CHARACTERIZING MULTIFRAGMENTATION

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ABSTRACT

We discuss various methods to characterize the fragment size distributions in nuclear multifragmentation. The goal is to find the best signals of a phase transition associated to multifragmentation. We review the concepts of scaling and critical exponents and we examine the possibility to determine them in finite nuclei. The fluctuations of the fragment size distribution and a possible signal of intermittency are also discussed.

1 Introduction

The main deexcitation mode of a nucleus at energies up to a few MeV/nucleon is light particle emission. The mechanism of this mode is sufficiently well understood theoretically to allow a precise study of the thermodynamics of nuclear matter at low temperature. At excitation energies of the order of five to ten MeV/nucleon, this mode is relayed by the emission of medium mass fragments. Unfortunately the mechanism of fragments production in nuclei is still poorly understood and a similar study is not yet feasible. This is particularly straitening because in this energy range one predicts a phase transition of nuclear matter similar to the liquid-gas phase transition of standard fluids[1]. Proving the existence (or the absence) of this transition and studying its properties is one of the major goals of heavy ion physics.

How nuclear fragment emission reveals the presence of a phase transition is still unclear. The emission of various intermediate mass fragments in the same event - what we will call multifragmentation- is one possible signature of a phase transition. This association[2] is suggested by simple fragmentation models, like the Fisher droplet model[3], percolation[4], lattice-gas[5] and statistical equilibrium[6] models. However this multi-fragment production is not enough to sign a phase transition. Refined analysis of the characteristics of the fragment size (mass or charge) distributions are needed.

Nuclear fragmentation is a statistical process in the sense that at a microscopic level the initial conditions are never completely determined. Even if one could repeat many times a nucleus-nucleus collision with exactly the same macroscopic conditions (bombarding energy, impact parameter, etc.), the internal degrees of freedom of the

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nuclei (positions and momenta of the nucleon...) will change from one collision to another. Because of this statistical nature one has to study first the mean (statistical average) distributions and secondly its fluctuations. These studies become, in principle, experimentally feasible with the arrival of a new generation of very efficient 4π fragment detectors[7].

The aim of this contribution is to offer an overview of various methods that have been proposed recently to characterize the fragment size distributions (FSD) in nuclear fragmentation. Section 2 is devoted to the statistical average of the FSD studied with the method of moments, to the determination of critical exponents and to the finite size effects. In Section 3 we deal with the fluctuations. We first study the statistics of the largest fragment and secondly we discuss the relevance of the concept of intermittency in nuclear fragmentation. We conclude in Section 4, with some warnings concerning the application of these methods to experimental data.

2 Mean Fragment Size Distributions

Let us consider the fragmentation of a system of size (charge or mass) S. From a theoretical point of view, the basic quantity in a fragmentation process is the probability $P\{n\}$ to observe a partition $\{n_1, n_2, ...n_S\}$. The experiments (or the numerical simulations) give only the frequency $f\{n\}$ of the partitions. Because of statistical fluctuations, $f\{n\}$ is only a rough representation of $P\{n\}$. On an other hand, in a first step we do not need the full information carried by $P\{n\}$. Hence we perform partial integrations on $f\{n\}$ (statistical averages). For example, the mean fragment size distributions $< n_s, \alpha >$ of partitions of class α are a natural compression of the information. The choice of α is very important. We will call α the control parameter. Let us discuss this point in some detail.

Consider a random mechanism that generates partitions of an integer number and some choice for the criteria of the statistical averages. Let's take for instance, partitions with the same multiplicity of fragments. For a given multiplicity m_0 the mean fragment size distribution $< n_s, m_0 >$ would look like the thick line of fig 1. The shape of the curve may of course depend on m_0. One could perform similar averages taking partitions according to other criteria, for example the same number of intermediate mass fragments (IMF), the mass of the largest fragment s_{max}, the sum of all observed fragments mass but $s = 1$ (S_{bound}), etc. On fig. 1 we have also shown the mean fragment size $\bar{s}$ and the width $\sigma^2_{[s]}$ of the mean distribution. The doted curve represents the fragment size distribution $n_s^{(j)}$ of an event (partition) j with same multiplicity m_0. Because of the finiteness of the system every realization is rather far from $< n_s >$. For a fixed size s it is interesting to look at the frequency distribution of $n_s^{(j)}$. One expects a curve centered around $< n_s >$ with some width $\sigma_{[n_s]}$ (see window of fig 1). We know that for a Bernouilli process the ratio $\frac{\sigma_{[n_s]}}{<n_s>}$ goes to zero as $1/\sqrt{m_0}$. Any deviation from this behaviour indicates the existence of the so called "non-statistical fluctuations". In

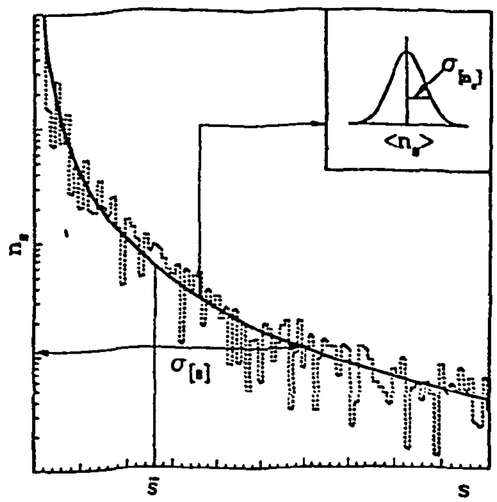

Figure 1: *Schematic representation of the various quantities discussed in the text. The mean fragment size distribution (thick line) with mean fragment size $\bar{s}$ and width $\sigma_{[s]}$. The dotted line is the distribution $n_s^{(j)}$ of a particular event j. For a fixed size s, the distribution of n_s is also represented, with mean value $< n_s >$ and width $\sigma_{[n_s]}$ (window).*

contrast with statistical fluctuations that are just a "noise", these fluctuations may contain interesting information. We will show some examples in Section 3.

2.1 *Moments of the Mean Fragment Distribution*

The data of a fragmentation experiment can be represented as a list of partitions $\{n_s^{(j)}\}$ (where $j = 1, 2 \cdots N$ is the event number) forming a matrix. This matrix can be examined by rows (horizontal analysis, event by event) or by columns (vertical analysis).

The vertical analysis will consist in the statistical average of events of same class α (same excitation energy, same multiplicity m_0, etc.) as discussed previously. We will denote this vertical average by the symbol $<>$.

We will consider the following definition of the mean fragment distribution,

$$< n_s >= \frac{1}{N} \sum_{j}^{N} (n_s^{(j)} - \delta(s - s_{max}^{(j)})), \tag{1}$$

where $s = 1, 2 \cdots, S$ and where N is the number of events of class α and where the contribution of the largest fragment $s_{max}^{(j)}$ produced event by event has been dropped out in order to keep the optimal information with U-shape distributions when calculating the moments.

This (mean) distribution can be characterized by its moments

$$m_k = \sum_s s^k < n_s > . \qquad (2)$$

With the above definitions m_0 is the mean multiplicity (minus one) and $m_1 = S - < s_{max} >$.

Usually, three or four moments suffice to characterize accurately a FSD that does not show too complicated structures. A fragmentation can then be represented by one point in the three or four-dimensional space of these moments. Often projections in a plane are simpler to interpret[8]. Another interesting combination of moments[9] is given by

$$\gamma_2 = \frac{m_2 m_0}{m_1^2}, \qquad (3)$$

which is related to the variance $\sigma^2_{[s]}$ of $< n_s >$ (width of this curve) by $\gamma_2 = \sigma^2 / < s >^2 + 1$. This quantity (that may be called the reduced width of $< n_s >$) has been frequently used to compare experimental and theoretical results[10, 11, 12, 13]. We stress that this quantity has *a priori* nothing to do with the statistical fluctuations of n_s that we will examine in Section 3.

2.2 *Scaling Hypothesis and Critical Exponents*

We will consider for a moment a system of infinite size and we will discuss latter finite size effects. When the system manifests a second order phase transition, the behaviour of $< n_s >$ is particularly interesting in the vicinity of a critical point[4, 5, 3]. The basic idea of a scaling theory of $< n_s >$ is that near the critical point there exists a typical largest cluster size $s_\xi \sim |\epsilon|^{-1/\sigma}$ of spatial extension $\xi \sim |\epsilon|^{-\nu}$, where ν and σ are two critical exponents and ϵ is the "distance" of the control variable to the critical point. For example $\epsilon = T - T_c$ in thermal phase transitions and $\epsilon = p - p_c$ in percolation theory. The hypothesis is then that for all small values of ϵ, the function $< n_s >$ will look the same if we plot them as a function of (s/s_ξ). Monte Carlo simulations suggest as general form $< n_s(\epsilon) > /S = s^{-\tau} f(s/s_\xi)$, with $f(0) = 1$. Or $< n_s(\epsilon) > /S = s^{-\tau} F(z)$, with $z = |\epsilon| s^\sigma$ and $F(0) = 1$.

The moments m_k can then be calculated near $\epsilon = 0$ by replacing the sums with integrals:

$$m_k = \sum_s s^k < n_s(\epsilon) >= (1/\sigma) \mid \epsilon \mid^{(\tau-k-1)/\sigma} \int_0^\infty \mid z \mid^{(1+k-\tau)/\sigma} 1/z F(z) dz.$$

Hence the singular $sing(m_k)$ part of the first three moments define the following relations between critical exponents : $sing(m_0) = |\epsilon|^{(2-\alpha)}$, with $2 - \alpha = (\tau - 1)/\sigma$; $sing(m_1) = |\epsilon|^\beta$, with $\beta = (\tau - 2)/\sigma$ and $sing(m_2) = |\epsilon|^{-\gamma}$, with $\gamma = -(\tau - 3)/\sigma$. We see that moments with $k > \tau - 1$ diverge when $|\epsilon| \to 0$.

The determination of these critical exponents from data is in principle straightforward. For example, first one localizes the critical point by looking for at the maximum of m_2 as a function of the control parameter. After that, one plots $\ln m_2$ versus $ln|\epsilon|$. One expects two parallel straight lines with same slope γ. The choice of the critical point is rather strict, small changes in its value destroying the linearity of the two lines.

Critical exponents are like the "fingerprints" of phase transitions. Each universal class (Liquid-gas, percolation, ferro-magnetic-paramagnetic...) has its own set of exponents. Comparing this universal values with experimental ones is the only serious way to characterize an unknown phase transition. In contrast, other quantities like the critical temperature or the critical density are not universal and do not serve to characterize the transition (for example, normal fluids have different critical temperatures, but same critical exponents).

2.3 *Control Parameter and Finite Size Effects*

The first worry one encounters when applying the concepts of critical phenomena to atomic nuclei is how to specify the control parameter ϵ. In principle, looking for a thermal phenomenon one would like to choose $\epsilon = T - T_c$, but unfortunately this information is not directly available from experiment. When looking for a "geometrical" phenomenon, like a in percolation fragmentation, one would took $\epsilon = p - p_c$, where p is the bond activation probability and p_c its critical value, but this quantity is even more inaccessible experimentally. One possible solution is to substitute the temperature (or p) by another quantity that is measurable and that is strongly correlated with it. Furthermore, if this correlation is *strictly linear* in the critical region, (this is the case of the control parameter p and the moment m_0 in percolation theory[10]) then taking $\epsilon = m_0 - m_{0crit}$ one can determine directly the correct values of the critical exponents. In thermal phase transitions, the relation between the temperature T and the multiplicity m_0 (or other control parameter) has to be carefully studied. We will discuss below a method that overcomes this difficulty.

	τ	β	γ	$1+\beta/\gamma$
Percolation	2.20	0.45	1.76	1.25
Lattice-gas	2.21	0.33	1.24	1.27
Statis. Equil.	~ 2.2	-	-	2.63 ± 0.07
$Au + emul.$	2.17 ± 0.1	-	-	1.2 ± 0.1
$Au + C$	2.14 ± 0.06	0.29 ± 0.2	1.4 ± 0.1	1.21 ± 0.1

Table 1: *Critical exponents for various systems. From refs : Percolation [4],Lattice-gas[16], Statistical Equilibrium Model[17], experimental data on 1 GeV/u Au fragmentation in emulsion [8] and in Au + C reactions[18].*

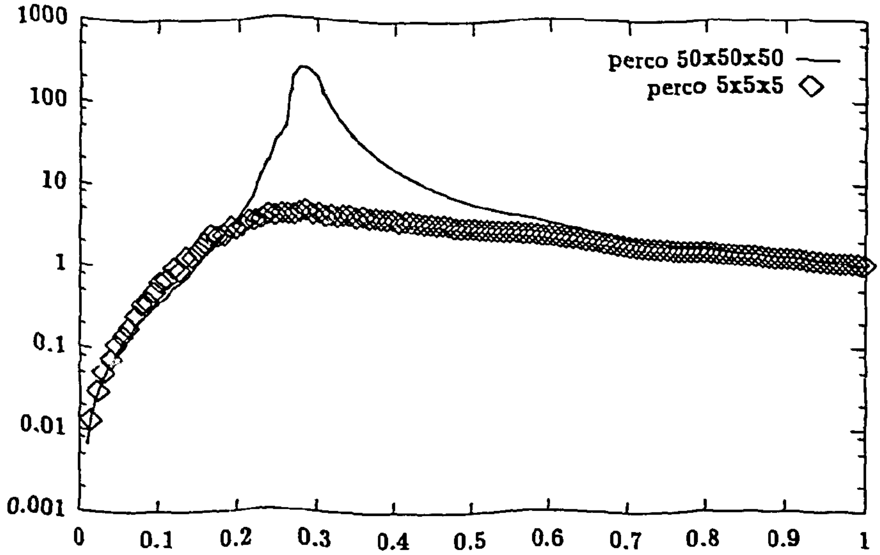

Figure 2: *Mean value of m_2/S as function of m_0/S for small (S = 125) and large (S = 125000) percolation systems.*

The second and more serious difficulty concerns the finite size of atomic nuclei. Strictly speaking, we cannot talk about critical behaviour in a finite size system because none of the moments m_k can diverge. Nevertheless, some aspects of this behaviour remain valid. The critical point is replaced by a "critical region", the width of which increases when decreasing the system size. In the middle of this region, finite and infinite systems behave very differently, but on both sides, these differences are much smaller. Hence it is in principle still possible to extract some information on the exponents by looking at these two regions, but avoiding the central one. This is shown in figure 2, where the quantity m_2/S is plotted as a function of the control parameter m_0/S for a large percolation cubic lattice and for a small one of the typical size of a nucleus. The similarity is even stronger for the quantity $(S - m_1)/S =< s_{max} > /S$ which plays the role of the "order parameter" in percolation-like theories. In infinite systems, this quantity is finite in the "percolating" phase and zero in the other. We see in figure 3 that for $m_0 \ll m_{0crit}$, small and large systems behave similarly but very differently elsewhere.

These two examples give an idea of the difficulties to extract accurate values for the exponents β and γ in nuclear fragmentation. The members of the EOS Collaboration[18] have tried to extract these exponents from their data on 1 GeV/nucleon Au projectiles fragmentation. The control parameter is the multiplicity m_0. The critical multiplicity is determined by looking at the best linearity of the curve $\ln < s_{max} >$ for $m_0 < m_{0crit}$ and at the best linearity and best parallelism of the two branches of m_2. These curves

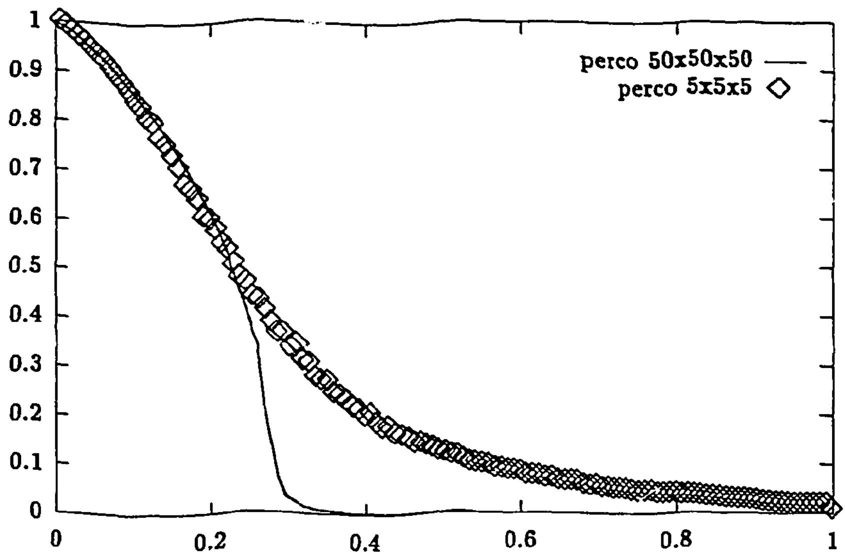

Figure 3: *Mean value of the largest fragment,* $< s_{max} > /S$ *as function of* m_0/S *for small* ($S = 125$) *and large* ($S = 125000$) *percolation systems.*

are drawn as a function of $ln| m_0 - m_{0crit} |$. The slopes β and γ are rather sensitive to the choice of m_{0crit}.

In an earlier work[8], the ratio of the exponents β/γ was determined more directly. Representing $\ln < s_{max} >$ versus $ln(m_2/m_1)$ for events of the same type (say, same m_0) one obtains a two-branch curve, the crossing point corresponding to the "critical point". The slope of the lower branch is $1 + \beta/\gamma$. The advantage of this method is that there is no need to fix m_{0crit} and no need for a linear relation between T or p and m_0. The price one pays is that only the ratio of the exponents is measured. Using the data of Waddington and Freier[19] on Au fragmentation in emulsion, it was concluded that this ratio is compatible with both percolation and liquid-gas predictions.

The exponent τ is in principle easier to determine, by using equation 7 right at the "critical point". The problem is again to define this "point". Here it is also possible to avoid this difficulty, by looking at the slope of $\log m_3/m_1$ versus $\log m_2/m_1$[8]. Unfortunately, most theories predict very similar values for τ. In any case, one always gets $\tau > 2$ from experiments in nuclear fragmentation.

In Table 1 we synthesize our present knowledge of critical exponents. We see that the two experimental determinations of β/γ [8, 18] are in good agreement with both liquid-gas and percolation predictions. When determined separately by the EOS collaboration [18] β and γ are in better agreement with the former prediction.

Also shown in Table 1 is the value of β/γ calculated with the statistical equi-

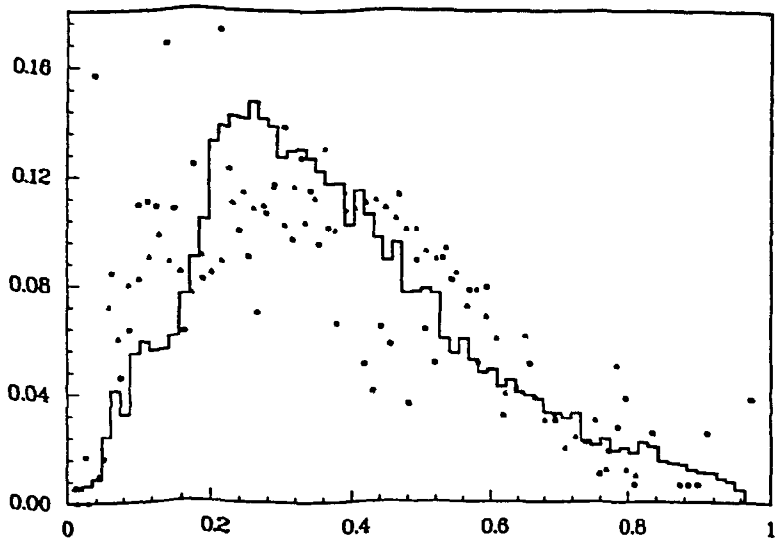

Figure 4: *Fluctuations of the size of the largest fragment s_{max} versus reduced multiplicity m_0/S. Experimental data on Au fragmentation, from EOS Collaboration* [20] *(triangles) and from Waddington and Freier data* [19, 8] *(circles). The percolation calculation is from Ref.*[10] *(histogram).*

librium model[17]. It differs from the two independent *experimental* determinations by a factor of two. This discrepancy deserves further theoretical considerations.

3 Fluctuations

Fluctuations in the fragmentation of a classical system of finite size arise from the lack of control on the initial microscopic conditions [2]. Even the macroscopic conditions cannot be fixed better than on the average (for instance the impact parameter in collisions). One could look directly at the S variances of n_s, $\sigma^2_{[n_s]} =< (n_s - < n_s >)^2 >$, but this information is overdetailed and as in the case of the mean value, it is more convenient to compress the information performing horizontal averages.

3.1 *Fluctuations of s_{max}*

The first moment m_1 is the simplest horizontal compression of the data. Its variance, $\sigma^2_{[m_1]}$, is obviously the same as the variance of s_{max} because $m_1 = S - s_{max}$. In the

[2] *We do not consider chaotic systems, for which a "precise" determination of the initial conditions is meaningless.*

percolation model one finds that the fluctuations of s_{max} are maximum exactly in the critical region (determined by the maximum of γ_2). This property seems to be also true (see Fig.4) for the experimental results of Au fragmentation[8, 20]. In any case the shape of $\sigma^2_{[m_1]}$ as function of m_0 varies significantly according to the model of fragmentation[24]. It can therefore be used as a discriminant observable to test the validity of theoretical models.

3.2 *Factorial Moments and Intermittency*

The pioneering work in this field was done by Ploszajczak and Tucholski[21]. Rather than studying first the general properties of $\sigma^2_{[n_s]}$, they have directly looked at a very refined property : the possible existence of intermittency in the FSD. In light of the work of Bialas and Peschanski[22] they have calculated the factorial moments

$$F_k(\delta) = \frac{\sum_s < n_s(\delta)(n_s(\delta)-1)\cdots(n_s(\delta)-k+1) >}{\sum_s < n_s(\delta) >^k} \tag{4}$$

where $\delta = 1, 2, \cdots$ is a window of fragment sizes and $n_s(\delta)$ the number of fragments in the sth bin of size δ. They have found a power law dependence of the F_k on δ in the experimental data of Waddington and Freier[19] and in the percolation model (near the threshold of percolation). This result was interpreted as the signal of an intermittency pattern linked to a phase transition. Let us first make a remark to simplify the coming discussion.

The summation over s as done in Eq. 9 has no clear theoretical interpretation. Moreover it is useless and it hides the various contributions to the sum. If one considers

$$F_{k,1}(\delta) = \frac{< n_1(\delta)(n_1(\delta)-1)\cdots(n_1(\delta)-k+1) >}{< n_1(\delta) >^k}$$

which is closely related to the variance of n_1 (for $k = 2$), one practically recovers the same results as with the sum. The reason is clear : the frequencies of the light fragments are largely dominant in the sum in the critical region, where the FSD is "singular" at the origin. Indeed, F_k as calculated in[21], gives information *only* on the fluctuation of the small sizes. More interestingly, the behaviour of the other $F_{k,s}$ with $s > 1$ is totally different, because there is no more intermittency signal.

The interpretation of the signal as indicating a true intermittency remains puzzling. Let us briefly explain why[23].

i) *on the basis of numerical simulations* (percolation model, trial model[25])

- At fixed multiplicity there is no intermittency signal at all.

- The signal vanishes when the size of the system grows (although one would expect an opposite trend for the signature of a phase transition).

- The signal disappears for $s > 1$. Hence it is only produced by the very light elements.

- The trial model[25], which apparently does not contain other correlations than those due to the mass conservation, can also give a very nice signal of intermittency.

ii) *on theoretical grounds*

The intermittency characterizes a property of the distribution of the random variable p, (here the probability to find a cluster with a mass $\in [1, 1+\delta]$), namely $\frac{<p^k>}{<p>^k} \sim \delta^{-f_k}$, ($f_k > 0$). It is a property of p *and not of* n. And p and n do not obey the same probability law. Even if the average values are equal ($<n> = m_0 <p>$), the variance of n is larger than that of $m_0 p$: because of the finite multiplicity of the system, there is an additional and unavoidable noise (the "statistical fluctuations") which superimposes itself to the unknown width of p (the "dynamical fluctuations"). Notice that a variable multiplicity would also introduce an additional width.

The factorial moments are only a tool to disentangle the dynamical fluctuations from the statistical ones. They permit to derive the moments of p from a combination of experimentally known moments of n. But this deconvolution, as done in Ref.[21], is only possible under two sharp conditions. (See the demonstration in the original paper[22].)

a) The independency of the m_0 realizations of p at each fragmentation.

b) The control over the multiplicity m_0.

If these two conditions are not fulfilled, we do not know how it is possible to infer the behaviour of $<p^k> / <p>^k$ from that of F_k. Now, condition a) is necessarily violated because of the conservation of mass and condition b) was never been taken into account in the literature.

For this reasons we would be very careful before identifying the intermittency signal found in ref.[21] with a proper intermittency. Our (provisional) guess would be the following : in fragmenting systems the only correlations produced by the mass conservation induce an intermittency signal when the mean size distribution is singular in $s^{-\tau}$ ($\tau \simeq 2$). This signal grows with the width of the multiplicity distribution (and cancels with m_0 fixed). Elements of our guess can be founded in[29, 25, 26, 14, 28].

Therefore, the signal observed in Ref.[21] would not necessarily indicate a phase transition. It does it *indirectly* because it is linked to the power law of the FSD, power law associated *in certain models* to a phase transition. To understand precisely the source of this intermittency-like signal remains a problem to solve.

4 Concluding remarks

This Contribution is a short review of the methods to study phase transitions in finite nuclei. The determination of a set of critical exponents associated to the mean fragment size distribution appears to be the unique way to achieve a definite classification of the transition. This program seems in principle realizable with the new generation of 4π charged particles detectors. However we will temperate this optimism with a few warnings concerning the analysis of experimental data.

We assumed implicitly in the preceeding discussions that the size S of the fragmenting system was invariant. This is not always the case in nuclear fragmentation. At high bombarding energies, intermediate mass fragments (the ones that determine mostly the critical behaviour) come from a "spectator" source, which size may change drastically with impact parameter. For example, in ALADIN experiments[11] on 600 MeV/nucleon Au projectiles bombarding Cu targets, for the most violent collisions that have been detected (lowest Z_{bound}) the emitting "spectator" source has on average the size of a Fe nucleus[27]. The influence of this variation of S on the critical exponents should be carefully studied.

At lower bombarding energies (less then 100 MeV/nucleon) we have the problems of the number and the size of emitting sources. As a function of the impact parameter, the reaction mechanism varies from deep inelastic to quasi-fusion and (maybe) total fusion. Even with a complete identification of the fragment momenta it is not possible to determine, event by event, the source of each fragment. This raises various difficulties. What is the multiplicity (or another control parameter) of each source ? What is the largest fragment s_{max} of each source ? (we recall that the subtraction of the largest fragment, at least in the "liquid" or "percolating" phase, is essential to calculate correctly the critical exponents). All these questions deserve a close examination.

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