means other possibly needed boundary conditions.

It could be noticed that the set of functional equations (8)-(10) is independent of the mean field solution or the group element Ω. Then, the superposition of solutions will be a new solution satisfying alternative boundary conditions. This property is used in this work for introducing specially constructed mean values of the generating functional Z^Ω over the group elements Ω, in order to recover the Lorentz invariance lost by the existence of the mean fields $\mathcal{A}(\Omega)$.

A point to be remarked in connection with (8) is that being a third order functional differential equation it would need the fixation of the boundary conditions up to second derivations at vanishing sources. But such derivatives are precisely the disconnected Green functions of the problem. Presently we don't know the exact number of boundary conditions needed. It looks us that this problem is in some way linked with the composite operator affective action concepts exposed in [14] and [15]. The limit in the arbitrariness of the initial conditions for (8) could create some constraints in the number of independent composite effective action arguments.

a) Mean value definitions.

For any function F of the fields $\mathcal{A}\in\mathcal{F}$ the mean value will be defined as

$$< F(\mathcal{A}^a_\mu(\Omega)) >= \frac{\int_V d\Omega F(\mathcal{A}^a_\mu(\Omega))}{\int_V d\Omega}, \qquad (12)$$

where $d\Omega$ is the product of the Haar measures corresponding to the space of parameters of the SU(3) and the disconnected sections of the SO(4) group.

It could be noticed again that the discussion given here should be useful in obtaining covariant Green functions starting from the fields (6) which were found in [11] not able to produce euclidean invariance by themselves in the SU(3) case.
From (12) directly follows that mean value of any function being invariant under SU(3) and the SO(4) transformation coincides with itself. Moreover because by a general SO(4) transformation we can change the sign of all the components of the $\mathcal{A}^a_\mu(\Omega)$ field, it follows that the mean value of a product of an odd number of fields vanish.

Let us consider now means of the product of two fields. Because in any reference system the resulting SO(4) tensor structure should be the same, as well as it should be case for the isotopic one, it follows

$$< \mathcal{A}^a_\mu(\Omega)\mathcal{A}^b_\nu(\Omega) >= C\delta^{ab}\delta_{\mu\nu}. \tag{13}$$

Then after taking the trace over SO(4) and SU(3) indices and considering the form (6) for the fields it follows

$$\begin{aligned} C &= \frac{1}{32} < \mathcal{A}^a_\mu(\Omega)\mathcal{A}^a_\mu(\Omega) > \\ &= \frac{3\lambda^2}{32}. \end{aligned} \tag{14}$$

Then

$$< \mathcal{A}^a_\mu(\Omega)\mathcal{A}^b_\nu(\Omega) >= \frac{3\lambda^2}{32}\delta^{ab}\delta_{\mu\nu}. \tag{15}$$

For the mean value of the field intensity from (15) follows

$$< G^a_{\mu\nu}(\mathcal{A}(\Omega) >= gf^{abc} < \mathcal{A}^b_\mu(\Omega)\mathcal{A}^c_\nu(\Omega) >= 0. \tag{16}$$

b) Invariant generating functional.

The SO(4) invariant generating functional can be now defined as

$$Z[j] =< Z^\Omega[j] > \tag{17}$$

That is, as the mean value of the functionals being associated to the fields $\mathcal{A}(\Omega) \in \mathcal{F}$ assuming a common value for the auxiliary sources j_i.This quantity should satisfy the same Schwinger equation but now with SO(4) invariant boundary conditions in the way

$$\left[\frac{\delta S}{\delta\phi^i}[\frac{\delta}{\delta j}] + j_i\right] Z[j] = 0, \tag{18}$$

$$Z[0] = 1, \tag{19}$$

$$\frac{\delta Z}{\delta j_i}[0] = 0,\ldots. \tag{20}$$

The fact that all the generating functionals used in the mean value have at least in the one loop approximation, an energy which depends of the common SO(4) invariant parameter $G_{\mu\nu}(\mathcal{A}(\Omega))G_{\mu\nu}(\mathcal{A}(\Omega))$, then indicates that the new functional is not equivalent to the one producing the usual perturbation expansion. In particular the one loop energy associated to Z exactly coincides with the same quantity related to any particular Z^Ω.

The additional boundary conditions on Z designed by the dots in (20) are supposed to be also linear in Z. This is a natural requirement because the specification of the mean value of an arbitrary product of field operator is a linear expression in Z.

It would be possible to concentrate the attention here in a closer investigation of the proposed $Z[j]$. However in the following we prefer to discuss the possibilities for obtaining non standard perturbative properties which could be associated to $Z[j]$ by considering its corresponding effective action functional.

§3 Gluon mass and condensate from special free propagators.

In starting let us introduce the effective action Γ associated to the generating functional (17) in the usual way

$$\Gamma[\phi] = \ln Z[j] - j_i\phi^i, \tag{21}$$

$$\phi^i = \frac{\delta \ln Z[j]}{\delta j_i} \tag{22}$$

The aim of this section consists in searching for tree and one loop level propagators being able to produce some basic properties which are expected for the QCD vacuum.

As the generating function (21) associated to the covariant ansatz for Z could satisfy also an usual loop (or power in $\hbar$) expansion, it seems convenient to inspect the degree of arbitrariness within the solutions of the loop expansion which could produce the expected QCD properties like gluon condensate and mass gap, for example.

The one loop expression for the effective action and the corresponding quantum lagrange equations can be written in the compact notation above as follows [21]:

$$\Gamma[\phi] = S[\phi] + \frac{1}{2}\ln DetD(\phi), \tag{23}$$

$$\Gamma_{,i}[\phi] = S_{,i}[\phi] + \frac{1}{2}S_{,ijk}D_{kj}, \tag{24}$$

where the functionals derivates over all the gluon and ghost fields of an arbitrary functional L are written as

$$\frac{\delta L[\phi]}{\delta\phi^i} = L_{,i}[\phi], \tag{25}$$

and the ϕ dependent propagator D is defined through

$$D_{ij} = -S^{-1}_{,ij}[\phi], \tag{26}$$

where S is the source independent part of the action (2).

We are interested in the case of vanishing mean fields as required by SO(4) invariance. Thus, in this case D is given by

$$D_{ij} = -S^{-1}_{,ij}[0]. \tag{27}$$

In terms of the basic gluon and ghost fields of the problem the only nonvanishing second derivates of the action are

$$\frac{\delta^2 S}{\delta A^a_\mu(x)\delta A^b_\nu(x')}[0] = \delta^{ab}\left(\partial^2_x\delta_{\mu\nu} - (1+\frac{1}{\alpha})\partial^x_\mu\partial^x_\nu\right)\delta(x-x'), \tag{28}$$

$$\frac{\delta^2 S}{\delta C^b(x')\delta\bar{C}^a(x)}[0] = \delta^{ab}\partial^2_x\delta(x-x'). \tag{29}$$

Then, the gluon and ghost propagators should be the inverse kernels of (28) and (29) respectively as in the normal perturbative expansion. However, at this point possible alternatives to the usual perturbation series seem to arise.

In particular, being (28) consisting of only derivate terms, the inverse kernel which is the gluon propagator, could contain space independent terms which are solutions of the homogeneous equations. As the propagator is a SO(4) tensor (and not a vector) such solutions could not lead necessary to a breaking the SO(4) invariance.

A natural idea that come to the mind in first considering this observation is the possible relevance of the off-diagonal long range order concept first introduced by Yang [22]. In addition, it could be noticed that being the gluons vector like particles a condensation of them with zero momentum and arbitrary physical polarization could give rise to a lorentz invariant condensate. Being the gluons self interacting the possibility exist for such condensates being dynamically enforced.

In accordance with the above remarks, let us consider free propagators of the form:

$$\begin{aligned} D^{ab}_{\mu\nu}(x,x') &= \frac{C}{(2\pi)^4}\delta_{\mu\nu}\delta^{ab} + \int \frac{dp}{(2\pi)^4}\frac{\delta^{ab}}{p^2}\{\delta_{\mu\nu} - (1+\alpha)\frac{p_\mu p_\nu}{p^2}\}exp(ipx) \\ &= \int \frac{dp}{(2\pi)^4} D^{ab}_{\mu\nu}(p)exp(ipx), \qquad (30) \\ D^{ab}_{G}(x,x') &= -\int \frac{dp}{(2\pi)^4}\frac{\delta^{ab}}{p^2}exp(ipx) \qquad (31) \end{aligned}$$

with

$$D^{ab}_{\mu\nu}(p) = C\delta_{\mu\nu}\delta^{ab}\delta(p) + \frac{\delta^{ab}}{p^2}\{\delta_{\mu\nu} - (1+\alpha)\frac{p_\mu p_\nu}{p^2}\}. \qquad (32)$$

After substituting (30) and (31) in the quantum lagrange equation (24) and taking as vanishing the gluon and ghost fields, in first order in $\hbar$ follows:

$$\Gamma_{,i}[0] = 0, \qquad (33)$$

where the usual tadpole graphs in (33) vanish in dimensional regularization. The contribution of the condensate part of the propagator vanish because only the three legs vertex contributes since the vertex is linear in the momenta and all them vanish due to the $\delta(p)$ and the momentum conservation. Thus, the Lagrange equations are satisfied in the considered approximation.

The inverse propagators is given by the usual one loop expression:

$$\Gamma_{,ij}[0] = S_{,ij}[0] + \frac{1}{2}\{S_{,lmij}[0]D_{ml} - S_{,lmi}[0]D_{mk}S_{,knj}[0]D_{nl}\}. \qquad (34)$$

The calculations of the second term in (34) produces the standard one loop polarization tensor expression with a correction depending on the condensate parameter. Substituting

the D_{ij} gluon-ghost propagator (30) and (31) in (34) and passing to the momentum representation the following expressions for the Fourier transform of the gluonic part of $\Gamma_{,ij}$ follows

$$\Gamma^{ab}_{\mu\nu}(p) = \delta^{ab}\{p^2\delta_{\mu\nu} - (1+\frac{1}{\alpha})p_\mu p_\nu\} - \Sigma^{ab}_{\mu\nu}(p), \qquad (35)$$

$$\Sigma^{ab}_{\mu\nu}(p) = \delta^{ab}\{\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\}(m^2 + \frac{5g^2p^2}{(4\pi)^2}\ln\frac{p^2}{\mu^2}) \qquad (36)$$

where the mass term m^2 is related to the condensate parameter C through

$$m^2 = \frac{3g^2}{(2\pi)^4}C(1-\alpha). \qquad (37)$$

It can be seen in (36) that the correction is purely transverse. That is, the scalar polarization remains being massless but the other three polarizations turn to be massive.

The expression (37) is gauge parameter dependent. However, as it has been stressed in [9],[10], in external fields problems a dependence on α can appear. It could be the case , since the perturbative discussion given here is expected to be connected with the external field problem , that the α dependence in (37) became a consequence of the same effect. Since the introduction α is not a necessary procedure, then, if the above situation is present, a reasonable way of doing is to consider the limit $\alpha \to 0$. Such a limit is equivalent to the quantization by using the Dirac's delta in the Lorentz condition which produces a normal perturbative expansion. In the phenomenological evaluations below the limit $\alpha \to 0$ will be considered.

It should be noticed also that the form of the chosen addition to the free propagator is a very particular one which has not been analyzed in its correspondence with a physical state of the interaction free problem. A more detailed investigation of the gauge dependence properties and the conditions for the chosen propagator to be physical will be considered elsewhere. It seems that a convenient way of discussing this problem is through the use of a BRST kind of discussion of the physical state conditions in different gauges.

Let us calculate now the mean value of the field intensity squared in the simplest approximation, that is the mean value in the perturbative ground state. This corresponds to evaluate

$$<0|S_g[\phi]|0> = \left[\frac{1}{N}\int \mathcal{D}\phi S_g[\phi]exp(S_T[\phi])\right]_{s=0} \qquad (38)$$

where

$$S_g[\phi] \equiv -\frac{1}{4}\int dx G^a_{\mu\nu}(x)G^a_{\mu\nu}(x). \qquad (39)$$

in the approximation in which all the interaction vertices are disregarded. Then

$$\begin{aligned} <0|S_g|0> = &\{\left[\frac{1}{2}S^g_{ij}\frac{\delta^2}{\delta j_i\delta j_j} + \frac{1}{3!}S^g_{ijk}\frac{\delta^3}{\delta j_i\delta j_j\delta j_k} + \frac{1}{4!}S^g_{ijkl}\frac{\delta^4}{\delta j_i\delta j_j\delta j_k\delta j_l}\right] \\ &exp(-\frac{i}{2}j_iD_{ij}j_j)\}_{s=0} + \ldots \qquad (40) \end{aligned}$$

The first and second terms in the squared brackets have zero contribution as evaluated in dimensional regularization at zero sources. The last term gives a non-vanishing contribution which corresponds to the condensate part of the propagators. It can be evaluated to be

$$< 0|S_g|0 >= -\frac{72g^2C^2}{(2\pi)^8}\int dx. \tag{41}$$

Then the gluon condensate parameter in this approximation is determined by

$$G^2 \equiv < G^a_{\mu\nu}(x)G^a_{\mu\nu}(x) >= \frac{288g^2C^2}{(2\pi)^8}. \tag{42}$$

After substituting (42) in (37) the following relation between the squared mass parameter and the gluon condensate follows

$$m^2 = \frac{1}{4\sqrt{2}}\sqrt{g^2G^2}. \tag{43}$$

Therefore the special kind of free propagators (30) (31) in the Feynman gauge lead in the one loop approximation, to a mass term for three gluon polarizations while the scalar polarization remains massless. The gluon condensate parameter in the considered interaction free state takes a positive value, that is, the condensate is chromomagnetic like. Both quantities become linked by (43) a fact which allow to make a phenomenological calculation of the mass term by assuming a current estimates for the gluon condensate parameter [1]

$$g^2G_V^2 \cong 0.5\ (GeV/c^2)^4. \tag{44}$$

Substituting (44) in (43) it follows the estimate

$$m \cong 0.35\ GeV/c^2 \tag{45}$$

for the mass term.

In connection with (45) we comment again on the gauge problem . The outcome (37) for the squared mass depends on the gauge parameter. We had selected to avoid the introduction of the α parameter by setting $\alpha \rightarrow 0$ in accordance with the gauge parameter dependence in external field problems. However, the need for this way of act is by now mean clear. Moreover, the symmetrical in all the components tensor form of the additional part of the propagator suggests that a gauge treating all the field components in equal footings (like the Feynman gauge) could be appropriate . The result in this case for the mass becomes $\sim 0.5\ GeV/c^2$.

It should be also noticed that the appearance of the mass term is not necessarily linked with the prediction of stable gluon excitations in the octet representation. This may be the case, because the usual momentum dependent contribution in (38) could produce an imaginary part of the polarization operator, hence leading to the damping of the charged massive gluon excitations. This looks as a way of obtaining consistency with the experimental absence of charged gluons. These questions clearly need to be considered in a more detailed discussion.

In order to investigate the preference for the perturbative state associated to the condensate, an evaluation was done of the contribution to the effective potential of all the one loop graphs only having insertions of the mass term in the polarizations tensor (36).

The dependence on the G^2 turned to be of the form

$$V(G^2) = \frac{G^2}{4} + \frac{3}{16\pi^2}g^2\frac{G^2}{32}\ln\frac{g^2G^2}{\mu^4} \tag{46}$$

where μ is the dimensional parameter included by the renormalization procedure. It could be selected to fix the minimum of V at the expected value of G^2.

The contribution (46) to the effective potential indicates the spontaneous generation of the condensate of G^2 from the usual perturbative vacuum. This occurs in close analogy with the analogous generation of the chromomagnetic fields. Relation (46) shows a coefficient of the logarithmic term being smaller than the corresponding to one loop potential for chromomagnetic fields. The inclusion of the remaining one loops insertions is however needed for a more definite comparison. It could be also expected that these one loop results become related to the mean value of the classical action over the $\mathcal{A}(\Omega)$ fields. Then, since the one loop results (in the presence of the mean fields) are relevant within the chromomagnetic field approach, they could change noticeably the above calculated values.

Therefore, we conclude that at this first level of approximation the selected gluon propagators seem to give motivating physical indications. However, they should be considered as merely signaling the need for further investigations which will be considered elsewhere.

§4 Conclusion

An ansatz for QCD generating functional in terms of the corresponding magnitudes associated to chromomagnetic field extremals is proposed. The functional is euclidean invariant and have vanishing mean fields and finite gluon condensate in the first approximations.

Its inspection began here by investigating the possibilities of obtaining the above mentioned physical properties already in the framework of the loop expansion. It is interesting that certain class of free-propagators showing the C.N. Yang's ODLRO seem to produce gluon mass terms in the polarization tensor as well as gluon condensates in the first one loop approximation.

In forthcoming works we intend to investigate the predictions for mass terms (and their necessary strong damping rates), energy and condensate following directly from the promediated chromomagnetic generating functional, an approach which seem to have the opportunity to give improved non perturbative information.

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