

NATIONAL INSTITUTE FOR FUSION SCIENCE

Neoclassical and Anomalous Transport in  
Axisymmetric Toroidal Plasmas with  
Electrostatic Turbulence

H. Sugama and W. Horton

(Received - Apr. 24, 1995)

NIFS-355

May 1995

RESEARCH REPORT  
NIFS Series

27001

NAGOYA, JAPAN

This report was prepared as a preprint of work performed as a collaboration research of the National Institute for Fusion Science (NIFS) of Japan. This document is intended for information only and for future publication in a journal after some rearrangements of its contents.

Inquiries about copyright and reproduction should be addressed to the Research Information Center, National Institute for Fusion Science, Nagoya 464-01, Japan.

# Neoclassical and anomalous transport in axisymmetric toroidal plasmas with electrostatic turbulence

H. Sugama

*National Institute for Fusion Science, Nagoya 464-01, Japan*

and

W. Horton

*Institute for Fusion Studies, The University of Texas at Austin*

*Austin, Texas 78712*

Neoclassical and anomalous transport fluxes are determined for axisymmetric toroidal plasmas with weak electrostatic fluctuations. The neoclassical and anomalous fluxes are defined based on the ensemble-averaged kinetic equation with the statistically averaged non-linear term. The anomalous forces derived from that quasilinear term induce the anomalous particle and heat fluxes. The neoclassical banana-plateau particle and heat fluxes and the bootstrap current are also affected by the fluctuations through the parallel anomalous forces and the modified parallel viscosities. The quasilinear term, the anomalous forces, and the anomalous particle and heat fluxes are evaluated from the fluctuating part of the drift kinetic equation. The averaged drift kinetic equation with the quasilinear term is solved for the plateau regime to derive the parallel viscosities modified by the fluctuations. The entropy production rate due to the anomalous transport processes is formulated and used to identify conjugate pairs of the anomalous fluxes and forces, which are connected by the matrix with the Onsager symmetry.

Keywords: neoclassical transport, anomalous transport, electrostatic turbulence, drift kinetic equation

## I. INTRODUCTION

Extensive theoretical and experimental studies on transport processes of magnetically confined plasmas have been performed over many years since it is crucially important to understand the transport rates for realizing controlled nuclear fusion. The neoclassical transport theory<sup>1-3</sup> is based on Coulomb collisions of particles moving in toroidal magnetic configurations. Particle and energy transport fluxes observed in most fusion devices exceed the predictions of the neoclassical theory and thus are called anomalous transport.<sup>4</sup> The anomalous transport is considered to result from the turbulent fluctuations caused by various instabilities existing in confined plasmas. Most of the theoretical works on the anomalous (or turbulent) plasma transport have been done separately from the neoclassical transport. Shaing<sup>5,6</sup> and Balescu,<sup>7</sup> however, have attempted to unify the neoclassical and anomalous transport theories. The present work treats this same problem from a different approach.

Here we investigate the weakly turbulent regime as in the theories of Shaing and Balescu. Only electrostatic fluctuations in the axisymmetric toroidal system are considered for simplicity. The principal difference between the present theory and the formulation of Shaing and Balescu lies in the way of dividing the physical variables into the average and fluctuating parts. Two-scale separation of spatio-temporally varying quantities is essentially important in treating both the neoclassical and turbulent effects. Variables treated by the neoclassical theory are spatio-temporally smooth and regarded as the ensemble-averaged parts while the fluctuating parts are treated mainly by the anomalous or turbulent transport theory. For example, the averaged flow is incompressible to the lowest order in the neoclassical theory, which is crucially important in deriving the neoclassical banana-plateau transports fluxes. This incompressibility is derived from the continuity equation with the slow temporal variation of the ensemble-average density neglected. On the other hand, the fluctuating part of

the flow can be compressible, which causes the ion sound wave, the ion temperature gradient driven mode and other effects that influence the anomalous transport. Here we emphasize that it matters significantly how the separation of variables into the average and fluctuating parts is defined. In our treatment, strict separation into the ensemble-average and fluctuating parts is done at the level of the kinetic equation. We define the average part of fluid variables such as densities, flow velocities, temperatures and heat flux from the average kinetic distribution function. These definitions are different from those given by ensemble-average of random fluid variables. For example, the flow velocities and temperatures given from the average kinetic distribution function (which we call the ‘kinetic definition’) deviate from the average of the random flow velocities and temperatures given from the random kinetic distribution function (which we call the ‘fluid definition’). In the works of Shaing and Balescu, clear definitions for the average and fluctuating parts are not written although they seem to obey the fluid definition. When the fluid definition is employed, the averaged fluid equations include many nonlinear terms with respect to the fluctuations of the fluid variables. On the other hand, by using the kinetic definition, each fluid equation contains only a single nonlinear term with respect to the fluctuations, and thus the complexities are reduced.

Furthermore, the kinetic definition makes clearer the division of the total transport into the neoclassical and anomalous parts. The averaged kinetic equation is a starting point of the neoclassical part of the theory, where the effects of the fluctuations are contained through the term including the statistically averaged quadratic nonlinearity. We define anomalous particle and heat fluxes from this term quite naturally according to the analogy to the definitions of the classical and neoclassical fluxes. This definition of the anomalous heat flux is different from that of Shaing<sup>5,6</sup> and Balescu.<sup>7</sup> Thus, this statistically averaged nonlinear term plays an essential role in the unification of the neoclassical and anomalous transport theories, and it is calculated by the quasilinear technique in the weakly turbulent regime.

Evaluation of this quasilinear term requires the fluctuating part of the kinetic distribution function. In our formulation, owing to the kinetic definition, the fluctuating part of the kinetic equation coincides with the standard drift or gyrokinetic equation for the plasma turbulence, and the fluctuation of the kinetic distribution function takes a well-known form used for microinstabilities. In contrast Shaing<sup>5,6</sup> and Balescu<sup>7</sup> employ Shaing's ansatz<sup>5-7</sup> for the kinetic distribution and derive the kinetic response to the fluctuations from a drift or gyrokinetic equation including non-diamagnetic flow dependence which we treat by the average part of the drift kinetic equation.

In the present formulation, the modified averaged parallel viscosities are obtained in the plateau regime from the solution of the averaged drift kinetic equation and the fluctuations affect the neoclassical banana-plateau transport fluxes. We find the new relation between the parallel viscosities and the average poloidal flows caused by the electrostatic fluctuations, which are not described by Shaing and Balescu since the quasilinear fluctuation effects on the average kinetic distribution are neglected by them. Physically, the parallel viscosity modification arises from the pressure anisotropy induced by the parallel velocity diffusion produced by the drift wave fluctuations. This anisotropy competes with the neoclassical anisotropy mechanism from the poloidal flow in producing the parallel viscosity.

We also analyze the Onsager relations and the entropy production functional in the anomalous transport system. Shaing argues that the Onsager relation holds for the anomalous transport coefficients<sup>6</sup> while Balescu claims that it does not.<sup>7-9</sup> Here, we emphasize that the Onsager relation is closely related to the entropy production. For example, in the classical process, the entropy production defined in terms of the collision operator is represented by the product of transport fluxes and thermodynamic forces and the Onsager symmetry is valid for the transport matrix which relates these fluxes and forces to each other.<sup>3</sup> Thus, conjugated pairs of the fluxes and forces are determined through the entropy production functional. According to the neoclassical and anomalous transport processes, there exist

two types of entropy production: one is derived from the collision operator as mentioned above, and the other is due to the transfer of energy and momentum through the anomalous processes. Since neither Shaing nor Balescu give clear expressions for the entropy production in the anomalous transport process from which the conjugate pairs of the anomalous transport fluxes and forces should be defined, their arguments on the Onsager relation seem to be incomplete. We define the anomalous entropy production in terms of the anomalous quasilinear term and thus give the conjugate flux-force pairs. The resulting expression of the anomalous entropy production coincides with that derived by Horton.<sup>10</sup> It is also shown that the Onsager symmetry is satisfied by the anomalous transport matrix connecting these anomalous fluxes and forces. However, this anomalous transport relation has the structure which is totally different from the classical or neoclassical one, since the anomalous transport matrix is also a highly nonlinear function of the forces such as the density and temperature gradients through the eigenfrequencies. Furthermore, in order to the complete transport relations, the spectrum of the potential fluctuations remains to be determined. The fluctuation spectrum is given by the nonlinear saturation mechanism although here we only treat the spectrum as given following the works of Shaing and Balescu. We estimate the anomalous transport and the parallel viscosities from the dispersion relation for the ion temperature gradient driven mode.

This work is organized as follows. In Sec.II, basic equations for the density, momentum, energy and energy fluxes are derived from the ensemble-averaged kinetic equation including the effects of the electrostatic fluctuations through the quasilinear term. In Sec.III, the neoclassical and anomalous transport fluxes are defined. In Sec.IV, the average and fluctuating parts of the drift kinetic equation are shown, which give the bases for the neoclassical and anomalous parts of our theory, respectively. In Sec.V, we derive the entropy production in the anomalous transport process and show the Onsager relation between the conjugate pairs of the anomalous transport fluxes and forces. In Sec.VI, the averaged drift kinetic equation

with the quasilinear term is solved for the plateau regime to give the average parallel viscosities. There, estimates are given for the anomalous transport and parallel viscosities from the dispersion relation for the ion temperature gradient driven mode. Finally, conclusions and discussion are given in Sec. VIII.

## II. BASIC EQUATIONS

We start from an ensemble-averaged kinetic equation for species  $a$ :

$$\frac{\partial f_a}{\partial t} + \mathbf{v} \cdot \nabla f_a + \frac{e_a}{m_a} \left( \mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} \right) \cdot \frac{\partial f_a}{\partial \mathbf{v}} = C_a + \mathcal{D}_a \quad (1)$$

where  $C_a$  is a collision term and  $\mathcal{D}_a$  is fluctuation-averaged nonlinear term defined by

$$\mathcal{D}_a = -\frac{e_a}{m_a} \left\langle \hat{\mathbf{E}} \cdot \frac{\partial \hat{f}_a}{\partial \mathbf{v}} \right\rangle_{ens} \quad (2)$$

$$\hat{\mathbf{E}} = -\nabla \hat{\phi} \quad (3)$$

Here  $\langle \cdot \rangle_{ens}$  denotes the ensemble average and we divided the distribution function (the electric field) into the ensemble-averaged part  $f_a(\mathbf{E})$  and the fluctuating part  $\hat{f}_a(\hat{\mathbf{E}})$ . Taking the moments of the kinetic equation yields the following fluid equations. The continuity equation:

$$\frac{\partial n_a}{\partial t} + \nabla \cdot (n_a \mathbf{u}_a) = 0. \quad (4)$$

The momentum balance equation:

$$m_a n_a \left( \frac{\partial}{\partial t} + \mathbf{u}_a \cdot \nabla \right) \mathbf{u}_a = n_a e_a \left( \mathbf{E} + \frac{1}{c} \mathbf{u}_a \times \mathbf{B} \right) - \nabla p_a - \nabla \cdot \boldsymbol{\pi}_a + \mathbf{F}_{a1} + \mathbf{K}_{a1} \quad (5)$$

The energy balance equation:

$$\frac{3}{2} \frac{\partial p_a}{\partial t} = -\nabla \cdot \left( \mathbf{q}_a + \frac{5}{2} p_a \mathbf{u}_a + \boldsymbol{\pi}_a \cdot \mathbf{u}_a \right) + \mathbf{u}_a \cdot \nabla p_a + \mathbf{u}_a \cdot (\nabla \cdot \boldsymbol{\pi}_a) + Q_a + H_a. \quad (6)$$

The energy flux equation:

$$\begin{aligned} m_a n_a \frac{\partial \mathbf{Q}_a}{\partial t} &= \frac{e_a}{m_a} \left[ \mathbf{E} \cdot \left( \frac{5}{2} p_a \mathbf{1} + \boldsymbol{\pi}_a + m_a n_a \mathbf{u}_a \mathbf{u}_a \right) + \frac{1}{c} \mathbf{Q}_a \times \mathbf{B} \right] \\ &+ \frac{T_a}{m_a} \left[ \frac{5}{2} (\mathbf{F}_{a1} + \mathbf{K}_{a1}) + (\mathbf{F}_{a2} + \mathbf{K}_{a2}) \right] - \nabla \cdot \mathbf{r}_a \end{aligned} \quad (7)$$



Here  $\mathbf{F}_{a1}$ ,  $Q_a$ , and  $\mathbf{F}_{a2}$  represents the collisional generation rates of momentum, heat, and heat flux, respectively, which are defined in Ref. 2. The density  $n_a$ , pressure  $p_a$ , flow velocity  $\mathbf{u}_a$ , heat flux  $\mathbf{q}_a$ , total energy flux  $\mathbf{Q}_a$ , viscosity tensor  $\boldsymbol{\pi}_a$ , and energy weighted stress tensor  $\mathbf{r}_a$  are all defined from the ensemble-averaged distribution function  $f_a$  in the standard way as given in Ref. 2. It should be noted that these ‘kinetic’ definitions of the average fluid variables are different from the conventional ‘fluid’ definitions: for example, the both definitions give the same average density and pressure although the average flow velocity in the latter is defined by the average of the random flow velocity given from the total kinetic distribution and is different from that in the former as

$$\left\langle \frac{\int d^3v (f_a + \hat{f}_a) \mathbf{v}}{\int d^3v (f_a + \hat{f}_a)} \right\rangle_{ens} \neq \frac{\int d^3v f_a \mathbf{v}}{\int d^3v f_a} \equiv \mathbf{u}_a \quad (8)$$

and similarly the different average temperatures are given depending on the definitions. By our kinetic definitions, the above fluid equations contain the nonlinear terms with respect to the kinetic fluctuations, which are all derived from  $\mathcal{D}_a$ , while, when the fluid definitions were used, there are large numbers of nonlinear terms with respect to the fluctuations at each level of the averaged fluid equations. For example, the averaged momentum balance equation is written in the fluid definition as

$$\begin{aligned} & m_a \frac{\partial}{\partial t} (n_a \mathbf{u}_a + \langle \hat{n}_a \hat{\mathbf{u}}_a \rangle_{ens}) \\ & + m_a \nabla \cdot (n_a \mathbf{u}_a \mathbf{u}_a + n_a \langle \hat{\mathbf{u}}_a \hat{\mathbf{u}}_a \rangle_{ens} + \langle \hat{n}_a \hat{\mathbf{u}}_a \rangle_{ens} \mathbf{u}_a + \mathbf{u}_a \langle \hat{n}_a \hat{\mathbf{u}}_a \rangle_{ens} + \langle \hat{n}_a \hat{\mathbf{u}}_a \hat{\mathbf{u}}_a \rangle_{ens}) \\ & = n_a e_a \left( \mathbf{E} + \frac{1}{c} \mathbf{u}_a \times \mathbf{B} \right) - \nabla p_a - \nabla \cdot \boldsymbol{\pi}_a + \mathbf{F}_{a1} + e_a \langle \hat{n}_a \hat{\mathbf{E}} \rangle_{ens} + \frac{e_a}{c} \langle \hat{n}_a \hat{\mathbf{u}}_a \rangle_{ens} \times \mathbf{B} \end{aligned} \quad (9)$$

where all the random fluid variables are defined from the total distribution function  $f_a + \hat{f}_a$  and divided into the average  $(\mathbf{u}_a, \dots)$  and fluctuating parts  $(\hat{\mathbf{u}}_a, \dots)$ .

The heat generation rate and forces resulting from the fluctuation term  $\mathcal{D}_a$  are given by

$$\begin{aligned}
H_a &= \int d^3v \mathcal{D}_a \frac{1}{2} m_a (\mathbf{v} - \mathbf{u}_a)^2 = e_a \langle (\hat{\Gamma}_a - \hat{n}_a \mathbf{u}_a) \cdot \hat{\mathbf{E}} \rangle_{ens} \\
\mathbf{K}_{a1} &= \int d^3v \mathcal{D}_a m_a \mathbf{v} = e_a \langle \hat{n}_a \hat{\mathbf{E}} \rangle_{ens} \\
\mathbf{K}_{a2} &= \int d^3v \mathcal{D}_a m_a \mathbf{v} \left( \frac{m_a v^2}{2T_a} - \frac{5}{2} \right) \\
&= \frac{e_a}{T_a} \left\langle \frac{5}{2} (\hat{p}_a - \hat{n}_a T_a) \hat{\mathbf{E}} + \hat{\boldsymbol{\pi}}_a \cdot \hat{\mathbf{E}} \right\rangle_{ens}
\end{aligned} \tag{10}$$

where we defined the fluctuating density, particle flux and pressure (scalar and stress parts) as

$$\begin{aligned}
\hat{n}_a &= \int d^3v \hat{f}_a \\
\hat{\Gamma}_a &= \int d^3v \hat{f}_a \mathbf{v} \\
\hat{p}_a &= \frac{1}{3} \int d^3v \hat{f}_a m_a v^2 \\
\hat{\boldsymbol{\pi}}_a &= \int d^3v \hat{f}_a m_a \left( \mathbf{v} \mathbf{v} - \frac{1}{3} v^2 \mathbf{1} \right)
\end{aligned} \tag{11}$$

Let us define an ordering parameter  $\Delta$  for the fluctuating variables as

$$\frac{\hat{f}_a}{f_a} \sim \frac{e_a \hat{\phi}}{T_a} \sim \frac{\hat{n}_a}{n_a} \sim \frac{k_{\parallel}}{k_{\perp}} \sim \Delta \tag{12}$$

where  $k_{\parallel} \sim L^{-1}$  ( $L$ : the scale length of the plasma equilibrium quantities) and  $k_{\perp}$  denotes the parallel and perpendicular wavenumbers of fluctuations, respectively. Then we have

$$\begin{aligned}
\mathbf{K}_{aj\perp} &\sim \Delta \\
\mathbf{K}_{aj\parallel} &\sim \Delta^2 \quad (j = 1, 2).
\end{aligned} \tag{13}$$

Another ordering parameter is a drift-ordering parameter  $\delta$  given by

$$\delta \sim \rho_a / L \tag{14}$$

where  $\rho_a$  is a thermal gyroradius. When  $\Delta \sim \delta$ , we should use the gyrokinetic equation to obtain the gyrophase dependence of  $\hat{f}_a$ . If  $\Delta \gg \delta$ , the lowest-order part of  $\hat{f}_a$  has no gyrophase dependence and is obtained from the drift-kinetic equation. In the latter case,  $\hat{\pi}_a$  has a Chew-Goldberger-Low (CGL) form:

$$\begin{aligned}\hat{\pi}_a &= (\hat{p}_{\parallel a} - \hat{p}_{\perp a}) \left( \mathbf{nn} - \frac{1}{3} \mathbf{1} \right) \\ \hat{p}_{\parallel a} &= \int d^3v \hat{f}_a m_a v_{\parallel}^2 \\ \hat{p}_{\perp a} &= \frac{1}{2} \int d^3v \hat{f}_a m_a v_{\perp}^2\end{aligned}\tag{15}$$

where  $\mathbf{n} = \mathbf{B}/B$ ,  $v_{\parallel} = \mathbf{v} \cdot \mathbf{n}$  and  $\mathbf{v}_{\perp} = \mathbf{v} - v_{\parallel} \mathbf{n}$ .

From the perpendicular components of Eqs. (5) and (7), we have the perpendicular classical and neoclassical fluxes of particles and heat of  $\mathcal{O}(\delta^2)$  and the anomalous perpendicular fluxes driven by  $\mathbf{K}_{a,j\perp}$  of  $\mathcal{O}(\delta\Delta)$ . In the parallel components of Eqs. (5) and (7), forces such as  $(\nabla \cdot \boldsymbol{\pi}_a)_{\parallel}$  and  $(\nabla \cdot \mathbf{r}_a)_{\parallel}$  are  $\mathcal{O}(\delta)$  while the parallel forces  $\mathbf{K}_{a,j\parallel}$  are  $\mathcal{O}(\Delta^2)$ . Thus, if we take  $\Delta \sim \delta$ , the order of the anomalous perpendicular fluxes is equal to that of classical and neoclassical perpendicular fluxes, and the fluctuation effects due to  $\mathbf{K}_{a,j\parallel}$  on the neoclassical banana-plateau fluxes and the bootstrap current are neglected. Hereafter, we employ the ordering  $\Delta \sim \delta^{1/2} \gg \delta$ . Then the anomalous perpendicular fluxes exceed the neoclassical ones and besides the banana-plateau fluxes and the bootstrap current are significantly affected by the fluctuations as shown in the following sections.

### III. NEOCLASSICAL AND ANOMALOUS FLUXES

In axisymmetric systems such as tokamaks, the magnetic field is given by

$$\mathbf{B} = I \nabla \zeta + \frac{1}{2\pi} \nabla \zeta \times \nabla \chi\tag{16}$$

where  $\zeta$  denotes the toroidal angle,  $\chi$  the poloidal magnetic flux and  $I$  the covariant toroidal component of the magnetic field which is a flux surface quantity due to no radial current

condition  $\nabla \chi \cdot \nabla \times \mathbf{B} = 0$ . Hereafter, we assume that the ensemble-averaged distribution function  $f_a$  is independent of the toroidal angle  $\zeta$  and that all the average fluid variables defined from  $f_a$  and the average electric field  $\mathbf{E}$  are axisymmetric.

The lowest-order parts of the continuity equation (4) and the energy balance equation (6) show that the divergence of the lowest-order flows  $\mathbf{u}_a$  and  $\mathbf{q}_a$  vanishes. Using this zero-divergence constraint and the lowest-order parallel components of Eqs. (5) and (7) with the friction-flow relations,<sup>2,3</sup> we have the relations:

$$\begin{aligned} \left[ \begin{array}{c} -\frac{1}{en_e} \langle B j_{\parallel} \rangle \\ \frac{2}{5} p_e^{-1} \langle B q_{\parallel e} \rangle \end{array} \right] &= \langle B^2 \rangle \left[ \begin{array}{c} u_{e\theta} - u_{i\theta} \\ \frac{2}{5} \frac{q_{e\theta}}{p_e} \end{array} \right] + \left[ \begin{array}{c} \langle B(V_{1e} - V_{1i}) \rangle \\ \langle B V_{2e} \rangle \end{array} \right] \\ &= -\frac{\tau_e}{n_e m_e} \left[ \begin{array}{cc} \tilde{\sigma}_{\parallel} & -\sqrt{\frac{2}{5}} \tilde{\alpha}_{\parallel} \\ -\sqrt{\frac{2}{5}} \tilde{\alpha}_{\parallel} & \frac{2}{5} \tilde{\kappa}_{\parallel}^e \end{array} \right] \left[ \begin{array}{c} n_e e \langle B E_{\parallel}^{(A)} \rangle - \langle \mathbf{B} \cdot \mathbf{K}_{e1} \rangle + \langle \mathbf{B} \cdot \nabla \cdot \boldsymbol{\pi}_e \rangle \\ -\langle \mathbf{B} \cdot \mathbf{K}_{e2} \rangle + \langle \mathbf{B} \cdot \nabla \cdot \boldsymbol{\Theta}_e \rangle \end{array} \right] \end{aligned} \quad (17)$$

and

$$\frac{2}{5} p_i^{-1} \langle B q_{\parallel i} \rangle = \frac{2}{5} \frac{q_{i\theta}}{p_i} \langle B^2 \rangle + \langle B V_{2i} \rangle = -\frac{2}{5} \tilde{\kappa}_{\parallel}^i \frac{\tau_i}{n_i m_i} (-\langle \mathbf{B} \cdot \mathbf{K}_{i2} \rangle + \langle \mathbf{B} \cdot \nabla \cdot \boldsymbol{\Theta}_i \rangle). \quad (18)$$

where  $\langle \cdot \rangle$  denotes the magnetic flux surface average, and  $\boldsymbol{\Theta}_a = (m_a/T_a)[r_a - \frac{1}{3}(l : r_a)] - \frac{5}{2} \boldsymbol{\pi}_a$ . Here  $u_{a\theta}$  and  $q_{a\theta}$  is the flux functions<sup>2</sup> defined by

$$u_{a\theta}(\psi) \equiv \frac{\mathbf{u}_a \cdot \nabla \theta}{\mathbf{B} \cdot \nabla \theta} \quad q_{a\theta}(\psi) \equiv \frac{\mathbf{q}_a \cdot \nabla \theta}{\mathbf{B} \cdot \nabla \theta} \quad (19)$$

with the poloidal angle  $\theta$  and an arbitrary flux label  $\psi$ . The dimensionless coefficients  $\tilde{\sigma}_{\parallel}$ ,  $\tilde{\alpha}_{\parallel}$ ,  $\tilde{\kappa}_{\parallel}^e$  and  $\tilde{\kappa}_{\parallel}^i$  are given in Ref. 3. The diamagnetic flow contributions  $V_{1a}$  and  $V_{2a}$  are defined by

$$\begin{aligned} V_{1a} &\equiv -\frac{2\pi I}{\chi'} \frac{c T_a}{e_a B} \left( \frac{p'_a}{p_a} + \frac{e_a \Phi'}{T_a} \right) \\ V_{2a} &\equiv -\frac{2\pi I}{\chi'} \frac{c T'_a}{e_a B} \end{aligned} \quad (20)$$

The species summation of the lowest-order parallel component of the momentum balance equation (5) gives

$$\langle \mathbf{B} \cdot \nabla \cdot \boldsymbol{\pi}_e \rangle + \langle \mathbf{B} \cdot \nabla \cdot \boldsymbol{\pi}_i \rangle = 0 \quad (21)$$

where we should note that the species summation of the anomalous forces  $\sum_a \mathbf{K}_{a1}$  vanishes due to the quasineutrality condition  $\sum_a e_a \hat{n}_a = 0$ . The parallel viscosities  $\langle \mathbf{B} \cdot \nabla \cdot \boldsymbol{\pi}_a \rangle$  and  $\langle \mathbf{B} \cdot \nabla \cdot \boldsymbol{\Theta}_a \rangle$  are calculated by solving the ensemble-averaged drift kinetic equation which contains the quasilinear fluctuation term as shown in the following sections. The source of the parallel viscosities consists of two parts : one is the same as in the conventional neoclassical theory<sup>2,3</sup> due to the poloidal flows and the other is the anomalous drive due to the quasilinear fluctuation term. Thus, the parallel viscosities are written as

$$\begin{aligned} \begin{bmatrix} \langle \mathbf{B} \cdot \nabla \cdot \boldsymbol{\pi}_a \rangle \\ \langle \mathbf{B} \cdot \nabla \cdot \boldsymbol{\Theta}_a \rangle \end{bmatrix} &= 3 \langle (\mathbf{n} \cdot \nabla B)^2 \rangle \begin{bmatrix} \mu_{a1} & \mu_{a2} \\ \mu_{a2} & \mu_{a3} \end{bmatrix} \begin{bmatrix} \frac{u_{a\theta}}{5} \\ \frac{2g_{a\theta}}{5p_a} \end{bmatrix} + \begin{bmatrix} Y_{a1} \\ Y_{a2} \end{bmatrix} \\ &= 3 \langle (\mathbf{n} \cdot \nabla B)^2 \rangle \begin{bmatrix} \mu_{a1} & \mu_{a2} \\ \mu_{a2} & \mu_{a3} \end{bmatrix} \begin{bmatrix} \frac{u_{a\theta} - W_{a1}}{5} \\ \frac{2g_{a\theta} - W_{a2}}{5p_a} \end{bmatrix}. \end{aligned} \quad (22)$$

Here the neoclassical viscosity coefficients  $\mu_{aj}$  ( $j = 1, 2, 3$ ) are given in Ref. 2 for the Pfirsch-Schlüter, plateau, and banana regimes, and written in Ref. 3 as

$$3 \langle (\mathbf{n} \cdot \nabla B)^2 \rangle \begin{bmatrix} \mu_{a1} & \mu_{a2} \\ \mu_{a2} & \mu_{a3} \end{bmatrix} = \frac{n_a m_a}{\tau_a} \langle B^2 \rangle \varphi \begin{bmatrix} \mu_{11}^a & \sqrt{\frac{5}{2}} \mu_{13}^a \\ \sqrt{\frac{5}{2}} \mu_{31}^a & \frac{5}{2} \mu_{33}^a \end{bmatrix} \quad (23)$$

for the banana regime, where  $\varphi$  and  $\mu_{jk}^a$  should be replaced with  $\varphi_{a*}$  and  $\lambda_{jk}$  for the plateau regime (see Ref. 3 for the definitions of these coefficients). In Eq. (22),  $Y_{aj}$  ( $j = 1, 2$ ) denote the anomalous parts of the parallel viscosities,  $W_{aj}$  ( $j = 1, 2$ ) represent the shifts of the poloidal flows due to the fluctuations, and they are calculated in the Sec. VI for the plateau regime.

Taking the flux surface averages of the toroidal components of Eqs. (5) and (7) yields the flux-surface-averaged radial particle and heat fluxes up to  $\mathcal{O}(\delta^2)$  as

$$\begin{aligned} \langle \Gamma_a \cdot \nabla \psi \rangle &= \langle \Gamma_a \cdot \nabla \psi \rangle_d + \langle \Gamma_a \cdot \nabla \psi \rangle_{PS} + \langle \Gamma_a \cdot \nabla \psi \rangle_{bp} \\ &+ \frac{2\pi I}{\chi'} n_a c \left\langle \frac{E_{\parallel}^{(A)}}{B} \left( \frac{B^2}{\langle B^2 \rangle} - 1 \right) \right\rangle + \left\langle \frac{n_a c}{B} \mathbf{E}^{(A)} \times \mathbf{n} \cdot \nabla \psi \right\rangle \\ &+ \langle \Gamma_a \cdot \nabla \psi \rangle_{PS}^{\text{anom}} + \langle \Gamma_a \cdot \nabla \psi \rangle^{\text{anom}} \end{aligned}$$

$$\begin{aligned}
\langle \mathbf{q}_a \cdot \nabla \psi \rangle &= \langle \mathbf{q}_a \cdot \nabla \psi \rangle_{cl} + \langle \mathbf{q}_a \cdot \nabla \psi \rangle_{PS} + \langle \mathbf{q}_a \cdot \nabla \psi \rangle_{bp} \\
&+ \langle \mathbf{q}_a \cdot \nabla \psi \rangle_{PS}^{anom} + \langle \mathbf{q}_a \cdot \nabla \psi \rangle^{anom}
\end{aligned} \tag{24}$$

where the first three in the right-hand sides represent the classical, Pfirsch-Schlüter and banana-plateau fluxes while the last two are the anomalous contributions defined later. The classical and Pfirsch-Schlüter fluxes are given in terms of the pressure and temperature gradients in the same way as in the case of no fluctuations and their transport relations are written in Refs. 1-3. However, the fluctuation effects appear in the anomalous fluxes as well as in the banana-plateau transport relations as shown later.

Anomalous terms  $\langle \mathbf{B} \cdot \mathbf{K}_{a_j} \rangle$  and  $W_{a_j}$  (or  $Y_{a_j}$ ) are incorporated into the neoclassical framework by defining the modified forces and poloidal flows as

$$\begin{aligned}
\langle B(V_{1e} - V_{1i}) \rangle^{(m)} &= \langle B(V_{1e} - V_{1i}) \rangle + \sqrt{\frac{2}{5}} \tilde{\alpha}_{\parallel} \frac{\tau_e}{n_e m_e} \langle \mathbf{B} \cdot \mathbf{K}_{e2} \rangle + \langle B^2 \rangle (W_{e1} - W_{i1}) \\
\langle BV_{2e} \rangle^{(m)} &= \langle BV_{2e} \rangle - \frac{2}{5} \tilde{\kappa}_{\parallel}^e \frac{\tau_e}{n_e m_e} \langle \mathbf{B} \cdot \mathbf{K}_{e2} \rangle + \langle B^2 \rangle W_{e2} \\
\langle BV_{2i} \rangle^{(m)} &= \langle BV_{2i} \rangle - \frac{2}{5} \tilde{\kappa}_{\parallel}^i \frac{\tau_i}{n_i m_i} \langle \mathbf{B} \cdot \mathbf{K}_{i2} \rangle + \langle B^2 \rangle W_{i2} \\
\langle BE_{\parallel}^{(A)} \rangle^{(m)} &= \langle BE_{\parallel}^{(A)} \rangle - (n_e e)^{-1} \langle \mathbf{B} \cdot \mathbf{K}_{e1} \rangle \\
u_{a\theta}^{(m)} &= u_{a\theta} - W_{a1} \\
\frac{2}{5} \frac{q_{a\theta}^{(m)}}{p_a} &= \frac{2}{5} \frac{q_{a\theta}}{p_a} - W_{a2}
\end{aligned} \tag{25}$$

If we use the above modified forces and poloidal flows, the neoclassical expressions for the parallel viscosities and accordingly for the banana-plateau particle and heat fluxes which are valid even in the presence of fluctuations. From Eq. (17), we can obtain the contributions of fluctuations to the parallel current. Then, from Eqs. (17), (18), (21), (22) and (23), we

obtain the banana-plateau transport equations as

$$\begin{bmatrix} \langle \Gamma_e \cdot \nabla \psi \rangle_{bp} \\ \langle \mathbf{q}_e \cdot \nabla \psi \rangle_{bp} \\ \langle \mathbf{q}_i \cdot \nabla \psi \rangle_{bp} \\ \langle B^2 \rangle^{-1/2} \langle B j_{\parallel} \rangle^{(m)} \end{bmatrix} = \begin{bmatrix} L_{11}^{bp} & L_{12}^{bp} & L_{13}^{bp} & L_{14}^{bp} \\ L_{21}^{bp} & L_{22}^{bp} & L_{23}^{bp} & L_{24}^{bp} \\ L_{31}^{bp} & L_{32}^{bp} & L_{33}^{bp} & L_{34}^{bp} \\ L_{41}^{bp} & L_{42}^{bp} & L_{43}^{bp} & L_{44}^{bp} \end{bmatrix} \begin{bmatrix} -n_e^{-1} (dP/d\psi)^{(m)} \\ -T_e^{-1} (dT_e/d\psi)^{(m)} \\ -T_i^{-1} (dT_i/d\psi)^{(m)} \\ \langle B^2 \rangle^{-1/2} \langle B E_{\parallel}^{(A)} \rangle^{(m)} \end{bmatrix} \quad (26)$$

where the fluxes in the left-hand side are defined by

$$\begin{aligned} \langle \Gamma_a \cdot \nabla \psi \rangle_{bp} &= -\frac{2\pi I c \langle \mathbf{B} \cdot \nabla \cdot \boldsymbol{\pi}_a \rangle}{\chi' e_a \langle B^2 \rangle} \\ T_a^{-1} \langle \mathbf{q}_a \cdot \nabla \psi \rangle_{bp} &= -\frac{2\pi I c \langle \mathbf{B} \cdot \nabla \cdot \boldsymbol{\Theta}_a \rangle}{\chi' e_a \langle B^2 \rangle} \\ \langle B j_{\parallel} \rangle^{(m)} &= \langle B j_{\parallel} \rangle - \frac{n_e e^2 \tau_e}{m_e} \tilde{\sigma}_{\parallel} \langle B E_{\parallel}^{(A)} \rangle^{(m)} - \sqrt{\frac{2}{5}} \tilde{\alpha}_{\parallel} \frac{e \tau_e}{m_e} \langle \mathbf{B} \cdot \mathbf{K}_{e2} \rangle \end{aligned} \quad (27)$$

and the modified pressure and temperature gradients are given from Eq. (25) as

$$\begin{aligned} \left( \frac{dP}{d\psi} \right)^{(m)} &= \frac{dP}{d\psi} + \frac{\chi' e}{2\pi I c} \left( \sqrt{\frac{2}{5}} \tilde{\alpha}_{\parallel} \frac{\tau_e}{m_e} \langle \mathbf{B} \cdot \mathbf{K}_{e2} \rangle + n_e \langle B^2 \rangle (W_{e1} - W_{i1}) \right) \\ \left( \frac{dT_a}{d\psi} \right)^{(m)} &= \frac{dT_a}{d\psi} + \frac{\chi' e_a}{2\pi I c} \left( \frac{2}{5} \tilde{\kappa}_{\parallel}^a \frac{\tau_a}{n_a m_a} \langle \mathbf{B} \cdot \mathbf{K}_{a2} \rangle - \langle B^2 \rangle W_{a2} \right). \end{aligned} \quad (28)$$

The transport matrix is given by

$$\begin{aligned} \begin{bmatrix} L_{11}^{bp} & L_{12}^{bp} & L_{13}^{bp} & L_{14}^{bp} \\ L_{21}^{bp} & L_{22}^{bp} & L_{23}^{bp} & L_{24}^{bp} \\ L_{31}^{bp} & L_{32}^{bp} & L_{33}^{bp} & L_{34}^{bp} \\ L_{41}^{bp} & L_{42}^{bp} & L_{43}^{bp} & L_{44}^{bp} \end{bmatrix} &= \frac{n_e \rho_e^2}{2T_e \tau_e} \left( \frac{2\pi I}{\chi'} \right)^2 \varphi \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{\frac{5}{2}} T_e & 0 & 0 \\ 0 & 0 & \sqrt{\frac{5}{2}} \frac{T_e}{Z_i} & 0 \\ 0 & 0 & 0 & -e \frac{\chi'}{2\pi I} |\Omega_e| \tau_e \end{bmatrix} \\ &\times \begin{bmatrix} l_{11}^{ee} & l_{13}^{ee} & l_{13}^{ei} & l_{1E}^e \\ l_{31}^{ee} & l_{33}^{ee} & l_{33}^{ei} & l_{3E}^e \\ l_{31}^{ie} & l_{33}^{ie} & A l_{33}^{ii} & l_{3E}^i \\ l_{E1}^e & l_{E3}^e & l_{E3}^i & l_{EE}^e \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{\frac{5}{2}} T_e & 0 & 0 \\ 0 & 0 & \sqrt{\frac{5}{2}} \frac{T_e}{Z_i} & 0 \\ 0 & 0 & 0 & -e \frac{\chi'}{2\pi I} |\Omega_e| \tau_e \end{bmatrix} \end{aligned} \quad (29)$$

where  $\rho_a = v_{Ta}/|\Omega_a|$  and  $\Omega_a = e_a \langle B^2 \rangle^{1/2} / m_a c$ . The dimensionless coefficients  $l_{11}^{ee}, \dots$  and  $A$  are defined in Ref. 3. From Eqs. (18), (22), (23) and (25), the parallel ion viscosity is written

as

$$\begin{aligned} \langle \mathbf{B} \cdot \nabla \cdot \boldsymbol{\pi}_i \rangle &= \frac{n_i m_i}{\tau_i} \langle B^2 \rangle \varphi \left[ \left( \mu_{11}^i - (\mu_{13}^i)^2 \tilde{\kappa}_{\parallel}^i \frac{\varphi}{1 + \tilde{\kappa}_{\parallel}^i \mu_{33}^i \varphi} \right) u_{i\theta}^{(m)} \right. \\ &\quad \left. - \sqrt{\frac{5}{2}} \frac{\mu_{13}^i}{1 + \tilde{\kappa}_{\parallel}^i \mu_{33}^i \varphi} \frac{\langle B V_{2i} \rangle^{(m)}}{\langle B^2 \rangle} \right] \end{aligned} \quad (30)$$

The Pfirsch-Schlüter type fluxes induced by the parallel fluctuation forces are given by

$$\begin{aligned} \langle \boldsymbol{\Gamma}_a \cdot \nabla \psi \rangle_{PS}^{\text{anom}} &= -\frac{2\pi I}{\chi'} \left\langle \frac{\mathbf{n} \cdot \mathbf{K}_{a1}}{m_a \Omega_a} \left( 1 - \frac{B^2}{\langle B^2 \rangle} \right) \right\rangle \\ T_a^{-1} \langle \mathbf{q}_a \cdot \nabla \psi \rangle_{PS}^{\text{anom}} &= -\frac{2\pi I}{\chi'} \left\langle \frac{\mathbf{n} \cdot \mathbf{K}_{a2}}{m_a \Omega_a} \left( 1 - \frac{B^2}{\langle B^2 \rangle} \right) \right\rangle. \end{aligned} \quad (31)$$

The anomalous transport induced by the perpendicular fluctuation forces are given by

$$\begin{aligned} \langle \boldsymbol{\Gamma}_a \cdot \nabla \psi \rangle^{\text{anom}} &= \left\langle \frac{\nabla \psi}{m_a \Omega_a} \cdot (\mathbf{K}_{a1} \times \mathbf{n}) \right\rangle \\ T_a^{-1} \langle \mathbf{q}_a \cdot \nabla \psi \rangle^{\text{anom}} &= \left\langle \frac{\nabla \psi}{m_a \Omega_a} \cdot (\mathbf{K}_{a2} \times \mathbf{n}) \right\rangle. \end{aligned} \quad (32)$$

The anomalous fluxes in Eq. (31) are negligibly smaller than those in Eq. (32) since  $k_{\parallel} \ll k_{\perp}$  is assumed.

In the following sections,  $\mathbf{K}_{aj}$  and  $W_{aj}$  are calculated so that we can evaluate all the anomalous contribution to the total transport by using Eqs. (26), (31), and (32). Before proceeding to that, we here discuss the general properties of the anomalous transport matrices from the linear thermodynamic point of view. We have shown in Eq. (26) the anomalous effects on the banana-plateau transport by modifying the forces and the parallel current. However, it is useful to express the anomaly in terms of the modification of the transport coefficients rather than the forces so that the various contributions to the total transport can be represented by the corresponding transport matrices. In order to obtain such expressions, we need to express  $\mathbf{K}_{aj}$  and  $W_{aj}$  in terms of the thermodynamic forces. Since the density and temperature gradients are the causes of the fluctuations,  $\mathbf{K}_{aj}$ ,  $W_{aj}$  and accordingly all types



of the anomalous transport fluxes are nonlinear functions of them as shown in the following sections, although we here assume that the  $K_{aj}$  and  $W_{aj}$  are approximated as their linear combinations. Noting that the density gradient is also given by the pressure and temperature gradients as

$$\nabla \ln n_e = \nabla \ln n_i = \nabla \ln P - \frac{1}{1 + T_i/Z_i T_e} \left( \nabla \ln T_e + \frac{T_i}{Z_i T_e} \nabla \ln T_i \right) \quad (33)$$

the  $K_{aj}$  and  $W_{aj}$  are assumed to be written as

$$\begin{bmatrix} \mathbf{K}_{aj} \\ W_{aj} \end{bmatrix} = \begin{bmatrix} \mathbf{f}_{aj}^{(1)} & \mathbf{f}_{aj}^{(2)} & \mathbf{f}_{aj}^{(3)} \\ w_{aj}^{(1)} & w_{aj}^{(2)} & w_{aj}^{(3)} \end{bmatrix} \begin{bmatrix} -n_e^{-1} dP/d\psi \\ -T_e^{-1} dT_e/d\psi \\ -T_i^{-1} dT_i/d\psi \end{bmatrix} \quad (a = e, i; j = 1, 2). \quad (34)$$

(The linear thermodynamic form of the anomalous transport will be also discussed in Sec. V.) Then, the deviation of the banana-plateau transport from its non-turbulent expression is expressed as

$$\begin{bmatrix} \langle \Gamma_e \cdot \nabla \psi \rangle_{bp}^{anom} \\ \langle \mathbf{q}_e \cdot \nabla \psi \rangle_{bp}^{anom} \\ \langle \mathbf{q}_i \cdot \nabla \psi \rangle_{bp}^{anom} \\ \langle B^2 \rangle^{-1/2} \langle B_{j\parallel} \rangle^{anom} \end{bmatrix} = \begin{bmatrix} L_{11}^{bpan} & L_{12}^{bpan} & L_{13}^{bpan} & 0 \\ L_{21}^{bpan} & L_{22}^{bpan} & L_{23}^{bpan} & 0 \\ L_{31}^{bpan} & L_{32}^{bpan} & L_{33}^{bpan} & 0 \\ L_{41}^{bpan} & L_{42}^{bpan} & L_{43}^{bpan} & 0 \end{bmatrix} \begin{bmatrix} -n_e^{-1} dP/d\psi \\ -T_e^{-1} dT_e/d\psi \\ -T_i^{-1} dT_i/d\psi \\ \langle B^2 \rangle^{-1/2} \langle BE_{\parallel}^{(A)} \rangle \end{bmatrix} \quad (35)$$

where the transport matrix is given by

$$\begin{bmatrix} L_{11}^{bpan} & L_{12}^{bpan} & L_{13}^{bpan} \\ L_{21}^{bpan} & L_{22}^{bpan} & L_{23}^{bpan} \\ L_{31}^{bpan} & L_{32}^{bpan} & L_{33}^{bpan} \\ L_{41}^{bpan} & L_{42}^{bpan} & L_{43}^{bpan} \end{bmatrix} = \begin{bmatrix} L_{11}^{bp} & L_{12}^{bp} & L_{13}^{bp} & L_{14}^{bp} \\ L_{21}^{bp} & L_{22}^{bp} & L_{23}^{bp} & L_{24}^{bp} \\ L_{31}^{bp} & L_{32}^{bp} & L_{33}^{bp} & L_{34}^{bp} \\ L_{41}^{bp} & L_{42}^{bp} & L_{43}^{bp} & L_{44}^{bp} \end{bmatrix} \begin{bmatrix} M_{e1}^{(1)} & M_{e1}^{(2)} & M_{e1}^{(3)} \\ M_{e2}^{(1)} & M_{e2}^{(2)} & M_{e2}^{(3)} \\ M_{i2}^{(1)} & M_{i2}^{(2)} & M_{i2}^{(3)} \\ M_E^{(1)} & M_E^{(2)} & M_E^{(3)} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ M_J^{(1)} & M_J^{(2)} & M_J^{(3)} \end{bmatrix} \quad (36)$$

with

$$M_{e1}^{(j)} = -\frac{\chi' e}{2\pi I c} \left( \sqrt{\frac{2}{5}} \tilde{\alpha}_{\parallel} \frac{\tau_e}{n_e m_e} \langle \mathbf{B} \cdot \mathbf{f}_{e2}^{(j)} \rangle + \langle B^2 \rangle (w_{e1}^j - w_{i1}^{(j)}) \right)$$

$$\begin{aligned}
M_{a2}^{(j)} &= -\frac{\chi'}{2\pi I cT_a} \left( \frac{2}{5} \tilde{\kappa}_{\parallel}^a \frac{\tau_a}{n_a m_a} \langle \mathbf{B} \cdot \mathbf{f}_{a2}^{(j)} \rangle - \langle B^2 \rangle w_{a2}^j \right) \\
M_E^{(j)} &= -(n_e e)^{-1} \langle B^2 \rangle^{-1/2} \langle \mathbf{B} \cdot \mathbf{f}_{e1}^{(j)} \rangle \\
M_j^{(j)} &= \frac{e\tau_e}{m_e} \langle B^2 \rangle^{-1/2} (-\tilde{\sigma}_{\parallel} \langle \mathbf{B} \cdot \mathbf{f}_{e1}^{(j)} \rangle + \tilde{\alpha}_{\parallel} \langle \mathbf{B} \cdot \mathbf{f}_{e2}^{(j)} \rangle) \\
&\quad (a = e, i; \quad j = 1, 2, 3). \tag{37}
\end{aligned}$$

Here the Onsager symmetry and the positive definiteness are no longer ensured for the transport matrix in Eq. (36) even if that is restricted to the  $3 \times 3$  (or  $2 \times 2$ ) matrix  $[L_{jk}^{bpan}]$  with  $j, k = 1, 2, 3$  (or  $j, k = 1, 2$ ). Similarly, the linear thermodynamic form of the anomalous particle and heat fluxes given in Eqs. (31) and (32) can be written as

$$\begin{bmatrix} \langle \Gamma_e \cdot \nabla \psi \rangle_{PS}^{anom} \\ \langle \mathbf{q}_e \cdot \nabla \psi \rangle_{PS}^{anom} \\ \langle \mathbf{q}_i \cdot \nabla \psi \rangle_{PS}^{anom} \end{bmatrix} = \begin{bmatrix} L_{11}^{anPS} & L_{12}^{anPS} & L_{13}^{anPS} \\ L_{21}^{anPS} & L_{22}^{anPS} & L_{23}^{anPS} \\ L_{31}^{anPS} & L_{32}^{anPS} & L_{33}^{anPS} \end{bmatrix} \begin{bmatrix} -n_e^{-1} dP/d\psi \\ -T_e^{-1} dT_e/d\psi \\ -T_i^{-1} dT_i/d\psi \end{bmatrix} \tag{38}$$

$$\begin{bmatrix} \langle \Gamma_e \cdot \nabla \psi \rangle^{anom} \\ \langle \mathbf{q}_e \cdot \nabla \psi \rangle^{anom} \\ \langle \mathbf{q}_i \cdot \nabla \psi \rangle^{anom} \end{bmatrix} = \begin{bmatrix} L_{11}^{an} & L_{12}^{an} & L_{13}^{an} \\ L_{21}^{an} & L_{22}^{an} & L_{23}^{an} \\ L_{31}^{an} & L_{32}^{an} & L_{33}^{an} \end{bmatrix} \begin{bmatrix} -n_e^{-1} dP/d\psi \\ -T_e^{-1} dT_e/d\psi \\ -T_i^{-1} dT_i/d\psi \end{bmatrix} \tag{39}$$

where the transport coefficients are given by

$$\begin{aligned}
L_{1j}^{anPS} &= -\frac{2\pi I}{\chi'} \left\langle \frac{\mathbf{n} \cdot \mathbf{f}_{e1}^{(j)}}{m_e \Omega_e} \left( 1 - \frac{B^2}{\langle B^2 \rangle} \right) \right\rangle \\
L_{2j}^{anPS} &= -\frac{2\pi I T_e}{\chi'} \left\langle \frac{\mathbf{n} \cdot \mathbf{f}_{e2}^{(j)}}{m_e \Omega_e} \left( 1 - \frac{B^2}{\langle B^2 \rangle} \right) \right\rangle \\
L_{3j}^{anPS} &= -\frac{2\pi I T_i}{\chi'} \left\langle \frac{\mathbf{n} \cdot \mathbf{f}_{i2}^{(j)}}{m_i \Omega_i} \left( 1 - \frac{B^2}{\langle B^2 \rangle} \right) \right\rangle \quad (j = 1, 2, 3) \tag{40}
\end{aligned}$$

$$\begin{aligned}
L_{1j}^{an} &= \left\langle \frac{\nabla \psi}{m_e \Omega_e} \cdot (\mathbf{f}_{e1}^{(j)} \times \mathbf{n}) \right\rangle \\
L_{2j}^{an} &= T_e \left\langle \frac{\nabla \psi}{m_e \Omega_e} \cdot (\mathbf{f}_{e2}^{(j)} \times \mathbf{n}) \right\rangle \\
L_{3j}^{an} &= T_i \left\langle \frac{\nabla \psi}{m_i \Omega_i} \cdot (\mathbf{f}_{i2}^{(j)} \times \mathbf{n}) \right\rangle \quad (j = 1, 2, 3). \tag{41}
\end{aligned}$$

Generally, these anomalous transport matrices do not satisfy the Onsager-type symmetry and the positive definiteness, and this broken symmetry in the anomalous transport is in agreement with Balescu's argument.<sup>7-9</sup>

Here, it should be noted that the pairs of the thermodynamic forces and the anomalous fluxes employed in Eqs. (35), (38), and (39) are chosen in the same way as in the case of the classical and neoclassical transport, which is because we intended to classify the classical, neoclassical and anomalous fluxes according to the corresponding transport matrices in the linear thermodynamic transport equations. However, there exist qualitative differences between the collision-induced (classical and neoclassical) transport and the turbulence-induced (anomalous) transport. One of them is the intrinsically nonlinear thermodynamic force dependence of the anomalous transport and another remarkable difference is that those two types of transport correspond to different structure of entropy production functional. The inner product of the collision-induced fluxes and the thermodynamic forces causes the collisional entropy production, by which the conjugated pairs of the fluxes and forces are identified. The Onsager symmetry of the classical and neoclassical transport matrices are derived from the self-adjointness of the collision operator. On the contrary, the anomalous transport results in another structure of entropy production.<sup>10</sup> The structure of the anomalous entropy production is given in Sec. V. The products of the anomalous fluxes and forces in Eqs. (35), (38), and (39) do not corresponding to this anomalous entropy production, and those pairs are not conjugate in that sense. The Onsager symmetry and the positive definiteness are not valid for the anomalous transport matrices connecting the conjugated pairs defined by the collision operator  $C_a$  as mentioned above. In Sec. V, for the operator  $\mathcal{D}_a$ , the conjugated pairs of the anomalous fluxes and forces are defined from the anomalous entropy production produced by the resonant wave-particle interactions. For the relation between these fluxes and driving forces, the Onsager symmetry holds.

## IV. DRIFT KINETIC EQUATION WITH ELECTROSTATIC FLUCTUATIONS

The drift kinetic equation<sup>11,12</sup> is given by

$$\frac{\partial \bar{f}_a}{\partial t} + (v_{\parallel} \mathbf{n} + \mathbf{v}_{da}) \cdot \nabla \bar{f}_a + e_a \left[ \frac{\partial \Phi}{\partial t} + (v_{\parallel} \mathbf{n} + \mathbf{v}_{da}) \cdot \mathbf{E}^{(A)} \right] \frac{\partial \bar{f}_a}{\partial E} = C_a(\bar{f}_a) \quad (42)$$

where  $\bar{f}_a$  is a gyroangle-averaged distribution function in the phase space of guiding center variables  $(\mathbf{x}, E, \mu)$  and  $\mathbf{v}_{da}$  is the guiding center drift velocity. The potential  $\Phi$  consists of the time-independent ensemble average part  $\Phi_0 = \langle \Phi \rangle_{ens}$  and the fluctuating part  $\hat{\phi}$ :

$$\Phi = \Phi_0 + \hat{\phi}, \quad \frac{\partial \Phi_0}{\partial t} = 0. \quad (43)$$

Then the energy variable  $E$  is divided into the average and fluctuating parts:

$$E = E_0 + e_a \hat{\phi}$$

$$E_0 = \frac{1}{2} m_a v^2 + e_a \Phi_0. \quad (44)$$

Hereafter let us use  $(\mathbf{x}, E_0, \mu)$  as independent guiding center variables instead of  $(\mathbf{x}, E, \mu)$ .

Then the drift kinetic equation is rewritten for  $\bar{f}_a(\mathbf{x}, E_0, \mu)$  as

$$\frac{\partial \bar{f}_a}{\partial t} + (v_{\parallel} \mathbf{n} + \mathbf{v}_{da0} + \hat{\mathbf{v}}_E) \cdot \nabla \bar{f}_a + e_a (v_{\parallel} \mathbf{n} + \mathbf{v}_{da0} + \hat{\mathbf{v}}_E) \cdot (\hat{\mathbf{E}} + \mathbf{E}^{(A)}) \frac{\partial \bar{f}_a}{\partial E_0} = C_a(\bar{f}_a) \quad (45)$$

where

$$\hat{\mathbf{E}} = -\nabla \hat{\phi}, \quad \hat{\mathbf{v}}_E = \frac{c}{B} \hat{\mathbf{E}} \times \mathbf{n}.$$

$$\mathbf{v}_{da0} = \frac{c T_a}{e_a B} \mathbf{n} \times \left( \frac{e_a}{T_a} \nabla \Phi_0 + x_{\perp a}^2 \nabla \ln B + 2 x_{\parallel a}^2 \mathbf{n} \cdot \nabla \mathbf{n} \right) \quad (46)$$

with  $x_{\perp} = v_{\perp} / v_{Ta}$ ,  $x_{\parallel} = v_{\parallel} / v_{Ta}$  and  $v_{Ta} = (2T_a / m_a)^{1/2}$ . Due to the electrostatic fluctuation terms, the solution  $\bar{f}_a$  of the drift kinetic equation also includes average and fluctuating parts:

$$\bar{f}_a = \langle \bar{f}_a \rangle_{ens} + \hat{\bar{f}}_a \quad (47)$$

Taking an ensemble average of the drift kinetic equation and retaining the terms up to  $\mathcal{O}(\delta)$ , we obtain

$$(\mathbf{v}_{\parallel} \mathbf{n} + \mathbf{v}_{da0}) \cdot \nabla \langle \bar{f}_a \rangle_{ens} + e_a v_{\parallel} E_{\parallel}^{(A)} \frac{\partial \langle \bar{f}_a \rangle_{ens}}{\partial E_0} = \langle C_a(\bar{f}_a) \rangle_{ens} + \bar{\mathcal{D}}_a \quad (48)$$

where

$$\bar{\mathcal{D}}_a = -e_a v_{\parallel} \left\langle \hat{E}_{\parallel} \frac{\partial \hat{f}_a}{\partial E_0} \right\rangle_{ens} \quad (49)$$

To the lowest order, we have

$$v_{\parallel} \mathbf{n} \cdot \nabla \langle \bar{f}_{a0} \rangle_{ens} = C_a(\langle \bar{f}_{a0} \rangle_{ens}). \quad (50)$$

As a solution of the lowest order equation, we use the Maxwellian distribution function

$$\bar{f}_{a0} = \langle \bar{f}_{a0} \rangle_{ens} = f_{aM} = \pi^{-3/2} n_a v_{Ta}^{-3} e^{-x_a^2} = n_a \left( \frac{m_a}{2\pi T_a} \right)^{3/2} \exp\left( \frac{e_a \Phi_0 - E_0}{T_a} \right) \quad (51)$$

where  $x_a = v/v_{Ta}$ . We should note that the lowest order distribution function  $\bar{f}_{a0}$  does not include a fluctuating part, i.e.,  $\hat{f}_{a0} = 0$ . To  $\mathcal{O}(\delta)$ , we have

$$v_{\parallel} \mathbf{n} \cdot \nabla \langle \bar{f}_{a1} \rangle_{ens} + \mathbf{v}_{da0} \cdot \nabla \bar{f}_{a0} + e_a v_{\parallel} E_{\parallel}^{(A)} \frac{\partial \bar{f}_{a0}}{\partial E_0} = C_a(\langle \bar{f}_{a1} \rangle_{ens}) + \bar{\mathcal{D}}_a. \quad (52)$$

In the right-hand side, we use the linearized collision operator and

$$\bar{\mathcal{D}}_a = -e_a v_{\parallel} \left\langle \hat{E}_{\parallel} \frac{\partial \hat{f}_{a1}}{\partial E_0} \right\rangle_{ens}. \quad (53)$$

The linearized drift kinetic equation for the fluctuating part  $\hat{f}_{a1}$  is written as

$$\frac{\partial \hat{f}_{a1}}{\partial t} + (v_{\parallel} \mathbf{n} + \mathbf{v}_{da0}) \cdot \nabla \hat{f}_{a1} + \hat{\mathbf{v}}_E \cdot \nabla \bar{f}_{a0} + e_a (v_{\parallel} \mathbf{n} + \mathbf{v}_{da0}) \cdot \hat{\mathbf{E}} \frac{\partial \bar{f}_{a0}}{\partial E_0} = C_a(\hat{f}_{a1}) \quad (54)$$

where higher order terms than  $\mathcal{O}(\delta)$  are neglected. Defining the nonadiabatic part  $\hat{h}_a$  by

$$\hat{f}_{a1} = -\frac{e_a \hat{\phi}}{T_a} f_{aM} + \hat{h}_a \quad (55)$$

we obtain

$$\left[ \frac{\partial}{\partial t} + (v_{\parallel} \mathbf{n} + \mathbf{v}_{da0}) \cdot \nabla + C_a \right] \hat{h}_a = \left( \frac{\partial}{\partial t} + \frac{c T_a}{e_a B} \mathbf{n} \times \nabla \ln f_{aM} \cdot \nabla \right) \frac{e_a \hat{\phi}}{T_a} f_{aM} \quad (56)$$

which has the well known form of the gyrokinetic equation<sup>11,13,14</sup> in the zero gyroradius limit. Using the Fourier representation for the rapid spatio-temporal variation of the fluctuating quantities as

$$\begin{bmatrix} \widehat{\phi} \\ \widehat{h}_a \end{bmatrix} = \sum_{\mathbf{k}} \begin{bmatrix} \widehat{\phi}_{\mathbf{k}} \\ \widehat{h}_{a\mathbf{k}} \end{bmatrix} \exp[i\mathbf{k} \cdot \mathbf{x} - i\omega_{\mathbf{k}}t] \quad (57)$$

and neglecting the bounce motion of the trapped ions that is justified in the plateau regime, we have

$$\widehat{h}_{a\mathbf{k}} = \frac{\omega - \omega_E - \omega_{*a}^T}{\omega - \omega_E - \omega_{D_a} - k_{\parallel}v_{\parallel} + i\nu_a} \frac{e_a \widehat{\phi}_{\mathbf{k}}}{T_a} f_{aM} \quad (58)$$

where  $C_a(\widehat{h}_{a\mathbf{k}})$  is replaced with  $\nu_a \widehat{h}_{a\mathbf{k}}$  and

$$\begin{aligned} \omega_E &= \mathbf{k} \cdot \frac{c}{B} \mathbf{n} \times \nabla \Phi_0 \\ \omega_{D_a} &= \mathbf{k} \cdot (\mathbf{v}_a \nabla B + \mathbf{v}_{acurv}) = \mathbf{k} \cdot \frac{cT_a}{e_a B} \mathbf{n} \times (x_{\perp a}^2 \nabla \ln B + 2x_{\parallel a}^2 \mathbf{n} \cdot \nabla \mathbf{n}) \\ \omega_{*a}^T &= \omega_{*a} \left[ 1 + \eta_a \left( x_a^2 - \frac{3}{2} \right) \right] \\ \omega_{*a} &= \mathbf{k} \cdot \frac{cT_a}{e_a B} \mathbf{n} \times \nabla \ln n_a \\ \eta_a &= d \ln T_a / d \ln n_a \end{aligned} \quad (59)$$

are used. Using Eqs. (53), (55), (57), and (58) and assuming that  $\nu_a \ll |\omega|$ , we have

$$\begin{aligned} \overline{\mathcal{D}}_a &= -e_a v_{\parallel} \sum_{\mathbf{k}} \text{Real} \left\langle \widehat{E}_{\parallel \mathbf{k}}^* \frac{\partial \widehat{h}_{a\mathbf{k}}}{\partial E_0} \right\rangle_{ens} \\ &= -\pi \sum_{\mathbf{k}} k_{\parallel} v_{\parallel} \frac{e_a^2 \langle |\widehat{\phi}_{\mathbf{k}}|^2 \rangle_{ens}}{T_a} \frac{\partial}{\partial E_0} \left[ (\omega - \omega_E - \omega_{*a}^T) \delta(\omega - \omega_E - \omega_{D_a} - k_{\parallel}v_{\parallel}) f_{aM} \right]. \end{aligned} \quad (60)$$

From Eqs. (10), (11), (55), (57), and (58), we can calculate the anomalous heat generation rate and forces as

$$H_a = \pi \frac{e_a^2}{T_a} \sum_{\mathbf{k}} \langle |\widehat{\phi}_{\mathbf{k}}|^2 \rangle_{ens} \int d^3v f_{aM} \delta(\omega - \omega_E - \omega_{D_a} - k_{\parallel}v_{\parallel}) (\omega - \omega_E - \omega_{*a}^T)$$

$$\begin{aligned}
& \times (k_{\parallel} v_{\parallel} - \omega_E - \omega_{*a} [1 + \eta_a]) \\
\mathbf{K}_{a1} &= \pi \frac{e_a^2}{T_a} \sum_{\mathbf{k}} \langle |\hat{\phi}_{\mathbf{k}}|^2 \rangle_{ens} \int d^3v f_{aM} \delta(\omega - \omega_E - \omega_{Da} - k_{\parallel} v_{\parallel}) (\omega - \omega_E - \omega_{*a}^T) \mathbf{k} \\
\mathbf{K}_{a2} &= \pi \frac{e_a^2}{T_a} \sum_{\mathbf{k}} \langle |\hat{\phi}_{\mathbf{k}}|^2 \rangle_{ens} \int d^3v f_{aM} \delta(\omega - \omega_E - \omega_{Da} - k_{\parallel} v_{\parallel}) (\omega - \omega_E - \omega_{*a}^T) \\
& \times \left[ \left( 3x_{\parallel a}^2 + x_{\perp a}^2 - \frac{5}{2} \right) k_{\parallel} \mathbf{n} + \left( x_{\parallel a}^2 + 2x_{\perp a}^2 - \frac{5}{2} \right) \mathbf{k}_{\perp} \right] \tag{61}
\end{aligned}$$

When the fluctuation spectrum  $\langle |\hat{\phi}_{\mathbf{k}}|^2 \rangle_{ens}$  and the dispersion relation  $\omega = \omega_{\mathbf{k}}$  are given, we can obtain from the above equations the parallel anomalous forces, which are necessary for the determination of the banana-plateau transport in Eq. (26) and the anomalous Pfirsch-Schlüter fluxes in Eq. (31). We can also obtain the anomalous fluxes from the perpendicular anomalous forces as

$$\begin{aligned}
\Gamma_a^{\text{anom}} &= \frac{c}{e_a B} \mathbf{K}_{a1} \times \mathbf{n} \\
\frac{1}{T_a} \mathbf{q}_a^{\text{anom}} &= \frac{c}{e_a B} \mathbf{K}_{a2} \times \mathbf{n}. \tag{62}
\end{aligned}$$

The flux surface averages of the radial components of Eq. (62) were already found in Eq. (32).

## V. ENTROPY PRODUCTION AND CONJUGATE PAIRS OF FLUXES AND FORCES IN ANOMALOUS TRANSPORT PROCESSES

We define the microscopic entropy  $S_{am}$  and macroscopic entropy  $S_{aM}$  per unit volume for species  $a$  by

$$S_{am} = - \int d^3v (\langle f_a \rangle + \hat{f}_a) \ln (\langle f_a \rangle + \hat{f}_a) \tag{63}$$

$$S_{aM} = - \int d^3v \langle f_a \rangle \ln \langle f_a \rangle. \tag{64}$$

In this section, the subscript for ensemble average is suppressed. We have the relation between these entropies by retaining terms up to  $\mathcal{O}(\Delta^2)$  as

$$S_{aM} = \langle S_{am} \rangle + \frac{1}{2} \int d^3v \frac{\langle \hat{f}_a^2 \rangle}{\langle f_a \rangle}. \quad (65)$$

Without collisions, the total microscopic entropy, i.e., the species summation and the spatial integration of  $S_{am}$  is conserved although the total macroscopic entropy can be increased by the turbulent or anomalous transport process. In this section, we are concerned with the entropy production by the turbulent process and neglect the collisional effect by assuming that the time scale of the turbulent fluctuations is much shorter than the collision time. Then we see from the average kinetic (Vlasov) equation that the conservation of  $S_{aM}$  is broken by  $\mathcal{D}_a$  and the entropy production rate due to the anomalous or turbulent process is defined by

$$\sigma_a^A = - \int d^3v \mathcal{D}_a \ln \langle f_a \rangle. \quad (66)$$

From the average and fluctuating parts of the Vlasov equation, we also obtain up to  $\mathcal{O}(\Delta^2)$

$$\sigma_a^A = \frac{1}{2} \int d^3v \left( \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) \frac{\langle \hat{f}_a^2 \rangle}{\langle f_a \rangle}. \quad (67)$$

Here we can see that  $\sigma_a^A$  is due to the spatio-temporal variation of the second term in the right-hand side of Eq. (65). Using  $\langle f \rangle \simeq f_{aM}$  and the linearized drift kinetic equation (54) for  $\hat{f}_a$  in the collisionless limit and assuming that the temporal variation of  $f_{aM}$  is much slower than that of  $\langle \hat{f}_a^2 \rangle$ , we have

$$\begin{aligned} \sigma_a^A &= \frac{1}{2} \int d^3v \frac{1}{f_{aM}} \left( \frac{\partial}{\partial t} + v_{\parallel} \mathbf{n} \cdot \nabla \right) \langle \hat{f}_a^2 \rangle = \int d^3v \left\langle \hat{f}_a \left[ -\hat{\mathbf{v}}_E \cdot \nabla \ln f_{aM} + \frac{e_a}{T_a} (v_{\parallel} \mathbf{n} + \mathbf{v}_{da0}) \cdot \hat{\mathbf{E}} \right] \right\rangle \\ &= \mathbf{J}_{a1} \cdot \mathbf{X}_{a1} + \mathbf{J}_{a2} \cdot \mathbf{X}_{a2} + J_{a3} X_{a3} \end{aligned} \quad (68)$$

where we defined conjugate pairs of forces

$$\mathbf{X}_{a1} = -\nabla \ln n_a, \quad \mathbf{X}_{a2} = -\nabla \ln T_a, \quad X_{a3} = 1/T_a \quad (69)$$



and fluxes

$$\begin{aligned}
\mathbf{J}_{a1} &= \Gamma_a^A = \int d^3v \langle \hat{f}_a \hat{\mathbf{v}}_E \rangle \\
&= \pi \sum_{\mathbf{k}} \frac{e_a^2 \langle |\hat{\phi}_{\mathbf{k}}|^2 \rangle}{T_a^2} \int d^3v f_{aM} \delta(\omega - \omega_E - \omega_{D_a} - k_{\parallel} v_{\parallel}) (\omega - \omega_E - \omega_{*a}^T) \frac{cT_a}{e_a B} \mathbf{k} \times \mathbf{n} \\
\mathbf{J}_{a2} &= \frac{1}{T_a} \mathbf{q}_a^A = \int d^3v \left( x_a^2 - \frac{3}{2} \right) \langle \hat{f}_a \hat{\mathbf{v}}_E \rangle \\
&= \pi \sum_{\mathbf{k}} \frac{e_a^2 \langle |\hat{\phi}_{\mathbf{k}}|^2 \rangle}{T_a^2} \int d^3v \left( x_a^2 - \frac{3}{2} \right) f_{aM} \delta(\omega - \omega_E - \omega_{D_a} - k_{\parallel} v_{\parallel}) (\omega - \omega_E - \omega_{*a}^T) \frac{cT_a}{e_a B} \mathbf{k} \times \mathbf{n} \\
J_{a3} &= e \int d^3v \langle \hat{f} \hat{\mathbf{E}} \rangle \cdot (v_{\parallel} \mathbf{n} + \mathbf{v}_{a\nabla B} + \mathbf{v}_{a\text{curv}}) \\
&= \pi \sum_{\mathbf{k}} \frac{e_a^2 \langle |\hat{\phi}_{\mathbf{k}}|^2 \rangle}{T_a} \int d^3v f_{aM} \delta(\omega - \omega_E - \omega_{D_a} - k_{\parallel} v_{\parallel}) (\omega - \omega_E - \omega_{*a}^T) (\omega - \omega_E). \quad (70)
\end{aligned}$$

Here  $\mathbf{J}_{a1} = \Gamma_a^A$  is the particle flux conjugate to the density gradient force  $\mathbf{X}_{a1} = -\nabla \ln n_a$  and is the same as the anomalous particle flux  $\Gamma_a^{\text{anom}}$  defined in Eq. (62).  $\mathbf{J}_{a2} = \mathbf{q}_a^A/T_a$  is the heat flux divided by the temperature conjugate to the temperature gradient force  $\mathbf{X}_{a2} = -\nabla \ln T_a$  although it is different from the anomalous flux  $\mathbf{q}_a^{\text{anom}}/T_a$  defined in Eq. (62). The flux  $J_{a3}$  is conjugate to the force  $X_{a3} = 1/T_a$  and represents the rate of the energy transfer from the electrostatic fluctuations to the particles moving along the guiding center orbits.

The positive definiteness of  $\sigma_a^A$  is shown as

$$\sigma_a^A = \pi \sum_{\mathbf{k}} \frac{e_a^2 \langle |\hat{\phi}_{\mathbf{k}}|^2 \rangle}{T_a^2} \int d^3v f_{aM} \delta(\omega - \omega_E - \omega_D - k_{\parallel} v_{\parallel}) (\omega - \omega_E - \omega_{*a}^T)^2 \geq 0. \quad (71)$$

The relation between the fluxes and forces are given by

$$\begin{bmatrix} \mathbf{J}_{a1} \\ \mathbf{J}_{a2} \\ J_{a3} \end{bmatrix} = \begin{bmatrix} L_{11}^a & L_{12}^a & L_{13}^a \\ L_{12}^a & L_{22}^a & L_{23}^a \\ L_{13}^a & L_{23}^a & L_{33}^a \end{bmatrix} \begin{bmatrix} \mathbf{X}_{a1} \\ \mathbf{X}_{a2} \\ X_{a3} \end{bmatrix} \quad (72)$$

where

$$L_{lm}^a = \pi \sum_{\mathbf{k}} \frac{e_a^2 \langle |\hat{\phi}_{\mathbf{k}}|^2 \rangle}{T_a^2} \int d^3v \left( x_a^2 - \frac{3}{2} \right)^{l+m-2} f_{aM} \delta(\omega - \omega_E - \omega_{D_a} - k_{\parallel} v_{\parallel}) \left( \frac{cT_a}{e_a B} \right)^2 (\mathbf{k} \times \mathbf{n})(\mathbf{k} \times \mathbf{n})$$

$$L_{l3}^a = \pi \sum_{\mathbf{k}} \frac{e_a^2 \langle |\widehat{\phi}_{\mathbf{k}}|^2 \rangle}{T_a} \int d^3v \left( x_a^2 - \frac{3}{2} \right)^{l-1} f_{aM} \delta(\omega - \omega_E - \omega_{D_a} - k_{\parallel} v_{\parallel}) (\omega - \omega_E) \frac{c T_a}{e_a B} (\mathbf{k} \times \mathbf{n})$$

$$L_{33}^a = \pi \sum_{\mathbf{k}} e_a^2 \langle |\widehat{\phi}_{\mathbf{k}}|^2 \rangle \int d^3v f_{aM} \delta(\omega - \omega_E - \omega_{D_a} - k_{\parallel} v_{\parallel}) (\omega - \omega_E)^2 \quad (l, m = 1, 2). \quad (73)$$

Thus the matrix relating the fluxes to the forces satisfies the Onsager symmetry and the positive definiteness although it is qualitatively different from the classical and neoclassical transport matrices in that the former matrix depends also on the forces through the eigenfrequencies, which are determined from the dispersion relation, and through the spectrum of the fluctuation amplitudes, which is given by the nonlinear saturation. In order to elucidate the origin of the Onsager symmetry and the positive definiteness satisfied by the anomalous transport equations (72), a general quasilinear formulation of the drift kinetic equation with the electrostatic fluctuations is developed in Appendix A. There, we derive a symmetric and positive definite five-dimensional diffusion tensor  $D_{\frac{1}{2}}^A$  which relates the anomalous fluxes to the gradient forces in the drift phase space. From that phase space diffusion tensor, the Onsager symmetry and the positive definiteness of the the anomalous transport matrix (73) are derived.

The effects of the finite gyroradius can be derived from using the gyro-kinetic equation and the results are easily obtained by including  $[J_0(k_{\perp} v_{\perp} / \Omega_a)]^2$  into the integrands in the matrix coefficients given by Eq. (73). As shown in Ref. 10, for stronger turbulent regime, the delta functions in the integrands are replaced with the resonance functions derived from the renormalized propagator which takes account of the turbulent resonance broadening.

In Ref. 9, Balescu derived the linear thermodynamic form of the anomalous transport and discussed the broken Onsager symmetry and the difficulty lying in the transport coefficients. The relation between his results and ours given in this section is shown in detail in Appendix B. There we find that, in the linear thermodynamic form employed by Balescu, the Onsager symmetry is hidden and that it is essential to the Onsager relation of Eq. (72)

to include  $J_{a3}$  and  $X_{a3}$  as the flux and force, which are not treated as such in his linear thermodynamic form.

## VI. ANOMALOUS EFFECTS ON THE PARALLEL VISCOSITIES

Now, let us find the solution of the averaged drift kinetic equation (52). Here, we consider a large aspect ratio tokamak in order to derive the approximate solution for the plateau regime, and, as shown in Appendix B, the solution is written as

$$\langle \bar{f}_{a1} \rangle_{ens} = \bar{F}_{a1} + \bar{g}_a^{(l=1)} - (\nu_a \mathcal{L}_a)^{-1} \langle \bar{\mathcal{D}}_a^{(l \geq 2)} \rangle + \bar{h}_a \quad (74)$$

where we denoted the contribution of the thermodynamic forces by given by

$$\bar{F}_{a1} = -\frac{2\pi I v_{||}}{\chi' \Omega_a} \frac{\partial \bar{f}_{a0}}{\partial \psi} = \frac{2v_{||}}{v_{Ta}^2} \left[ V_{1a} + \left( x_a^2 - \frac{5}{2} \right) V_{2a} \right] \bar{f}_{a0}, \quad (75)$$

the poloidal flow part by

$$\bar{g}_a^{(l=1)} = \frac{2v_{||}}{v_{Ta}^2} B \left[ u_{a\theta} + \frac{2q_{a\theta}}{5p_a} \left( x_a^2 - \frac{5}{2} \right) \right] \bar{f}_{a0}, \quad (76)$$

the part contributing to the parallel viscosity by

$$\begin{aligned} \bar{h}_a = \epsilon \bar{\nu}_a^{-1/3} \left\{ \frac{x_a}{v_{Ta}} B \left[ u_{a\theta} + \frac{2q_{a\theta}}{5p_a} \left( x_a^2 - \frac{5}{2} \right) \right] \bar{f}_{a0} + \frac{1}{\nu_a} \int_0^1 \langle \bar{\mathcal{D}}_a^{(l \geq 2)} \rangle d\xi \right\} \\ \times \int_0^\infty d\tau \sin(\theta - \bar{\nu}_a^{-1/3} \xi \tau) e^{-r^3/6} \end{aligned} \quad (77)$$

and the definition of  $(\nu_a \mathcal{L}_a)^{-1} \langle \bar{\mathcal{D}}_a^{(l \geq 2)} \rangle$  is described in Appendix B. Here,  $\xi \equiv v_{||}/v$  denotes the cosine of the pitch angle. Fluctuation effects on the solution are explicitly included through  $\langle \bar{\mathcal{D}}_a^{(l \geq 2)} \rangle$  in the third and fourth terms in Eq. (74). The third term  $-(\nu_a \mathcal{L}_a)^{-1} \langle \bar{\mathcal{D}}_a^{(l \geq 2)} \rangle$  results from the balance between the collisional pitch angle scattering and the quasilinear anisotropic deformation of the distribution in the velocity space. The fourth term  $\bar{h}_a$  represents the distribution of the resonant particles ( $|\xi| \ll 1$ ). The anisotropic distribution

(or  $\xi$ -dependence) in the velocity space caused by the second term (poloidal flow) and that by the third term (quasilinear effect) give the sources of the resonant particles that are responsible for the neoclassical and anomalous parallel viscosities, which are shown in Eq.(77). Since the quasilinear term  $\langle \bar{\mathcal{D}}_a \rangle$  contains a delta function of  $\xi$ , all the Legendre function components with  $l = 2, 3, 4, \dots$  homogeneously contribute to  $\langle \bar{\mathcal{D}}_a^{(l \geq 2)} \rangle$  although its even parts with  $l = 2, 4, 6, \dots$  vanish in the integral  $\int_0^1 \langle \bar{\mathcal{D}}_a^{(l \geq 2)} \rangle d\xi$ . Then, noting that only  $\bar{h}_a$  contributes to the flux averaged parallel viscosities, we obtain

$$\begin{aligned} \begin{bmatrix} \langle \mathbf{B} \cdot \nabla \cdot \boldsymbol{\pi}_a \rangle \\ \langle \mathbf{B} \cdot \nabla \cdot \boldsymbol{\Theta}_a \rangle \end{bmatrix} &= \begin{bmatrix} \langle \int d^3v m_a v_{\parallel}^2 \mathbf{B} \cdot \nabla \bar{h}_a \rangle \\ \langle \int d^3v m_a v_{\parallel}^2 \left(x_a^2 - \frac{5}{2}\right) \mathbf{B} \cdot \nabla \bar{h}_a \rangle \end{bmatrix} \\ &= \frac{\sqrt{\pi}}{2} \epsilon^2 n_a m_a \omega_{Ta} B_0^2 \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{13}{4} \end{bmatrix} \begin{bmatrix} u_{a\theta} \\ \frac{2}{5} \frac{q_{a\theta}}{p_a} \end{bmatrix} + \begin{bmatrix} Y_{a1} \\ Y_{a2} \end{bmatrix} \\ &= \frac{\sqrt{\pi}}{2} \epsilon^2 n_a m_a \omega_{Ta} B_0^2 \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{13}{4} \end{bmatrix} \begin{bmatrix} u_{a\theta} - W_{a1} \\ \frac{2}{5} \frac{q_{a\theta}}{p_a} - W_{a2} \end{bmatrix} \end{aligned} \quad (78)$$

where the anomalous contributions from the quasilinear fluctuation term are given by

$$\begin{aligned} \begin{bmatrix} Y_{a1} \\ Y_{a2} \end{bmatrix} &= \frac{\sqrt{\pi}}{2} \epsilon^2 n_a m_a \omega_{Ta} v_{Ta} \tau_{aa} B_0 \int_0^\infty dx_a e^{-x_a^2} \frac{x_a^4}{\tau_{aa} \nu_a(x_a)} \begin{bmatrix} 1 \\ \left(x_a^2 - \frac{5}{2}\right) \end{bmatrix} \int_0^1 \frac{\langle \bar{\mathcal{D}}_a^{(l \geq 2)} \rangle}{f_{aM}} d\xi \\ \begin{bmatrix} W_{a1} \\ W_{a2} \end{bmatrix} &= \frac{v_{Ta} \tau_{aa}}{B_0} \int_0^\infty dx_a e^{-x_a^2} \frac{x_a^4}{\tau_{aa} \nu_a(x_a)} \begin{bmatrix} \left(-\frac{3}{2} + \frac{1}{6} x_a^2\right) \\ \left(1 - \frac{1}{3} x_a^2\right) \end{bmatrix} \int_0^1 \frac{\langle \bar{\mathcal{D}}_a^{(l \geq 2)} \rangle}{f_{aM}} d\xi \end{aligned} \quad (79)$$

where the quasilinear term  $\bar{\mathcal{D}}_a^{(l \geq 2)}$  is given by

$$\bar{\mathcal{D}}_a^{(l \geq 2)} = \bar{\mathcal{D}}_a - \frac{1}{2} \int_{-1}^1 \bar{\mathcal{D}}_a d\xi - \frac{3\xi}{2} \int_{-1}^1 \xi \bar{\mathcal{D}}_a d\xi$$

$$\begin{aligned} \bar{\mathcal{D}}_a &= \pi f_{aM} \sum_{\mathbf{k}} \frac{e_a^2 \langle |\hat{\phi}_{\mathbf{k}}|^2 \rangle_{ens}}{T_a^2} \frac{k_{\parallel}}{|k_{\parallel}|} \\ &\times \left[ (\omega - \omega_E - \omega_{*a}^{T'}) \alpha \delta(\xi - \alpha) - \frac{1}{2x_a^2} (\omega - \omega_E - \omega_{*a}^T) \left( 1 + \frac{2}{x_a \xi} \frac{\omega_{acurv}}{k_{\parallel} v_{Ta}} \right) \delta'(\xi - \alpha) \right] \end{aligned} \quad (80)$$

with

$$\omega_{*a}^{T'} = \omega_{*a} \left[ 1 + \eta_a \left( x_a^2 - \frac{5}{2} \right) \right], \quad \omega_{acurv} = \mathbf{k} \cdot \mathbf{v}_{acurv}, \quad \alpha = \frac{\omega - \omega_E - \omega_{Da}}{k_{\parallel} v}$$

Here, if the anomalous contributions  $Y_a$ , or  $W_a$ , vanish, Eq. (78) reduces to the conventional plateau parallel viscosities.

As seen from Eqs. (61), (79) and (80), the anomalous effects are negligibly small when the phase velocities of the fluctuations in the reference frame moving with the guiding center particles are much larger than the thermal velocity, i.e.,  $|(\omega - \omega_E - \omega_{Da})/k_{\parallel}| \gg v_{Ta}$ . Thus we now investigate the unstable modes with  $|(\omega - \omega_E - \omega_{Da})/k_{\parallel}| \sim v_{Ta}$  to give the detailed expressions of the anomalous transport and the parallel viscosities. In the case of the electron drift wave driven by the density gradient  $dn_a/d\psi < 0$  with  $\eta_a \simeq 0$  and  $\omega_{Da} \simeq 0$ , the anomalous effects are small since  $v_{Ti} \ll |(\omega - \omega_E)/k_{\parallel}| \ll v_{Te}$  is required for the unstable modes and the anomalous contributions  $\mathbf{K}_{ej}$ ,  $Y_{ej}$  and  $W_{ej}$  ( $j = 1, 2$ ) are all proportional to  $|(\omega - \omega_E - \omega_{*e})/\omega_{*e}| \simeq \frac{1}{2}k_{\perp}^2\rho_i^2(1 + Z_i T_e/T_i) \ll 1$ , where  $Z_i$  is the ion charge number.

The most relevant fluctuations that resonantly exchange energy-momentum between the ions and the fluctuation is the ion-temperature gradient driven turbulence. Numerous studies of the stability and quasilinear fluxes from this form of drift wave turbulence are available. For the small perpendicular wavenumbers satisfying  $k_{\perp}\rho_i \ll 1$  or  $\omega_{Di} \ll k_{\parallel}v_{Ti}$ , we obtain the slab ion temperature gradient (ITG) driven modes<sup>15,16</sup> where the instabilities with  $|(\omega - \omega_E)/k_{\parallel}| \sim v_{Ti}$  are found as shown in Appendix C. The relationship of the small  $\omega_{Di}$  limit to the strong toroidal regime of  $\omega_{Di} \sim k_{\parallel}v_{Ti}$  is developed in Kim and Horton<sup>17</sup> and Kim *et al.*<sup>18</sup> When  $Z_i T_e/T_i = 1$  and  $dT_i/d\psi < 0$ , we can write the anomalous effects for ions from Appendix C as

$$T_i^{-1} \langle \mathbf{q}_i \cdot \nabla \psi \rangle^{\text{anom}} = 2.61 n_i \frac{cT_i}{e_i B_0} \sum_{\mathbf{k}} |\mathbf{k} \times \mathbf{n} \cdot \nabla \psi| \frac{e_i^2 \langle |\hat{\phi}_{\mathbf{k}}|^2 \rangle_{\text{ens}}}{T_i^2}$$

$$\langle \mathbf{B} \cdot \mathbf{K}_{i2} \rangle = -1.30 p_i B_0 \sum_{\mathbf{k}} \sigma_{*i} k_{\parallel} \frac{e_i^2 \langle |\hat{\phi}_{\mathbf{k}}|^2 \rangle_{\text{ens}}}{T_i^2}$$

$$\begin{aligned}
Y_{ij} &= C_{yj} \epsilon^2 p_i \omega_{Ti} \tau_{ii} B_0 \sum_{\mathbf{k}}' \sigma_{*i} k_{\parallel} \frac{e_i^2 \langle |\hat{\phi}_{\mathbf{k}}|^2 \rangle_{ens}}{T_i^2} \quad (j = 1, 2) \\
W_{ij} &= C_{wj} \frac{v_{Ti}^2 \tau_{ii}}{B_0} \sum_{\mathbf{k}}' \sigma_{*i} k_{\parallel} \frac{e_i^2 \langle |\hat{\phi}_{\mathbf{k}}|^2 \rangle_{ens}}{T_i^2} \quad (j = 1, 2)
\end{aligned} \tag{81}$$

where  $\Sigma'$  represents the summation over the wavenumber region where  $|\omega_{*e} \eta_i / k_{\parallel} v_{Ti}| \sim 2$  and  $\sigma_{*i}$  is the sign of the ion diamagnetic drift frequency. The dimensionless constants in Eq. (81) are given by

$$C_{y1} = -3.05, \quad C_{y2} = 0.85, \quad C_{w1} = 1.94, \quad C_{w2} = -0.45. \tag{82}$$

We should remark on the symmetry properties of the eigenfrequency and the fluctuation spectrum with respect to the parallel wavenumber  $k_{\parallel}$ . It is found that the eigenfrequency given is an even function of  $k_{\parallel}$ , which results from our use of the Maxwellian distribution with no flow velocity as an equilibrium. If we assume that the spectrum  $\langle |\hat{\phi}_{\mathbf{k}}|^2 \rangle_{ens}$  is also even in  $k_{\parallel}$ , the parallel anomalous forces  $\langle \mathbf{B} \cdot \mathbf{K}_{a_j} \rangle$  and the anomalous effects on the parallel viscosities  $Y_{a_j}$  and  $W_{a_j}$  vanish. These are confirmed by noting that the wavenumber spectra of  $\langle \mathbf{B} \cdot \mathbf{K}_{a_j} \rangle$ ,  $Y_{a_j}$  and  $W_{a_j}$  are odd functions of  $k_{\parallel}$ . Thus, in this case, the banana-plateau transport is not modified by the fluctuations. The  $k_{\parallel}$ -symmetry of the dispersion relation is broken, for example, if we take account of sheared flows in the equilibrium distribution function, although they are neglected here by the  $\delta$ -ordering.

Using  $m_e/m_i \ll 1$  and Eq. (21), we have  $\langle \mathbf{B} \cdot \nabla \cdot \boldsymbol{\pi}_i \rangle \simeq 0$ , from which we obtain the poloidal flow velocity  $u_P \equiv B_P u_{i\theta}$  as

$$u_P = -\frac{1}{2} \frac{c}{e_i B_T} \frac{dT_i}{dr} + 1.95 \epsilon q^{-1} v_{Ti}^2 \tau_{ii} \sum_{\mathbf{k}}' \sigma_{*i} k_{\parallel} \frac{e_i^2 \langle |\hat{\phi}_{\mathbf{k}}|^2 \rangle_{ens}}{T_i^2} \tag{83}$$

where Eqs. (18), (20), (22), (78), and (81) are used. In the right-hand side, the first term represents the ion temperature gradient driven poloidal flow in the plateau regime given by the conventional neoclassical theory, the second is due to  $\langle \mathbf{B} \cdot \mathbf{K}_{i2} \rangle$  and  $Y_{ij}$  (or  $W_{ij}$ ). As

previously mentioned, the second term vanishes if the spectrum  $\langle |\widehat{\phi}_{\mathbf{k}}|^2 \rangle_{ens}$  is even in  $k_{\parallel}$ . We can see that first term contributes to the direction of the electron diamagnetic rotation with  $dn_e/dr < 0$  and  $dT_i/dr < 0$  assumed, while the sign of the second term depends on the wavenumber spectrum. The ratio of the anomalous poloidal flow to the ion temperature gradient driven flow has the same order of magnitude as the ratio of the anomalous parallel current to the pressure gradient driven bootstrap current and that is estimated for the plateau regime from Eq. (83) as

$$\frac{u_P^{anom}}{u_P^{neo}} \sim \frac{j_{\parallel}^{anom}}{j_{\parallel}^{neo}} \sim \epsilon(\omega_{T_i}, \tau_{ii}) \left( \frac{L_{T_i}}{\rho_i} \right) \sum_{\mathbf{k}}' \sigma_{\bullet, k_{\parallel}} R \frac{e_i^2 \langle |\widehat{\phi}_{\mathbf{k}}|^2 \rangle_{ens}}{T_i^2} \quad (84)$$

where a dimensionless numerical constant is omitted and  $L_{T_i} = |d \ln T_i / dr|^{-1}$  is used. When we write the parallel wavenumber as that  $k_{\parallel} = (m - nq) / Rq$  with the poloidal and toroidal mode numbers  $(m, n)$ , Eq. (84) implies that the anomalous rotation and the anomalous parallel currents have opposite signs on different sides of the mode rational surfaces as shown by Shaing.<sup>6</sup> As shown by Dong *et al.*,<sup>19</sup> in the presence of the parallel shear flow which breaks the radial symmetry, the peak of the fluctuating potential shifts radially. In that case, the anomalous forces on the both sides of the mode rational surface do not cancel out and they generate the net rotation and current. If we use  $\sigma_{\bullet, k_{\parallel}} R \sim 1$  and  $\sum_{\mathbf{k}}' e_i^2 \langle |\widehat{\phi}_{\mathbf{k}}|^2 \rangle_{ens} / T_i^2 \sim \Delta^2 \sim \delta \sim \rho_i / L_{T_i}$ , the ratio in Eq. (84) reduces to  $\epsilon(\omega_{T_i}, \tau_{ii})$  and we have  $\epsilon \ll \epsilon(\omega_{T_i}, \tau_{ii}) \ll \epsilon^{-1/2}$  for the plateau regime. Thus, the anomalous contribution to the transport along the magnetic flux surface is expected to become dominant in the weak collisional plateau regime. For comparison, let us consider the anomalous contribution to the anomalous heat flux across the magnetic flux surface. Using Eq.(81), the ratio of the anomalous to the banana-plateau heat flux is estimated for the plateau regime as

$$\frac{q_i^{anom}}{q_i^{neo}} \sim \frac{\chi_i^{anom}}{\chi_i^{neo}} \sim \frac{1}{q} \left( \frac{L_{T_i}}{\rho_i} \right) \sum_{\mathbf{k}}' k_{\perp} R \frac{e^2 \langle |\widehat{\phi}_{\mathbf{k}}|^2 \rangle_{ens}}{T_i^2}. \quad (85)$$

From the ordering  $k_{\perp} / k_{\parallel} \sim \Delta^{-1} \gg 1$ , this ratio is much larger than the ratio given by Eq. (84) and reduces to  $\Delta^{-1}$ .

## VII. CONCLUSIONS AND DISCUSSION

In this work, we have investigated the neoclassical and anomalous transport in axisymmetric toroidal systems with electrostatic fluctuations. The total transport is clearly separated into the neoclassical and anomalous parts by using the kinetic definitions which give the average fluid variables from the average kinetic distribution function. The neoclassical banana-plateau transport fluxes modified by the fluctuations were shown in Eq. (26), where the parallel fluctuation-induced forces  $\langle \mathbf{B} \cdot \mathbf{K}_{a_j} \rangle$ , ( $a = 1, 2; j = i, e$ ) and the corrections  $Y_{a_j}$  (or  $W_{a_j}$ ) to the parallel viscosities due to the fluctuations appear in the definitions of the modified thermodynamic forces and the parallel current. The anomalous transport fluxes were defined by Eq. (32) or Eq. (62) in terms of the perpendicular components of the fluctuation-induced forces  $\mathbf{K}_{a_j}$  in the similar way to the definition of the classical transport fluxes by the collisional friction forces  $\mathbf{F}_{a_j}$ . Thus, the anomalous fluxes are defined compactly in terms of  $\mathbf{K}_{a_j}$  in our treatment, which give expressions different from the anomalous heat fluxes defined by Shaing and by Balescu. The parallel components of  $\mathbf{K}_{a_j}$  produce the Pfirsch-Schlüter like anomalous fluxes as given by Eq. (31) although they are negligibly smaller than those in Eq. (32) for the fluctuations with  $k_{\parallel} \ll k_{\perp}$ .

The fluctuation-induced forces  $\mathbf{K}_{a_j}$  were defined by Eq. (10) in terms of the statistically nonlinear term  $\mathcal{D}_a$  which appears in the ensemble-averaged kinetic equation and they were calculated from the solution of the fluctuating part of the linear drift kinetic equation as given in Eq. (55). The anisotropic distribution in the velocity space caused by the quasilinear fluctuation source gives the corrections to the parallel viscosities  $Y_{a_j}$  (or  $W_{a_j}$ ), which were obtained for the plateau regime in Eq. (79) from the solution of the ensemble-averaged drift kinetic equation. Thus from  $\mathbf{K}_{a_j}$  and  $Y_{a_j}$  (or  $W_{a_j}$ ), we can evaluate the neoclassical and anomalous transport fluxes when the fluctuation spectrum  $\langle |\phi_{\mathbf{k}}|^2 \rangle_{ens}$  and the frequency  $\omega_{\mathbf{k}}$  are specified. The results using the slab ITG mode dispersion relation for the fluctuations



with the small perpendicular wavenumbers were shown in Sec. VI.

Neither Shaing nor Balescu described the anomalous contributions  $Y_{a_j}$  or  $W_{a_j}$  to the parallel viscosities since they did not take account of the ensemble-averaged drift kinetic equation with the quasilinear fluctuation term. The anomalous effects on the banana-plateau fluxes appear through  $\langle \mathbf{B} \cdot \mathbf{K}_{a_j} \rangle$  and  $Y_{a_j}$  (or  $W_{a_j}$ ). If  $\langle |\phi_{\mathbf{k}}|^2 \rangle_{ens}$  is even in  $k_{\parallel}$ ,  $\langle \mathbf{B} \cdot \mathbf{K}_{a_j} \rangle$  and  $Y_{a_j}$  (or  $W_{a_j}$ ) vanish. The spectra of  $\mathbf{K}_{a_j\perp}$  are larger than those of  $\mathbf{K}_{a_j\parallel}$  by an order of  $k_{\perp}/k_{\parallel} (\gg 1)$  so that the anomalous effects on the perpendicular transport is much larger than those on the parallel transport.

The entropy production in the anomalous transport process was given in Eqs. (66)–(68) and its positive definiteness was shown. Then, we identified conjugated pairs of the anomalous fluxes and forces and found the Onsager symmetry satisfied by the transport matrix connecting them. This matrix is a highly nonlinear function of the forces such as the density and temperature gradients through the eigenfrequencies and the fluctuation spectrum.

The magnetic flux surface average of Eqs. (4) and (6) yields the basic equations used for the particle and energy transport analyses. The results of our work suggest that the modified neoclassical fluxes as well as the anomalous fluxes should be included in the total transport fluxes and that the anomalous heat generation terms  $H_a$  should be added into the energy transport equations. The anomalous particle fluxes in Eqs. (31) and (32) are intrinsically ambipolar due to the quasineutrality condition  $\sum_a e_a \hat{n}_a = 0$  which is used for the dispersion relation. Thus, in the axisymmetric systems even with the electrostatic fluctuations, the ambipolarity gives no constraint to determine the average radial electric field. In the conventional neoclassical transport theory for the axisymmetric systems, the radial electric field does not affect the particle and heat fluxes and it is not required for the transport analyses. In our case, the radial electric field affects the anomalous energy exchange between the electrons and the ions through the dependence of  $H_a$  on the  $\mathbf{E} \times \mathbf{B}$  drift frequency  $\omega_E$ .

However, the anomalous fluxes and the dispersion relations contain  $\omega_E$  only in the form of  $(\omega - \omega_E)$  as seen in Eqs. (61) and (D1), which implies the Doppler shift and no explicit dependence of the anomalous fluxes on  $\omega_E$ . Further investigations for the determination of the radial electric field and the fluctuation spectrum  $\langle |\phi_{\mathbf{k}}|^2 \rangle_{ens}$  are required. We are also considering the direct extension of this work to the case of the nonaxisymmetric system with the magnetic fluctuations, which will be reported elsewhere.

## ACKNOWLEDGMENTS

The authors thank Dr. H. Sanuki, Dr. N. Nakajima, Prof. M. Wakatani, and Prof. R. D. Hazeltine for useful discussions, and Prof. M. Okamoto for his encouragement of this work. This work is supported in part by the U.S. Dept. of Energy contract no. DE-FG05-80ET-53088 and in part by the National Science Foundation under Grant No. PHY89-04035.

## APPENDIX A: DERIVATION OF THE ANOMALOUS ENTROPY PRODUCTION FUNCTIONAL

In this Appendix, the anomalous transport equations given by Eqs. (72) and (73) are derived from general quasilinear formulation of the drift kinetic equation with the electrostatic fluctuations in order to elucidate the origin of the Onsager symmetry and the positive definiteness satisfied by the matrix connecting the conjugate pairs of the anomalous fluxes and forces.

We start from the drift kinetic equation given in Ref. 12:

$$\frac{\partial f}{\partial t} = Lf \quad (\text{A1})$$

where

$$L \equiv -\dot{\mathbf{Z}} \cdot \frac{\partial}{\partial \mathbf{Z}} = - \left( \mathbf{v}_g \cdot \frac{\partial}{\partial \mathbf{x}} + \dot{w} \frac{\partial}{\partial w} + \dot{\mu} \frac{\partial}{\partial \mu} \right). \quad (\text{A2})$$

Here, the subscript for particle species is suppressed and the collision term is neglected as in Sec. V. The gyroangle-averaged distribution  $f$  is regarded as a function of the phase space variable  $\mathbf{Z} \equiv (\mathbf{x}, w, \mu)$  where the kinetic energy  $w = \frac{1}{2}mv^2$  is used as an independent velocity space variable instead of the total energy  $E = \frac{1}{2}mv^2 + e\Phi$ . The detailed expressions of  $\dot{\mathbf{Z}} \equiv (\mathbf{v}_g, \dot{w}, \dot{\mu})$  are given from Ref. 12 and are not shown here. The Jacobian for the phase space variable  $(\mathbf{Z}, \varphi)$  ( $\varphi$ : the gyroangle) is given by

$$J \equiv \frac{\partial(\mathbf{x}, \mathbf{v})}{\partial(\mathbf{Z}, \varphi)} = \frac{\partial(\mathbf{v})}{\partial(w, \mu, \varphi)} = \frac{B}{m^2|v_{\parallel}|}. \quad (\text{A3})$$

The phase space flow  $\dot{\mathbf{Z}}$  depends linearly on the electric field and it is naturally divided into the ensemble-averaged part and the fluctuating part with respect to the turbulent electrostatic field as

$$\dot{\mathbf{Z}} = \langle \dot{\mathbf{Z}} \rangle + \hat{\dot{\mathbf{Z}}} \quad \text{or} \quad \begin{cases} \mathbf{v}_g = \langle \mathbf{v}_g \rangle + \hat{\mathbf{v}}_g \\ \dot{w} = \langle \dot{w} \rangle + \hat{\dot{w}} \\ \dot{\mu} = \langle \dot{\mu} \rangle + \hat{\dot{\mu}} \end{cases} \quad (\text{A4})$$

where the subscript for the ensemble average is also suppressed as in Sec. V. The incompressibility of the collisionless particle motion in the  $(\mathbf{x}, \mathbf{v})$ -space or the Liouville's theorem is reflected in the following zero-divergence constraints for the guiding center motion

$$\frac{1}{J} \frac{\partial}{\partial \mathbf{Z}} \cdot (J \dot{\mathbf{Z}}) = \frac{1}{J} \frac{\partial}{\partial \mathbf{Z}} \cdot (J \langle \dot{\mathbf{Z}} \rangle) = \frac{1}{J} \frac{\partial}{\partial \mathbf{Z}} \cdot (J \hat{\dot{\mathbf{Z}}}) = 0. \quad (\text{A5})$$

Here we find that  $\langle \dot{\mathbf{Z}} \rangle$  and  $\hat{\dot{\mathbf{Z}}}$  satisfy the zero-divergence conditions separately. (It is noted that the phase space flow contained in the drift kinetic equation (42) satisfies this type of zero-divergence constraint only approximately since the higher order correction to the parallel velocity and the temporal variation of the magnetic moment described in Ref. 12 are neglected in Eq. (42).) According to the division of  $\dot{\mathbf{Z}}$  in Eq. (A4), the differential operator  $L$  is divided similarly as

$$L = L_0 + \hat{L} \quad (\text{A6})$$

where

$$\begin{aligned} L_0 &\equiv -\langle \dot{\mathbf{Z}} \rangle \cdot \frac{\partial}{\partial \mathbf{Z}} = -\frac{1}{J} \frac{\partial}{\partial \mathbf{Z}} \cdot J \langle \dot{\mathbf{Z}} \rangle \\ \hat{L} &\equiv -\hat{\dot{\mathbf{Z}}} \cdot \frac{\partial}{\partial \mathbf{Z}} = -\frac{1}{J} \frac{\partial}{\partial \mathbf{Z}} \cdot J \hat{\dot{\mathbf{Z}}}. \end{aligned} \quad (\text{A7})$$

Noting that the distribution function  $f$  also consists of the ensemble average and fluctuating parts as

$$f = \langle f \rangle + \hat{f} \quad (\text{A8})$$

we obtain from Eq. (A1) the ensemble-averaged drift kinetic equation

$$\left( \frac{\partial}{\partial t} - L_0 \right) \langle f \rangle = \langle \hat{L} \hat{f} \rangle. \quad (\text{A9})$$

Here,  $\langle \hat{L} \hat{f} \rangle$  does not completely coincide with  $\mathcal{D}$  defined in Sec. II which is easily seen from the fact that the velocity space integration of the latter exactly vanishes while that of the former does not. The fluctuating part of the drift kinetic equation is given in the linearized

form as

$$\left(\frac{\partial}{\partial t} - L_0\right) \hat{f} = \hat{L} \langle f \rangle. \quad (\text{A10})$$

Using the initial condition  $\hat{f}(t = -\infty) = 0$ ,  $\hat{f}$  is given by

$$\begin{aligned} \hat{f}(t) &= - \int_{-\infty}^t d\tau e^{(t-\tau)L_0} \dot{\hat{Z}}(\tau, \mathbf{Z}) \cdot \frac{\partial \langle f \rangle}{\partial \mathbf{Z}}(\tau, \mathbf{Z}) \\ &= - \int_0^\infty d\tau e^{\tau L_0} \dot{\hat{Z}}(t - \tau, \mathbf{Z}) \cdot \frac{\partial \langle f \rangle}{\partial \mathbf{Z}}(t - \tau, \mathbf{Z}) \\ &= - \int_0^\infty d\tau \dot{\hat{Z}}[t - \tau, \mathbf{Z}(-\tau)] \cdot \frac{\partial \langle f \rangle}{\partial \mathbf{Z}}[t - \tau, \bar{\mathbf{Z}}(-\tau)]. \end{aligned} \quad (\text{A11})$$

where  $\bar{\mathbf{Z}}(t)$  is defined as the solution of the ordinary differential equation

$$\frac{d\bar{\mathbf{Z}}}{dt} = \langle \dot{\mathbf{Z}} \rangle(\bar{\mathbf{Z}}) \quad (\text{A12})$$

with the initial condition  $\bar{\mathbf{Z}}(t = 0) = \mathbf{Z}$ . Here we assume that, within the correlation time of the electrostatic fluctuations,  $(\partial \langle f \rangle / \partial \bar{\mathbf{Z}})[t - \tau, \bar{\mathbf{Z}}(-\tau)]$  in Eq. (A11) varies only slightly and is replaced by  $(\partial \langle f \rangle / \partial \mathbf{Z})(t, \mathbf{Z})$ . (This assumption is questionable if  $\langle f \rangle$  contains the gyroangle dependence through which  $\langle f \rangle [t - \tau, \bar{\mathbf{Z}}(-\tau)]$  has the time scale comparable to or shorter than the fluctuation time scale. For this reason, we started from the drift kinetic equation.) Then the nonlinear (or quasilinear) term in the right-hand side of Eq. (A9) is written as

$$\langle \hat{L} \hat{f} \rangle = -\frac{1}{J} \frac{\partial}{\partial \mathbf{Z}} \cdot (J \mathbf{J}_2^A) \quad (\text{A13})$$

where the anomalous particle flux  $\mathbf{J}_2^A$  in the  $\mathbf{Z}$ -space is given by

$$\begin{aligned} \mathbf{J}_2^A(t) &\equiv \left\langle \dot{\hat{Z}}(t) \hat{f}(t) \right\rangle \\ &= - \int_0^\infty d\tau \left\langle \dot{\hat{Z}}(t, \mathbf{Z}) \dot{\hat{Z}}(t - \tau, \bar{\mathbf{Z}}(-\tau)) \right\rangle \cdot \frac{\partial \langle f \rangle}{\partial \mathbf{Z}}(t, \mathbf{Z}) \\ &= \langle f \rangle D_2^A \cdot \mathbf{X}_Z \end{aligned} \quad (\text{A14})$$

which are regarded as the anomalous transport equations represented in the  $\mathbf{Z}$ -space. Here the  $\mathbf{Z}$ -space gradient force  $\mathbf{X}_Z$  and the  $\mathbf{Z}$ -space anomalous diffusion tensor  $D_Z^A$  are defined by

$$\mathbf{X}_Z \equiv -\frac{\partial \ln \langle f \rangle}{\partial \mathbf{Z}} \quad (\text{A15})$$

$$D_Z^A \equiv \int_0^\infty d\tau \langle \hat{\mathbf{V}}(0) \hat{\mathbf{V}}(\tau) \rangle$$

$$\hat{\mathbf{V}}(\tau) \equiv \hat{\mathbf{Z}}[-\tau, \bar{\mathbf{Z}}(-\tau)] \quad (\text{A16})$$

where the stationary electrostatic turbulence is assumed to eliminate the explicit time dependence of  $D_Z^A$ . Furthermore, assuming the spatially homogeneous turbulence within the turbulence scale length, we have

$$\langle \hat{\mathbf{V}}(t) \hat{\mathbf{V}}(t + \tau) \rangle = \langle \hat{\mathbf{V}}(0) \hat{\mathbf{V}}(\tau) \rangle. \quad (\text{A17})$$

Then we find that, for an arbitrary vector  $\mathbf{a} = (a_j)_{j=1, \dots, 5}$ , the  $\mathbf{Z}$ -space anomalous diffusion tensor  $D_Z^A$  satisfies

$$\mathbf{a} \cdot D_Z^A \cdot \mathbf{a} = \lim_{T \rightarrow \infty} \frac{1}{2T} \left\langle \left( \mathbf{a} \cdot \int_0^T d\tau \hat{\mathbf{V}}(\tau) \right)^2 \right\rangle \geq 0 \quad (\text{A18})$$

from which we obtain the positive definiteness of the anomalous entropy production locally defined in the  $\mathbf{Z}$ -space as

$$\sigma_Z^A \equiv \mathbf{J}_Z^A \cdot \mathbf{X}_Z = \langle f \rangle \mathbf{X}_Z \cdot D_Z^A \cdot \mathbf{X}_Z \geq 0. \quad (\text{A19})$$

For the stationary and locally homogeneous electrostatic turbulence, we can define the correlation function  $F$  for the fluctuating scalar electrostatic potential  $\hat{\phi}$  as

$$F(t_1 - t_2, \mathbf{x}_1 - \mathbf{x}_2) \equiv \langle \hat{\phi}(t_1, \mathbf{x}_1) \hat{\phi}(t_2, \mathbf{x}_2) \rangle = F(t_2 - t_1, \mathbf{x}_2 - \mathbf{x}_1) \quad (\text{A20})$$

from which we find that

$$\begin{aligned} \langle \hat{\mathbf{E}}(t_1, \mathbf{x}_1) \hat{\mathbf{E}}(t_2, \mathbf{x}_2) \rangle &= - \frac{\partial^2 F(t_1 - t_2, \mathbf{x})}{\partial \mathbf{x} \partial \mathbf{x}} \Big|_{\mathbf{x}=\mathbf{x}_1-\mathbf{x}_2} = - \frac{\partial^2 F(t_2 - t_1, \mathbf{x})}{\partial \mathbf{x} \partial \mathbf{x}} \Big|_{\mathbf{x}=\mathbf{x}_2-\mathbf{x}_1} \\ &= \langle \hat{\mathbf{E}}(t_2, \mathbf{x}_2) \hat{\mathbf{E}}(t_1, \mathbf{x}_1) \rangle. \end{aligned} \quad (\text{A21})$$

This symmetry property leads to

$$\langle \hat{V}(\tau)\hat{V}(0) \rangle = \langle \hat{V}(0)\hat{V}(\tau) \rangle \quad (\text{A22})$$

which in turn reduces to the Onsager-type symmetry of the the  $\mathbf{Z}$ -space anomalous diffusion tensor  $D_Z^A$

$$(D_Z^A)^T = D_Z^A \quad (\text{A23})$$

where the superscript  $T$  denotes the transpose of the tensor. The symmetry (A23) and the positive definiteness (A19) of the anomalous diffusion tensor due to the particle-fluctuation interaction are the analogy to those of the linearized collision operator used in the classical and neoclassical transport theories.

The anomalous effect due to  $\langle \hat{L}\hat{f} \rangle$  on the temporal evolution of the macroscopic entropy  $S_M \equiv -\int d^3v \langle f \rangle \ln \langle f \rangle$  defined in Sec. V is given by

$$\begin{aligned} -\int d^3v (\ln \langle f \rangle + 1) \langle \hat{L}\hat{f} \rangle &= \int d^3v \frac{1}{J} (\ln \langle f \rangle + 1) \frac{\partial}{\partial \mathbf{Z}} \cdot (J \mathbf{J}_Z^A) \\ &= -\frac{\partial}{\partial \mathbf{x}} \cdot \mathbf{J}_S^A + \sigma^A \end{aligned} \quad (\text{A24})$$

where  $\mathbf{J}_S^A$  is the anomalous entropy flux given by

$$\mathbf{J}_S^A \equiv -\int d^3v (\ln \langle f \rangle + 1) \langle \hat{f} \hat{\mathbf{v}}_g \rangle \quad (\text{A25})$$

and  $\sigma^A$  is the anomalous entropy production defined as

$$\sigma^A \equiv \int d^3v \sigma_Z^A = \int d^3v \mathbf{J}_Z^A \cdot \mathbf{X}_Z \geq 0. \quad (\text{A26})$$

The local functional  $\sigma^A$  in the  $\mathbf{x}$ -space is given by the velocity space integral of  $\sigma_Z^A$  and is the same as given in Sec. V as is seen later. The anomalous entropy production arises from the resonant exchange of energy-momentum between the particles and the fluctuations as we now make clear.

In order to compare the formulation here to the results in Sec. V, we use the Maxwellian distribution function  $f_M$  as  $\langle f \rangle$ . Then, we have  $\partial f_M / \partial \mu = 0$  and Eq. (A26) is rewritten as

$$\sigma^A = - \int d^3v \left[ \langle \hat{f} \hat{v}_g \rangle \cdot \frac{\partial \ln f_M}{\partial \mathbf{x}} + \langle \hat{f} \hat{w} \rangle \frac{\partial \ln f_M}{\partial w} \right] = \mathbf{J}_1 \cdot \mathbf{X}_1 + \mathbf{J}_2 \cdot \mathbf{X}_2 + J_3 X_3 \quad (\text{A27})$$

which is found to be the same as Eq. (68) by noting that

$$- \left[ \hat{v}_g \cdot \frac{\partial \ln f_M}{\partial \mathbf{x}} + \hat{w} \frac{\partial \ln f_M}{\partial w} \right] = \hat{v}_E \cdot \left[ \mathbf{X}_1 + \left( x^2 - \frac{3}{2} \right) \mathbf{X}_2 \right] + e \hat{\mathbf{E}} \cdot (v_{\parallel} \mathbf{n} + \mathbf{v}_{\nabla B} + \mathbf{v}_{\text{curv}}) X_3. \quad (\text{A28})$$

The forces  $(\mathbf{X}_1, \mathbf{X}_2, X_3)$  and the fluxes  $(\mathbf{J}_1, \mathbf{J}_2, J_3)$  are defined in Sec. V and are related to the forces  $\mathbf{X}_Z$  and the fluxes  $\mathbf{J}_Z^A$  as

$$- \begin{bmatrix} \partial \ln f_M / \partial \mathbf{x} \\ \partial \ln f_M / \partial w \end{bmatrix} = \begin{bmatrix} \mathbf{l}_3 & (x^2 - \frac{3}{2}) \mathbf{l}_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \\ X_3 \end{bmatrix} \quad (\text{A29})$$

$$\begin{bmatrix} \mathbf{J}_1 \\ \mathbf{J}_2 \\ J_3 \end{bmatrix} = \int d^3v \begin{bmatrix} \mathbf{l}_3 & 0 \\ (x^2 - \frac{3}{2}) \mathbf{l}_3 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \langle \hat{f} \hat{v}_g \rangle \\ \langle \hat{f} \hat{w} \rangle \end{bmatrix} \quad (\text{A30})$$

where  $\mathbf{l}_3$  denotes the  $3 \times 3$  unit matrix. The anomalous transport equations in the  $\mathbf{Z}$ -space are written as

$$\begin{bmatrix} \langle \hat{f} \hat{v}_g \rangle \\ \langle \hat{f} \hat{w} \rangle \end{bmatrix} = -f_M D_4^A \begin{bmatrix} \partial \ln f_M / \partial \mathbf{x} \\ \partial \ln f_M / \partial w \end{bmatrix} \quad (\text{A31})$$

where  $D_4^A$  is the  $4 \times 4$  matrix reduced from  $D_2^A$  and is also symmetric and positive definite. Then we obtain the anomalous transport equations for the forces  $(\mathbf{X}_1, \mathbf{X}_2, X_3)$  and the fluxes  $(\mathbf{J}_1, \mathbf{J}_2, J_3)$  as

$$\begin{bmatrix} \mathbf{J}_1 \\ \mathbf{J}_2 \\ J_3 \end{bmatrix} = \mathbf{L}^A \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \\ X_3 \end{bmatrix} \quad (\text{A32})$$

where

$$\mathbf{L}^A = \int d^3v f_M \begin{bmatrix} \mathbf{l}_3 & 0 \\ (x^2 - \frac{3}{2}) \mathbf{l}_3 & 0 \\ 0 & 1 \end{bmatrix} D_4^A \begin{bmatrix} \mathbf{l}_3 & (x^2 - \frac{3}{2}) \mathbf{l}_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (\text{A33})$$

which shows that the matrix  $\mathbf{L}^A$  preserves the Onsager-type symmetry and the positive definiteness of  $D_4^A$ .



Using the Fourier representation for the fluctuations as in Eq. (57) and the approximation that  $\bar{\mathbf{x}}(-\tau) \simeq \mathbf{x} - (v_{\parallel} \mathbf{n} + \mathbf{v}_{d0})\tau$  within the correlation time of the fluctuations, we have

$$\langle \hat{\mathbf{E}}(0, \mathbf{x}) \hat{\mathbf{E}}(-\tau, \bar{\mathbf{x}}(-\tau)) \rangle = \sum_{\mathbf{k}} k k \langle |\hat{\phi}_{\mathbf{k}}|^2 \rangle \cos[(\omega - \omega_E - \omega_D - k_{\parallel} v_{\parallel})\tau] \quad (\text{A34})$$

and

$$D_4^A = \pi \langle |\hat{\phi}_{\mathbf{k}}|^2 \rangle \delta(\omega - \omega_E - \omega_D - k_{\parallel} v_{\parallel}) \times \left[ \begin{array}{cc} \left(\frac{c}{B}\right)^2 (\mathbf{k} \times \mathbf{n})(\mathbf{k} \times \mathbf{n}) & \frac{c}{B} (\mathbf{k} \times \mathbf{n}) e(\omega_D + k_{\parallel} v_{\parallel}) \\ e(\omega_D + k_{\parallel} v_{\parallel}) \frac{c}{B} (\mathbf{k} \times \mathbf{n}) & e^2(\omega_D + k_{\parallel} v_{\parallel})^2 \end{array} \right]. \quad (\text{A35})$$

Substituting Eq. (A35) into Eq. (A33), we obtain the same expressions as in Eq. (73).

## APPENDIX B: RELATION BETWEEN EQ. (72) AND THE LINEAR THERMODYNAMIC FORM OF THE ANOMALOUS TRANSPORT IN REF. (9)

Here, we derive the linear thermodynamic form of the anomalous transport equations (72). For that purpose, we use the drift wave dispersion relation for the eigenfrequency in Eq. (73) and linearize Eq. (72) with respect to the density and temperature gradient forces in the same way as in Ref. 9. (Since  $k_{\perp}\rho_s \sim \Delta \ll 1$  in our ordering, the anomalous fluxes by the electron drift wave is small as discussed in Sec. VI although here its dispersion relation is used for comparison to the results of the linear thermodynamic transport in Ref. 9.) We should note that, as in Ref. 9, we still treat the fluctuation spectrum  $\langle |\hat{\phi}_{\mathbf{k}}|^2 \rangle$  treated as given, and that we do not regard  $X_{a3} = 1/T_a$  as a thermodynamic force here. ( $X_{a3} = 1/T_a$  measures a velocity space gradient of the kinetic distribution function and really causes the anomalous entropy production although it is difficult to take the limit of  $X_{a3} \rightarrow 0$  in the linearizing procedure since it still exists even for the spatially uniform Maxwellian distribution which is a complete equilibrium state for the collisional (classical and neoclassical) processes.) Assuming the large-aspect-ratio system, we neglect  $\omega_{D_a}$  and use the following frequency ordering

$$\frac{|\omega - \omega_E|}{|k_{\parallel}|v_{Te}} \sim \frac{|k_{\parallel}|v_{Ti}}{|\omega - \omega_E|} \sim \lambda \ll 1. \quad (\text{B1})$$

Using the quasineutrality condition, we obtain the dispersion relation to the lowest order in  $\lambda$  as

$$\omega - \omega_E = A_0(1 - M\eta_i)\omega_{*e} = A_0 \frac{cT_e}{eB} (\mathbf{k} \times \mathbf{n}) \cdot (\mathbf{X}_{e1} - M\mathbf{X}_{i2}) \quad (\text{B2})$$

where  $A_0 = \Gamma_0/[1 + \theta(1 - \Gamma_0)]$ ,  $\Gamma_0 = e^{-b}I_0(b)$ ,  $M = b[1 - I_1(b)/I_0(b)]$ ,  $b = k_{\perp}^2 v_{Ti}^2 / 2\Omega_i^2$ ,  $\theta = Z_i T_e / T_i$ ,  $Z_i$  is the ion charge number, and  $I_n(b)$  are the modified Bessel functions. Here the finite gyroradius effect is retained through the  $b$ -dependence.

First, we consider the anomalous fluxes for the electrons. To the lowest order in  $\lambda$ ,  $\delta(\omega - \omega_E - \omega_{De} - k_{\parallel}v_{\parallel})$  in Eq. (73) for the electrons is replaced with  $\delta(k_{\parallel}v_{\parallel})$ . We also find that  $J_{e3}$  does not enter the linear thermodynamic transport equations since it is  $\mathcal{O}(X^2)$  where  $X$  denotes the order of the density and temperature gradient forces:  $\mathbf{X}_{a1} \sim \mathbf{X}_{a2} \sim X$ . As mentioned before,  $X_{a3} = 1/T_a$  is not regarded as a thermodynamic force so that  $X_{a3}$  is  $\mathcal{O}(X^0)$ . Then, we obtain the linearized form of the electron anomalous particle and heat fluxes as

$$\begin{bmatrix} \mathbf{J}_{e1} \\ \mathbf{J}_{e2} \end{bmatrix} = \begin{bmatrix} L_{11}^{ee} & L_{12}^{ee} & L_{12}^{e1} \\ L_{21}^{ee} & L_{22}^{ee} & L_{22}^{e1} \end{bmatrix} \begin{bmatrix} \mathbf{X}_{e1} \\ \mathbf{X}_{e2} \\ \mathbf{X}_{i2} \end{bmatrix} \quad (\text{B3})$$

where the transport coefficients are given by

$$\begin{aligned} L_{jk}^{ee} &= \lim_{\mathbf{X} \rightarrow 0} \left( L_{jk}^e + \frac{1}{T_e} \frac{\partial L_{j3}^e}{\partial \mathbf{X}_{ek}} \right) \\ L_{j2}^{e1} &= \lim_{\mathbf{X} \rightarrow 0} \frac{1}{T_e} \frac{\partial L_{j3}^e}{\partial \mathbf{X}_{i2}} \quad (j, k = 1, 2). \end{aligned} \quad (\text{B4})$$

Here,  $\lim_{\mathbf{X} \rightarrow 0}$  stands for the limit of  $\mathbf{X}_{e1}, \mathbf{X}_{e2}, \mathbf{X}_{i2} \rightarrow 0$ . Using the dispersion relation (B2), the anomalous transport matrix in Eq. (B3) is written as

$$\begin{aligned} \begin{bmatrix} L_{11}^{ee} & L_{12}^{ee} & L_{12}^{e1} \\ L_{21}^{ee} & L_{22}^{ee} & L_{22}^{e1} \end{bmatrix} &= \pi^{1/2} n_e \left( \frac{cT_e}{eB} \right)^2 \sum_{\mathbf{k}} \frac{e^2 \langle |\hat{\phi}_{\mathbf{k}}|^2 \rangle}{T_e^2} \frac{1}{|k_{\parallel}| v_{Te}} \\ &\times \begin{bmatrix} (1 - A_0)(\mathbf{k} \times \mathbf{n})(\mathbf{k} \times \mathbf{n}) & -\frac{1}{2}(\mathbf{k} \times \mathbf{n})(\mathbf{k} \times \mathbf{n}) & A_0 M(\mathbf{k} \times \mathbf{n})(\mathbf{k} \times \mathbf{n}) \\ -\frac{1}{2}(1 - A_0)(\mathbf{k} \times \mathbf{n})(\mathbf{k} \times \mathbf{n}) & \frac{5}{4}(\mathbf{k} \times \mathbf{n})(\mathbf{k} \times \mathbf{n}) & -\frac{1}{2}A_0 M(\mathbf{k} \times \mathbf{n})(\mathbf{k} \times \mathbf{n}) \end{bmatrix} \end{aligned} \quad (\text{B5})$$

which is in complete agreement with the electron anomalous transport equations in Ref. 9 to the lowest order in  $\lambda$  while it should be noted that the definition of the heat flux in Ref. 9 is different from ours. Now, we find that the symmetry  $L_{12}^{ee} = L_{21}^{ee}$  is no longer valid since the nonsymmetric additional terms  $(1/T_e)(\partial L_{j3}^e/\partial \mathbf{X}_{ek})$  enter these coefficients. Thus, the broken symmetry in the linear thermodynamic anomalous transport coefficients claimed by Balescu has been confirmed again. We also see that the neglect of  $J_{e3}$  in the electron anomalous

entropy production  $\sigma_e^A = \mathbf{J}_{e1} \cdot \mathbf{X}_{e1} + \mathbf{J}_{e2} \cdot \mathbf{X}_{e2} + J_{e3} X_{e3}$  breaks its positive definiteness since all of  $\mathbf{J}_{e1} \cdot \mathbf{X}_{e1}$ ,  $\mathbf{J}_{e2} \cdot \mathbf{X}_{e2}$ , and  $J_{e3} X_{e3}$  is of the same order  $\mathcal{O}(X^2)$  while  $J_{e3}$  ( $= \mathcal{O}(X^2)$ ) is necessarily neglected in the linear thermodynamic anomalous transport. In order to ensure the positive definiteness of the anomalous entropy production, it is indispensable to retain the energy transfer from the turbulent fields to the particles represented by  $J_{e3}$  which however has been rarely taken account of in conventional anomalous transport theories.

Next, let us consider the anomalous ion fluxes. The anomalous ion particle flux is given from the anomalous electron particle flux as  $\mathbf{J}_{i1} = Z_i^{-1} \mathbf{J}_{e1}$  which results from using the quasineutrality condition as the dispersion relation. To the lowest order in  $\lambda$ ,  $\delta(\omega - \omega_E - \omega_{Di} - k_{\parallel} v_{\parallel})$  in Eq. (73) for the ions is replaced with  $\delta(\omega - \omega_E) = \delta(A_0(1 - M\eta_i)\omega_{*e})$ . From this term, it is difficult to obtain the linear thermodynamic form of the anomalous ion heat flux even to the lowest order in  $\lambda$ . Furthermore, the  $k_{\perp}$ -spectrum of this anomalous ion heat flux has a singularity at  $M(b)\eta_i = 1$  ( $b = k_{\perp}^2 v_{Ti}^2 / 2\Omega_i^2$ ). These difficulties about the linear thermodynamic form of the anomalous ion heat flux using the approximation (B1) are the same as clarified by Balescu in Ref. 9.

## APPENDIX C: SOLUTION OF QUASILINEAR EQUATION IN THE PLATEAU REGIME

Here, we find the solution of the averaged drift kinetic equation (52). Putting

$$\langle \bar{f}_{a1} \rangle_{ens} = \bar{F}_{a1} + \bar{g}_a \quad (C1)$$

where

$$\bar{F}_{a1} = -\frac{2\pi I v_{\parallel}}{\lambda' \Omega_a} \frac{\partial \bar{f}_{a0}}{\partial \psi} = \frac{2v_{\parallel}}{v_{Ta}^2} \left[ V_{1a} + \left( x_a^2 - \frac{5}{2} \right) V_{2a} \right] \bar{f}_{a0}. \quad (C2)$$

Equation (52) is rewritten as

$$v_{\parallel} \mathbf{n} \cdot \nabla \bar{g}_a - C_a^T(\bar{g}_a) = -e_a v_{\parallel} E_{\parallel}^{(A)} \frac{\partial \bar{f}_{a0}}{\partial E_0} + C_a^T(\bar{F}_a) + C_a^F + \bar{\mathcal{D}}_a \quad (C3)$$

where  $C_a^T$  and  $C_a^F$  are the test and field particle parts of the linear collision operator, respectively. The  $l = 1$  part in the Legendre polynomial expansion of  $\langle \bar{f}_{a1} \rangle_{ens}$  as a function of  $\xi$  is written in the 13M approximation<sup>3</sup> as

$$\langle \bar{f}_{a1} \rangle_{ens}^{(l=1)} = \frac{2v_{\parallel}}{v_{Ta}^2} \left[ u_{\parallel a} + \frac{2}{5} \frac{q_{\parallel a}}{p_a} \left( x_a^2 - \frac{5}{2} \right) \right] \bar{f}_{a0} \quad (C4)$$

and that of  $\bar{g}_a$  is given as

$$\bar{g}_{a1}^{(l=1)} = \frac{2v_{\parallel}}{v_{Ta}^2} B \left[ u_{a\theta} + \frac{2}{5} \frac{q_{a\theta}}{p_a} \left( x_a^2 - \frac{5}{2} \right) \right] \bar{f}_{a0}. \quad (C5)$$

We find

$$(v_{\parallel} \mathbf{n} \cdot \nabla \bar{g}_a)^{(l=0)} \equiv \frac{1}{2} \int_{-1}^1 d\xi v_{\parallel} \mathbf{n} \cdot \nabla \bar{g}_a = 0. \quad (C6)$$

Then the  $l = 0$  part of Eq. (C3) is given by

$$-C_a^T(\bar{g}_a^{(l=0)}) = \bar{\mathcal{D}}_a^{(l=0)} + C_a^F{}^{(l=0)}. \quad (C7)$$

Subtracting Eq. (C7) from Eq. (C3) yields

$$v_{\parallel} \mathbf{n} \cdot \nabla \bar{g}_a - \nu_a \mathcal{L}_a \bar{g}_a = \xi F(v, \psi, \theta) + C_a^{(l \geq 2)} + \bar{\mathcal{D}}_a^{(l \geq 2)} \quad (C8)$$

where  $\nu_a \mathcal{L}_a$  denotes the pitch angle scattering part of the collision operator and

$$C'_a = C_a^T(\bar{g}_a) + C_a^F - \nu_a \mathcal{L}_a \bar{g}_a. \quad (\text{C9})$$

The superscript ( $l \geq 2$ ) represents the sum of the Legendre polynomial components with  $l \geq 2$ . Here  $F(v, \psi, \theta)$  is an isotropic function in the velocity space and its functional form will not affect the results of the following analysis.

Let us put

$$\bar{g}_a = \bar{g}_a^{(l=1)} - (\nu_a \mathcal{L}_a)^{-1} \langle \bar{\mathcal{D}}_a^{(l \geq 2)} \rangle + \bar{h}_a \quad (\text{C10})$$

where  $(\nu_a \mathcal{L}_a)^{-1}$  represents the inverse of the pitch angle scattering operator. Here  $\bar{g}_a^{(l=1)}$  and  $(\nu_a \mathcal{L}_a)^{-1} \langle \bar{\mathcal{D}}_a^{(l \geq 2)} \rangle$  do not contribute to the parallel viscosities. Neglecting  $C_a^{(l \geq 2)}$  compared to the pitch angle scattering term in Eq. (C8), we have

$$\begin{aligned} & \left[ \xi \frac{\partial}{\partial \theta} - \frac{1}{2} \frac{\partial \ln B}{\partial \theta} (1 - \xi^2) \frac{\partial}{\partial \xi} - \frac{\bar{v}_a}{2} \frac{\partial}{\partial \xi} (1 - \xi^2) \frac{\partial}{\partial \xi} \right] \bar{h}_a \\ &= \frac{\partial \ln B}{\partial \theta} \left\{ \frac{1 - 3\xi^2}{2} \frac{2x_a}{v_{Ta}} B \left[ u_{a\theta} + \frac{2q_{a\theta}}{5p_a} \left( x_a^2 - \frac{5}{2} \right) \right] \bar{f}_{a0} - \frac{1}{\nu_a} \int_{-1}^{\xi} \langle \bar{\mathcal{D}}_a^{(l \geq 2)} \rangle d\xi \right\} \\ &+ (\omega_{Ta} x_a)^{-1} \left\{ \bar{\mathcal{D}}_a^{(l \geq 2)} - \langle \bar{\mathcal{D}}_a^{(l \geq 2)} \rangle \right\} \\ &+ \xi \left\{ (\omega_{Ta} x_a)^{-1} F - \frac{2\bar{v}_a}{v_{Ta}} x_a B \left[ u_{a\theta} + \frac{2q_{a\theta}}{5p_a} \left( x_a^2 - \frac{5}{2} \right) \right] \bar{f}_{a0} \right\} \end{aligned} \quad (\text{C11})$$

where

$$\begin{aligned} \bar{v}_a &= (\omega_{Ta} \tau_{aa})^{-1} \frac{\tau_{aa} \nu_a(x_a)}{x_a} \\ \omega_{Ta} &= \frac{\chi'}{2\pi B \sqrt{g}} v_{Ta} \simeq \frac{v_{Ta}}{Rq}. \end{aligned} \quad (\text{C12})$$

Here the collision frequency  $\tau_{aa}$  and the energy dependent collision frequency  $\nu_a(x_a)$  are defined in Ref. 2, and we have used the Jacobian  $\sqrt{g} = (\nabla\psi \times \nabla\theta \cdot \nabla\zeta)^{-1}$ , the major radius  $R = |\nabla\zeta|^{-1}$  and the safety factor  $q$ . In the large aspect ratio system, we have  $\partial \ln B / \partial \theta \simeq$

$\epsilon \sin \theta$  with the inverse aspect ratio  $\epsilon$  and we also assume that  $\left\{ \overline{\mathcal{D}}_a^{(l \geq 2)} - \langle \overline{\mathcal{D}}_a^{(l \geq 2)} \rangle \right\} / \langle \overline{\mathcal{D}}_a^{(l \geq 2)} \rangle \sim \epsilon$ . For the plateau regime  $\epsilon^{3/2} \ll (\omega_{T_a} \tau_{aa})^{-1} \ll 1$ , the ordering  $\xi \sim \bar{\nu}_a^{1/3} \ll 1$  and the perturbation expansion with respect to  $\epsilon / \bar{\nu}_a^{2/3}$  are used<sup>5</sup> to solve Eq. (C11) and the lowest order solution is given by

$$\begin{aligned} \bar{h}_a = & \epsilon \bar{\nu}_a^{-1/3} \left\{ \frac{x_a}{v_{T_a}} B \left[ u_{a\theta} + \frac{2}{5} \frac{q_{a\theta}}{p_a} \left( x_a^2 - \frac{5}{2} \right) \right] \bar{f}_{a0} + \frac{1}{\nu_a} \int_0^1 \langle \overline{\mathcal{D}}_a^{(l \geq 2)} \rangle d\xi \right\} \\ & \times \int_0^\infty d\tau \sin(\theta - \bar{\nu}_a^{-1/3} \xi \tau) e^{-\tau^3/6}. \end{aligned} \quad (\text{C13})$$

## APPENDIX D: ANOMALOUS EFFECTS BY THE ITG MODES WITH $(\omega - \omega_E)/k_{\parallel}v_{Ti} = O(1)$

The dispersion relation for ion-temperature gradient (ITG) modes given by Eq. (48) is given in Kim *et al.*<sup>15,16</sup> here we simply analyze the limit of small  $\omega_D/k_{\parallel}v_{Ti}$ . Assuming that the response of the electrons to the electrostatic fluctuations is approximately adiabatic, the drift kinetic equation and the quasineutrality condition yield the following dispersion relation

$$1 + \frac{\omega_{*e}}{k_{\parallel}v_{Te}}Z(\zeta) + \frac{Z_i T_e}{T_i}[1 + \zeta Z(\zeta)] + \frac{\omega_{*e}\eta_h}{k_{\parallel}v_{Ti}}\left[\zeta + \left(\zeta^2 - \frac{1}{2}\right)Z(\zeta)\right] = 0 \quad (D1)$$

where  $Z(\zeta)$  is a plasma dispersion function and  $\zeta = (\omega - \omega_E)/k_{\parallel}v_{Ti}$ . Furthermore we assume that the density gradient is small  $\omega_{*a} \simeq 0$ . From the above dispersion relation, we find that instabilities occur when

$$\left|\frac{\omega_{*e}\eta_h}{k_{\parallel}v_{Ti}}\right| > \left[2\left(1 + \frac{Z_i T_e}{T_i}\right)\right]^{1/2}. \quad (D2)$$

The adiabatic response of the electrons, which is assumed in the dispersion relation in Eq. (D1), yields no anomalous contributions to the heat generation rate, the particle and heat transport fluxes and the parallel viscosities for electrons. Then the ion particle flux also vanishes according to the ambipolarity resulting from the quasineutrality. Thus, we treat the anomalous contributions to the heat generation rate, the heat flux and the parallel viscosities for ions only.

For  $|\omega_{*e}\eta_h/k_{\parallel}v_{Ti}| \gg 1$ , we obtain the typical slab ITG mode instability with

$$\omega - \omega_E \simeq |k_{\parallel}^2 c_s^2 \omega_{*i} \eta_i|^{1/3} (\sigma_{*i} + \sqrt{3}i)/2. \quad (D3)$$

Here  $c_s \equiv (Z_i T_e/m_i)^{1/2}$  denotes the ion sound velocity and

$$\sigma_{*i} \equiv \frac{\omega_{*i}\eta_h}{|\omega_{*i}\eta_i|} = \frac{\mathbf{k} \times \mathbf{n} \cdot \nabla\psi}{|\mathbf{k} \times \mathbf{n} \cdot \nabla\psi|} \frac{\partial T_i / \partial \psi}{|\partial T_i / \partial \psi|} \quad (D4)$$



the sign of the ion diamagnetic frequency. Then, we have  $|(\omega - \omega_E)/k_{\parallel}v_{Ti}| \gg 1$  and therefore the fluctuation-induced forces  $\mathbf{K}_{ij}$  and parallel viscosities  $Y_{ij}$  ( $j = 1, 2$ ) for ions are small. Now, let us consider the unstable modes with  $|(\omega - \omega_E)/k_{\parallel}v_{Ti}| \simeq O(1)$ . These modes exist near the marginal point  $|\omega_{*e}\eta_i/k_{\parallel}v_{Ti}| \sim [2(1 + Z_i T_e/T_i)]^{1/2}$  and their real frequencies are given by

$$\frac{\omega - \omega_E}{|k_{\parallel}v_{Ti}|} \sim \sigma_{*i} \left[ \frac{1}{2} \left( 1 + \frac{Z_i T_e}{T_i} \right) \right]^{1/2} \equiv \sigma_{*i} \zeta_0. \quad (\text{D5})$$

Then, from Eqs. (61) and (79), we have  $\langle H_i \rangle \simeq 0$  and

$$T_i^{-1} \langle \mathbf{q}_i \cdot \nabla \psi \rangle^{\text{anom}} = -4\pi^{1/2} \frac{n_e e c}{Z_i T_e} \zeta_0 \exp(-\zeta_0^2) \times \sum_{\mathbf{k}}' \left\langle \frac{1}{B} |\mathbf{k} \times \mathbf{n} \cdot \nabla \psi| \langle |\hat{\phi}_{\mathbf{k}}|^2 \rangle_{\text{ens}} \right\rangle \frac{\partial T_i / \partial \psi}{|\partial T_i / \partial \psi|} \quad (\text{D6})$$

$$\langle \mathbf{B} \cdot \mathbf{K}_{i2} \rangle = -2\pi^{1/2} \frac{n_e e^2}{T_e} \zeta_0 \exp(-\zeta_0^2) \sum_{\mathbf{k}}' \sigma_{*i} \langle k_{\parallel} B \langle |\hat{\phi}_{\mathbf{k}}|^2 \rangle_{\text{ens}} \rangle \quad (\text{D7})$$

$$\begin{aligned} \begin{bmatrix} Y_{i1} \\ Y_{i2} \end{bmatrix} &= \pi^{3/2} \epsilon^2 p_i \omega_{Ti} \tau_{ii} B_0 \zeta_0 \int_0^{\infty} dx_i e^{-x_i^2} \frac{x_i^4}{\tau_{ii} \nu_i(x_i)} \left[ \left( x_i^2 - \frac{5}{2} \right) \right] \\ &\times \sum_{\mathbf{k}}' \sigma_{*i} k_{\parallel} \left[ y_1(x_i, \zeta_0) + \frac{2T_i}{Z_i T_e} y_2(x_i, \zeta_0) \right] \frac{e_i^2 \langle |\hat{\phi}_{\mathbf{k}}|^2 \rangle_{\text{ens}}}{T_i^2} \end{aligned} \quad (\text{D8})$$

$$\begin{aligned} \begin{bmatrix} W_{i1} \\ W_{i2} \end{bmatrix} &= \pi \frac{v_{Ti}^2 \tau_{ii}}{B_0} \zeta_0 \int_0^{\infty} dx_i e^{-x_i^2} \frac{x_i^4}{\tau_{ii} \nu_i(x_i)} \left[ \begin{pmatrix} -\frac{3}{2} + \frac{1}{6} x_i^2 \\ 1 - \frac{1}{3} x_i^2 \end{pmatrix} \right] \\ &\times \sum_{\mathbf{k}}' \sigma_{*i} k_{\parallel} \left[ y_1(x_i, \zeta_0) + \frac{2T_i}{Z_i T_e} y_2(x_i, \zeta_0) \right] \frac{e_i^2 \langle |\hat{\phi}_{\mathbf{k}}|^2 \rangle_{\text{ens}}}{T_i^2} \end{aligned} \quad (\text{D9})$$

where  $\Sigma'$  represents the summation over the wavenumber region where  $|\omega_{*e}\eta_i/k_{\parallel}v_{Ti}| \sim [2(1 + Z_i T_e/T_i)]^{1/2}$  and the functions  $y_1$  and  $y_2$  are defined by

$$y_1(x, \gamma) = \frac{1}{8\gamma} \delta(x - \gamma) + \frac{1}{8x^2} (4\gamma x - 6\gamma^2 - 3) H(x - \gamma)$$

$$\begin{aligned}
y_2(x, \gamma) &= \frac{1}{8\gamma} \left( \gamma^2 - \frac{3}{2} \right) \delta(x - \gamma) \\
&+ \frac{1}{16x^2} \{ \gamma(8x^3 - 20x) - (12\gamma^2 + 6)x^2 + 30\gamma^2 + 9 \} H(x - \gamma). \quad (\text{D10})
\end{aligned}$$

When  $Z_i T_e / T_i = 1$ , we obtain Eq. (S1) from Eqs. (D6)–(D9).

## References

- <sup>1</sup>F. L. Hinton and R. D. Hazeltine, *Rev. Mod. Phys.* **42**, 239 (1976).
- <sup>2</sup>S. P. Hirshman and D. J. Sigmar, *Nucl. Fusion* **21**, 1079 (1981).
- <sup>3</sup>R. Balescu, *Transport Processes in Plasmas* (2 vols.) (North-Holland, Amsterdam, 1988).
- <sup>4</sup>P. C. Liewer, *Nucl. Fusion* **25**, 543 (1985).
- <sup>5</sup>K. C. Shaing, *Phys. Fluids* **31**, 8 (1988).
- <sup>6</sup>K. C. Shaing, *Phys. Fluids* **31**, 2249 (1988).
- <sup>7</sup>R. Balescu, *Phys. Fluids B* **2**, 2100 (1990).
- <sup>8</sup>R. Balescu, *Phys. Fluids B* **3**, 564 (1991).
- <sup>9</sup>R. Balescu, *Phys. Fluids B* **4**, 91 (1992).
- <sup>10</sup>W. Horton, *Plasma Phys.* **22**, 345 (1980).
- <sup>11</sup>R. D. Hazeltine and J. D. Meiss, *Plasma Confinement* (Addison-Wesley, Redwood City, California, 1992), p.113.
- <sup>12</sup>R. D. Hazeltine, *Plasma Phys.* **15**, 77 (1973).
- <sup>13</sup>P. H. Rutherford and E. A. Frieman, *Phys. Fluids B* **11**, 569 (1968).
- <sup>14</sup>J. B. Taylor and R. J. Hastie, *Plasma Phys.* **10**, 479 (1968).
- <sup>15</sup>W. Horton and R. K. Varma, *Phys. Fluids* **15**, 620 (1972).
- <sup>16</sup>W. Horton, *Phys. Rep.* **192**, 1 (1990).

<sup>17</sup>J.Y. Kim and W. Horton, *Physics of Fluids B* **3**, 1167 (1991).

<sup>18</sup>J.Y. Kim, Y. Kishimoto, W. Horton, and T. Tajima, *Phys. of Plasmas* **1**, 927 (1994).

<sup>19</sup>J. Q. Dong, W. Horton, R. D. Bengston, and G. X. Li, *Phys. of Plasmas* **1**, 3250 (1994).

## Recent Issues of NIFS Series

- NIFS-306 S. Murakami, M. Okamoto, N. Nakajima, K.Y. Watanabe, T. Watari, T. Mutoh, R. Kumazawa and T. Seki,  
*Monte Carlo Simulation for ICRF Heating in Heliotron/Torsatrons;*  
Sep. 1994 (IAEA-CN-60/D-P-I-14)
- NIFS-307 Y. Takeiri, A. Ando, O. Kaneko, Y. Oka, K. Tsumori, R. Akiyama, E. Asano, T. Kawamoto, T. Kuroda, M. Tanaka and H. Kawakami,  
*Development of an Intense Negative Hydrogen Ion Source with a Wide-Range of External Magnetic Filter Field;* Sep. 1994
- NIFS-308 T. Hayashi, T. Sato, H.J. Gardner and J.D. Meiss,  
*Evolution of Magnetic Islands in a Heliac;* Sep. 1994
- NIFS-309 H. Amo, T. Sato and A. Kageyama,  
*Intermittent Energy Bursts and Recurrent Topological Change of a Twisting Magnetic Flux Tube;* Sep.1994
- NIFS-310 T. Yamagishi and H. Sanuki,  
*Effect of Anomalous Plasma Transport on Radial Electric Field in Torsatron/Heliotron;* Sep. 1994
- NIFS-311 K. Watanabe, T. Sato and Y. Nakayama,  
*Current-profile Flattening and Hot Core Shift due to the Nonlinear Development of Resistive Kink Mode;* Oct. 1994
- NIFS-312 M. Salimullah, B. Dasgupta, K. Watanabe and T. Sato,  
*Modification and Damping of Alfvén Waves in a Magnetized Dusty Plasma;* Oct. 1994
- NIFS-313 K. Ida, Y. Miura, S -I. Itoh, J.V. Hofmann, A. Fukuyama, S. Hidekuma, H. Sanuki, H. Idei, H. Yamada, H. Iguchi, K. Itoh,  
*Physical Mechanism Determining the Radial Electric Field and its Radial Structure in a Toroidal Plasma;* Oct. 1994
- NIFS-314 Shao-ping Zhu, R. Horiuchi, T. Sato and The Complexity Simulation Group,  
*Non-Taylor Magnetohydrodynamic Self-Organization;* Oct. 1994
- NIFS-315 M. Tanaka,  
*Collisionless Magnetic Reconnection Associated with Coalescence of Flux Bundles;* Nov. 1994
- NIFS-316 M. Tanaka,  
*Macro-EM Particle Simulation Method and A Study of Collisionless Magnetic Reconnection;* Nov. 1994
- NIFS-317 A. Fujisawa, H. Iguchi, M. Sasao and Y. Hamada,

- Second Order Focusing Property of 210° Cylindrical Energy Analyzer;*  
Nov. 1994
- NIFS-318 T. Sato and Complexity Simulation Group,  
*Complexity in Plasma - A Grand View of Self- Organization;* Nov. 1994
- NIFS-319 Y. Todo, T. Sato, K. Watanabe, T.H. Watanabe and R. Horiuchi,  
*MHD-Vlasov Simulation of the Toroidal Alfvén Eigenmode;* Nov. 1994
- NIFS-320 A. Kageyama, T. Sato and The Complexity Simulation Group,  
*Computer Simulation of a Magnetohydrodynamic Dynamo II;* Nov. 1994
- NIFS-321 A. Bhattacharjee, T. Hayashi, C.C.Hegna, N. Nakajima and T. Sato,  
*Theory of Pressure-induced Islands and Self-healing in Three-dimensional Toroidal Magnetohydrodynamic Equilibria;* Nov. 1994
- NIFS-322 A. Iiyoshi, K. Yamazaki and the LHD Group,  
*Recent Studies of the Large Helical Device;* Nov. 1994
- NIFS-323 A. Iiyoshi and K. Yamazaki,  
*The Next Large Helical Devices;* Nov. 1994
- NIFS-324 V.D. Pustovitov  
*Quasisymmetry Equations for Conventional Stellarators;* Nov. 1994
- NIFS-325 A. Taniike, M. Sasao, Y. Hamada, J. Fujita, M. Wada,  
*The Energy Broadening Resulting from Electron Stripping Process of a Low Energy Au<sup>-</sup> Beam;* Dec. 1994
- NIFS-326 I. Viniar and S. Sudo,  
*New Pellet Production and Acceleration Technologies for High Speed Pellet Injection System "HIPEL" in Large Helical Device;* Dec. 1994
- NIFS-327 Y. Hamada, A. Nishizawa, Y. Kawasumi, K. Kawahata, K. Itoh, A. Ejiri, K. Toi, K. Narihara, K. Sato, T. Seki, H. Iguchi, A. Fujisawa, K. Adachi, S. Hidekuma, S. Hirokura, K. Ida, M. Kojima, J. Koong, R. Kumazawa, H. Kuramoto, R. Liang, T. Minami, H. Sakakita, M. Sasao, K.N. Sato, T. Tsuzuki, J. Xu, I. Yamada, T. Watari,  
*Fast Potential Change in Sawteeth in JIPP T-IIU Tokamak Plasmas;*  
Dec. 1994
- NIFS-328 V.D. Pustovitov,  
*Effect of Satellite Helical Harmonics on the Stellarator Configuration;*  
Dec. 1994
- NIFS-329 K. Itoh, S-I. Itoh and A. Fukuyama,  
*A Model of Sawtooth Based on the Transport Catastrophe;* Dec. 1994
- NIFS-330 K. Nagasaki, A. Ejiri,

*Launching Conditions for Electron Cyclotron Heating in a Sheared Magnetic Field; Jan. 1995*

- NIFS-331 T.H. Watanabe, Y. Todo, R. Horiuchi, K. Watanabe, T. Sato,  
*An Advanced Electrostatic Particle Simulation Algorithm for Implicit Time Integration; Jan. 1995*
- NIFS-332 N. Bekki and T. Karakisawa,  
*Bifurcations from Periodic Solution in a Simplified Model of Two-dimensional Magnetoconvection; Jan. 1995*
- NIFS-333 K. Itoh, S.-I. Itoh, M. Yagi, A. Fukuyama,  
*Theory of Anomalous Transport in Reverse Field Pinch; Jan. 1995*
- NIFS-334 K. Nagasaki, A. Isayama and A. Ejiri  
*Application of Grating Polarizer to 106.4GHz ECH System on Heliotron-E; Jan. 1995*
- NIFS-335 H. Takamaru, T. Sato, R. Horiuchi, K. Watanabe and Complexity Simulation Group,  
*A Self-Consistent Open Boundary Model for Particle Simulation in Plasmas; Feb. 1995*
- NIFS-336 B.B. Kadomtsev,  
*Quantum Telegraph : is it possible?; Feb. 1995*
- NIFS-337 B.B.Kadomtsev,  
*Ball Lightning as Self-Organization Phenomenon; Feb. 1995*
- NIFS-338 Y. Takeiri, A. Ando, O. Kaneko, Y. Oka, K. Tsumori, R. Akiyama, E. Asano, T. Kawamoto, M. Tanaka and T. Kuroda,  
*High-Energy Acceleration of an Intense Negative Ion Beam; Feb. 1995*
- NIFS-339 K. Toi, T. Morisaki, S. Sakakibara, S. Ohdachi, T. Minami, S. Morita, H. Yamada, K. Tanaka, K. Ida, S. Okamura, A. Ejiri, H. Iguchi, K. Nishimura, K. Matsuoka, A. Ando, J. Xu, I. Yamada, K. Narihara, R. Akiyama, H. Idei, S. Kubo, T. Ozaki, C. Takahashi, K. Tsumori,  
*H-Mode Study in CHS; Feb. 1995*
- NIFS-340 T. Okada and H. Tazawa,  
*Filamentation Instability in a Light Ion Beam-plasma System with External Magnetic Field; Feb. 1995*
- NIFS-341 T. Watanabe, G. Gnudi,  
*A New Algorithm for Differential-Algebraic Equations Based on HIDM; Feb. 13, 1995*
- NIFS-342 Y. Nejoh,  
*New Stationary Solutions of the Nonlinear Drift Wave Equation;*

Feb. 1995

- NIFS-343 A. Ejiri, S. Sakakibara and K. Kawahata,  
*Signal Based Mixing Analysis for the Magnetohydrodynamic Mode Reconstruction from Homodyne Microwave Reflectometry*; Mar.. 1995
- NIFS-344 B.B.Kadomtsev, K. Itoh, S.-I. Itoh  
*Fast Change in Core Transport after L-H Transition*; Mar. 1995
- NIFS-345 W.X. Wang, M. Okamoto, N. Nakajima and S. Murakami,  
*An Accurate Nonlinear Monte Carlo Collision Operator*; Mar. 1995
- NIFS-346 S. Sasaki, S. Takamura, S. Masuzaki, S. Watanabe, T. Kato, K. Kadota,  
*Helium I Line Intensity Ratios in a Plasma for the Diagnostics of Fusion Edge Plasmas*; Mar. 1995
- NIFS-347 M. Osakabe,  
*Measurement of Neutron Energy on D-T Fusion Plasma Experiments*;  
Apr. 1995
- NIFS-348 M. Sita Janaki, M.R. Gupta and Brahmananda Dasgupta,  
*Adiabatic Electron Acceleration in a Cnoidal Wave*; Apr. 1995
- NIFS-349 J. Xu, K. Ida and J. Fujita,  
*A Note for Pitch Angle Measurement of Magnetic Field in a Toroidal Plasma Using Motional Stark Effect*; Apr. 1995
- NIFS-350 J. Uramoto,  
*Characteristics for Metal Plate Penetration of a Low Energy Negative Muonlike or Pionlike Particle Beam*: Apr. 1995
- NIFS-351 J. Uramoto,  
*An Estimation of Life Time for A Low Energy Negative Pionlike Particle Beam*: Apr. 1995
- NIFS-352 A. Taniike,  
*Energy Loss Mechanism of a Gold Ion Beam on a Tandem Acceleration System*: May 1995
- NIFS-353 A. Nishizawa, Y. Hamada, Y. Kawasumi and H. Iguchi,  
*Increase of Lifetime of Thallium Zeolite Ion Source for Single-Ended Accelerator*: May 1995
- NIFS-354 S. Murakami, N. Nakajima, S. Okamura and M. Okamoto,  
*Orbital Aspects of Reachable  $\beta$  Value in NBI Heated Heliotron/Torsatrons*; May 1995