

Conf. 950420--11



WAPD-R(B)-8520

From : Reactor Technology
Date : November 9, 1994
Subject: An Approximate Algorithm for the Flux from a Rectangular Volume Source

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The attached report is transmitted for information and is entitled "An Approximate Algorithm for the Flux from a Rectangular Volume Source." This work is founded on analysis done by Ono and Tsuru and by Wallace and reported in References (a) and (b). The work described here extends the range and applicability of one of the flux calculation algorithms given in Reference (b) and should be filed with this document. The attached paper was reviewed by Dr. Kalman Shure and has been approved by NR for submission to a future ANS meeting. Please refer any questions to me.

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WP:940205A.WP5

Attachment: WAPD-T-3013

- References:
- (a) H. Ono and A. Tsuru, "An Approximate Calculation Method of Flux for Spherical and Cylindrical Sources With a Slab Shield," J. Nuc. Sci. and Tech., 2, #6, pp. 229-235, June 1976
 - (b) O. J. Wallace, "Improved Approximate Formulas for Flux From Cylindrical and Rectangular Sources," WAPD-TM-1623, March 1993

**AN APPROXIMATE ALGORITHM FOR THE FLUX FROM A
RECTANGULAR VOLUME SOURCE**

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Contract No. DE-AC11-93PN38195

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ABSTRACT

An exact semi-analytic formula for the flux from a rectangular surface source with a slab shield has been derived and the required function table has been calculated. This formula is the basis for an algorithm which gives a good approximation for the flux from a rectangular volume source. No other hand calculation method for this source geometry is available in the literature.

AN APPROXIMATE ALGORITHM FOR THE FLUX FROM A RECTANGULAR VOLUME SOURCE

O. J. Wallace

I. INTRODUCTION

Hand calculation methods involving semi-analytic approximations of exact flux formulas continue to be useful in shielding calculations since they enable shield design personnel to make quick estimates of dose rates, check calculations made by more exact and time-consuming methods, and rapidly determine the scope of problems. They are also a valuable teaching tool.

An approximate method for finding the flux at a point opposite a rectangular volume source with an intervening slab shield would be a useful tool for such hand calculations, but the only such approximations currently available apply only to a point opposite a corner or the center of a face of a square source. However, work based on the method and formulas of Ono and Tsuru (Ref. (1)), and reported in Refs. (2), (3), and (4), has been done which gives an exact semi-analytic formula for the flux at a point opposite the corner or center of a rectangular surface source with an intervening slab shield. This semi-analytic flux formula becomes the basis for a good approximation for the rectangular volume source case.

II. THE EXACT SEMI-ANALYTIC FORMULA FOR THE FLUX FROM A RECTANGULAR SURFACE SOURCE

This formula is derived in Reference (2) and discussed in Reference (4). See Figure 1, which shows a rectangular surface source with a rectangular volume source behind it. The flux at a point similar to F from the surface source is given by:

$$\Phi_F = \frac{S_a}{\pi} \int_0^{\theta_0} \int_0^{\phi_0} \sec \phi e^{-b \sec \phi \sec \theta} d\phi d\theta \quad (1)$$

where

$$\phi_0 = \tan^{-1} \frac{l_1}{2a} \text{ and } \theta_0 = \tan^{-1} \frac{l_2}{2a \sec \phi}.$$

Define the function

$$R_0(\phi_0, l_2/a, b) = \int_0^{\phi_0} \int_0^{\theta_0} \sec \phi e^{-b \sec \phi \sec \theta} d\phi d\theta. \quad (2)$$

The smoothed or easily interpolated form of this function is

$$\bar{R}_0(\phi_0, l_2/a, b) = \frac{e^b}{\phi_0} R_0(\phi_0, l_2/a, b) \quad (3)$$

and is tabulated in Table 1. In terms of this function,

$$\Phi_F = \frac{S_a}{\pi} \bar{R}_o(\phi_o, \ell_2/a, b) e^{-b} \phi_o. \quad (4)$$

III. RECTANGULAR VOLUME SOURCE APPROXIMATION

If a rectangular volume source has self-attenuation, the first four mean-free-paths which lie closest to an external detector point supply more than 98% of the flux seen by the detector point. Consider this portion of the source (or less if the source is thin) and use a Laquerre quadrature to locate and weight the flux from plane rectangular sources within the source volume. The sum of these fluxes will be a good approximation of the flux from the volume source, as in the following example:

See Figure 1. The interval of interest here is $|d-c| = \frac{b}{\mu_s}$ where $b \leq 4$ mfp, $\mu_s = 0.15$, and d is the coordinate of the source face closest to the detector point F . Apply a six-point Laquerre quadrature to the interval (d, c) and discard the two least important points. This gives an approximation using four abscissae or planes.

The pairs of Laquerre abscissae v_i and weights w_i which locate and weight these plane sources are respectively (0.8002, 0.459), (0.3045, 0.417), (0.0502, 0.113) and (0.0031, 0.010). The abscissae x_i and weights w_i for the interval (d, c) are then

$$\begin{aligned} x_i &= v_i (c - d) + d. \\ w_i &= w_i |c - d| \end{aligned}$$

In these equations and all others given below, the index i is assumed to have the values 1, 2, 3, and 4.

The corresponding total attenuation distances including the distance b' through an intervening slab shield are

$$b_{ri} = \mu_s (d - x_i) + b'$$

and the distances from the source to the detector point are

$$a_i = a + (d - x_i)$$

which gives the angles

$$\phi_{oi} = \tan^{-1} \frac{\ell_1}{2a_i}$$

and the ratios

$$r_i = \frac{\ell_2}{a_i}.$$

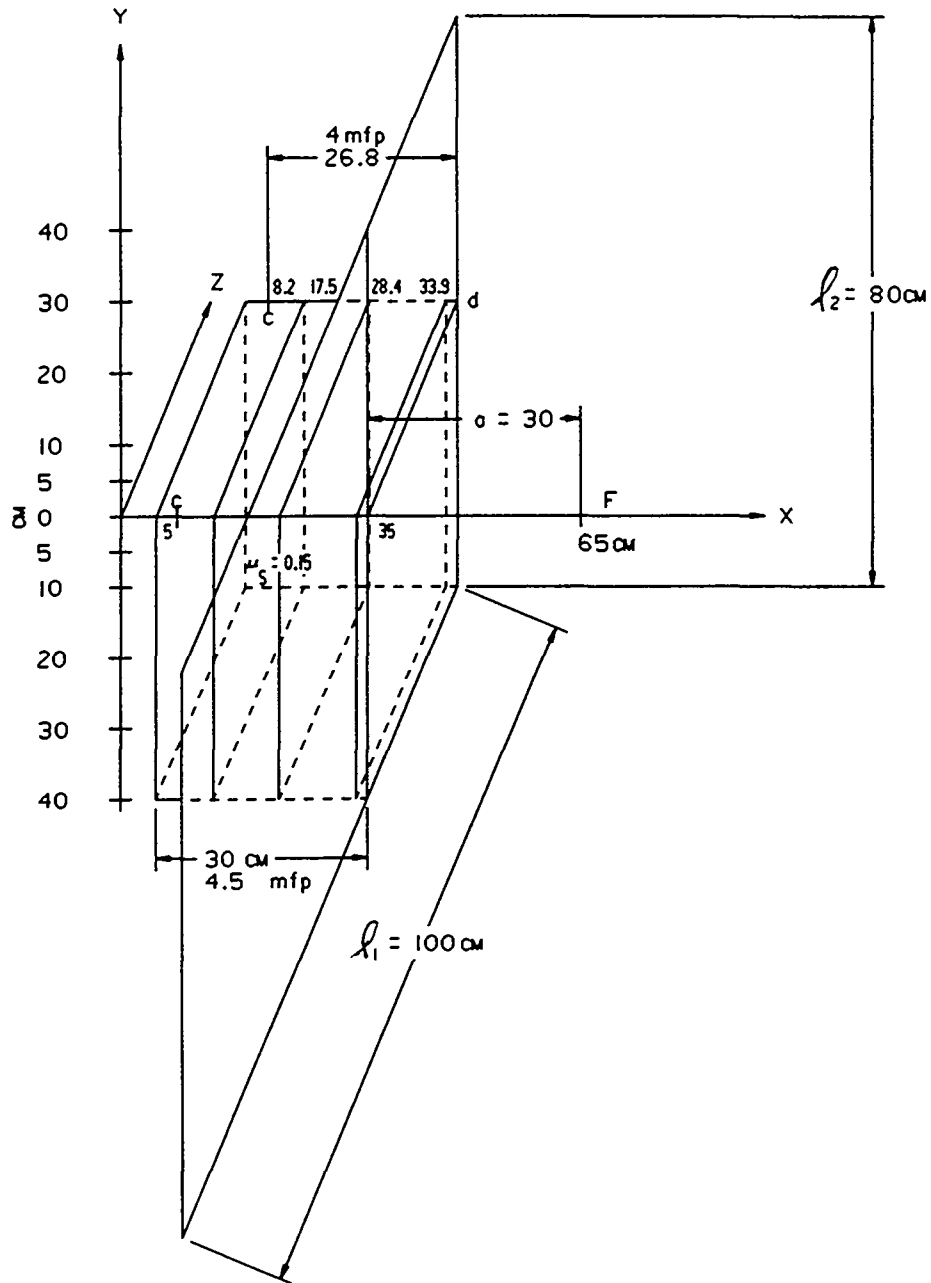


FIGURE 1

A Rectangular Surface Source and a
Rectangular Volume Source

Then the flux at the detector point is given approximately by the weighed sum of four applications of equation (4).

$$\phi_p = \frac{1}{\pi} \sum_{i=1}^4 S_{ai} w_i \overline{R_o} \left(\phi_{oi}, \frac{\ell_2}{a_i}, b_{it} \right) e^{-b_{it}} \phi_{oi} \quad (5)$$

For the example in Figure 1, the coordinates of d and c are 35 and 8.2 cm for $b = 4$ mfp and the abscissae x_i are 13.5, 26.8, 33.7 and 34.92. The corresponding weights w_i are 12.4, 11.3, 3.02 and 0.27.

$\ell_1 = 100$ cm, $\ell_2 = 80$ cm, and the distances a_i are 51.5, 38.2, 31.3 and 30.08. Assuming a slab shield of thickness $b' = 1$ mfp, the attenuation distances b_{it} are 3.22, 1.24, 0.202 and 0.01245 mfp. The four values of the angle ϕ_o are given by

$\phi_{oi} = \tan^{-1} \frac{\ell_1}{2a_i}$ and the ratios by $r_i = \ell_2/a_i$. Then for the quarter rectangular volume source in Figure 1 with $S_a = 10^6$, Equation (5) gives

$$\phi_F = (0.0036 + 0.0455 + 0.0490 + 0.0061) \times 10^6 = 1.042 \times 10^5$$

A check with a point kernel computer program gives $\phi_F = 0.986 \times 10^5$, which is satisfactory accuracy.

A cruder and more conservative approximation to the flux from a rectangular surface source may be found by first finding the flux from the corresponding surface source and then applying a multiplier which takes account of the source self-attenuation, but not the variation in the ratios ℓ_1/a and ℓ_2/a .

The multiplier is found by performing a Laquerre quadrature over the interval $b = \mu_s t = \mu_s |d-c|$ of the source, where $b \leq 4$ mfp, to find the source output. The corresponding attenuation distances are $b_i = \mu_s(d-x_i)$ and the weights are

$$w_i = \omega_i b / \mu_s.$$

Then the required multiplier is $R(b)/\mu_s$ where

$$R(b) = b \sum \omega_i e^{-b_i}$$

It is the approximate ratio between the flux from the surface source and the flux from the corresponding volume source in terms of the source attenuation μ_s assuming that the source per unit volume is the same numerically as the source per unit area. $R(b)$ is plotted in Figure 2. It is conservative by less than a factor of two if $a \geq 1/3 \text{ Max}(\ell_1, \ell_2)$ and becomes very conservative for smaller a , since the ratios ℓ_1/a and ℓ_2/a are considered to be the same for all terms in the sum.

These approximations for the flux from a rectangular volume source are new. No other such approximate formulas are available in the literature.

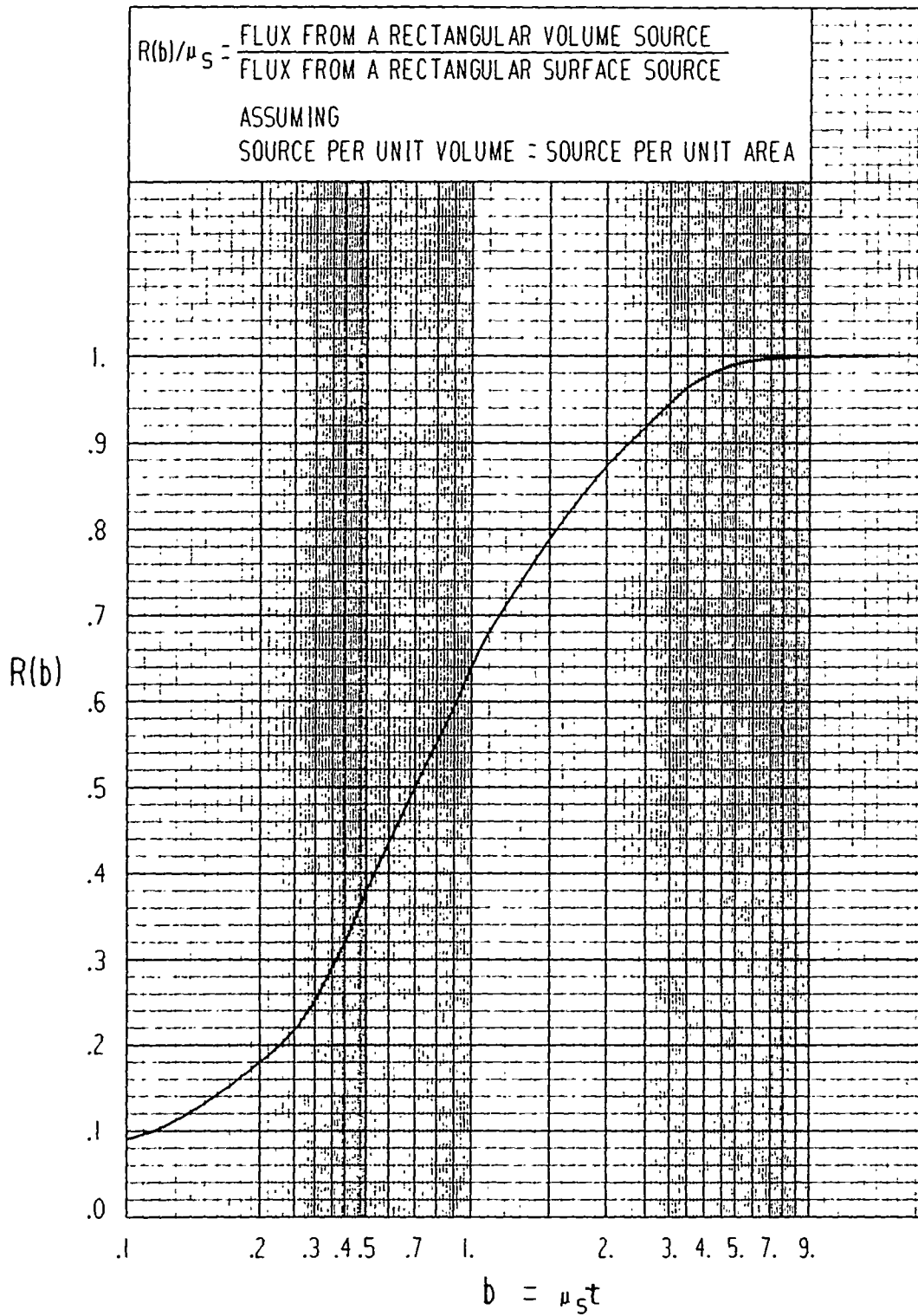


FIGURE 2

The Function $R(b)$

TABLE 1

The Smoothed Function $\bar{R}_0 \left(\phi_0, \frac{\ell_2}{a}, b \right) = R_0 \left(\phi_0, \frac{\ell_2}{a}, b \right) e^{-b/\phi_0}$

Table with 13 columns for L2/A, B (MFP), O (RAD) and 13 columns for x values (0.10 to 10.00). Rows include values for 0.1745, 0.04383, 0.08727, 0.13090, 0.17453, 0.26180, 0.34907, 0.43833, 0.52380, 0.61087, 0.78540, 0.87268, 1.04720, 1.57080.

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IV. REFERENCES

- (1) H. Ono and A. Tsuru, "An Approximate Calculation Method of Flux for Spherical and Cylindrical Sources with a Slab Shield," J. Nuc. Sci. and Tech., 2, #6, pp. 229-235 (June 1965).
- (2) O. J. Wallace, "Semi-Analytic Flux Formulas for Shielding Calculations," WAPD-TM-1197, May 1976.
- (3) O. J. Wallace, "Analytic Flux Formulas and Tables of Shielding Functions," WAPD-TM-1453, June 1981.
- (4) O. J. Wallace and Saeed Bokharee, "Improved Approximate Formulas for Flux from Cylindrical and Rectangular Sources", WAPD-TM-1623, April, 1993.

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