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UNE METHODE ELEMENTS FINIS ADAPTATIVE POUR LES CALCULS D'ECOULEMENTS TURBULENTS

*AN ADAPTATIVE FINITE ELEMENT METHOD FOR TURBULENT FLOW SIMULATIONS*

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# *AN ADAPTATIVE FINITE ELEMENT METHOD FOR TURBULENT FLOW SIMULATIONS*

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**SYNTHÈSE :**

Après avoir présenté dans les grandes lignes les méthodes de discrétisation en temps et en espace utilisées dans le code de thermohydraulique N3S développé au LNH, on décrit les facilités offertes par la version périphérique "Maillage Adaptatif" qui est composée de deux parties distinctes : le calcul d'indicateurs d'erreur d'une part et le développement d'un module de découpage d'éléments surfaciques et volumiques qui a pour vocation d'être utilisé par les codes ASTER de mécanique du solide, TRIFOU d'électromagnétisme développés également à la DER.

Les indicateurs d'erreur mis en oeuvre dans N3S sont décrits. Il s'agit d'un indicateur de projection permettant de quantifier l'erreur en espace commise dans un calcul d'écoulement que celui-ci soit laminaire ou turbulent ainsi que d'un indicateur représentant le résidu sur chaque élément des équations de Navier-Stokes.

La méthode de découpage des triangles en quatre sous-triangles et des tétraèdres en huit sous-tétraèdres est ensuite présentée avec ses contraintes et avantages. Elle est illustrée d'exemples traduisant les performances du module développé.

La dernière partie est consacrée au cas bidimensionnel de l'écoulement derrière une marche descendante.

**EXECUTIVE SUMMARY :**

After outlining the space and time discretization methods used in the N3S thermal hydraulic code developed at EDF/NHL, we describe the possibilities of the peripheral version, the Adaptative Mesh, which comprises two separate parts : the error indicator computation and the development of a module subdividing elements usable by the solid dynamics code ASTER and the electromagnetism code TRIFOU also developed by R&DD.

The error indicators implemented in N3S are described. They consist of a projection indicator quantifying the space error in laminar or turbulent flow calculations and a Navier-Stokes residue indicator calculated on each element.

The method for subdivision of triangles into four sub-triangles and tetrahedra into eight sub-tetrahedra is then presented with its advantages and drawbacks. It is illustrated by examples showing the efficiency of the module.

The last part concerns the 2D case of flow behind a backward-facing step.

# An adaptive finite element method for turbulent flow simulations

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## 1 Fluid dynamics code N3S

The main problems that the finite element code N3S deals with are thermohydraulic problems concerning flows with heat transfers in the various components of nuclear power plants such as head adapter, reactor vessel, steam generator pipes.

N3S has also been applied to various studies in flow configurations where the fluid is either compressible or incompressible such as external aerodynamics (flow around vehicles or in the urban environment). Specific applications required complementary developments like turbomachinery, flow around cooling towers.

The code solves Navier-Stokes equations in 2D, 2D axisymmetric or 3D geometries for laminar or turbulent problems. The turbulence is solved using a k-$\epsilon$ model and the wall boundary conditions follow the Reichardt law.

### 1.1 Time discretization

The Navier-Stokes equations are time discretized using a fractional step method [4]. The convection step consists of calculating the convected velocity field $\widetilde{u}^{n+1}$ which is given at the instant $t^n = n\Delta t$ by

$$\widetilde{u}^{n+1}(x) = u^n(X_x(t^n))$$

where $X_x(t^n)$ is the foot of the characteristic curve transported by $u^n$ which at the instant $t^{n+1}$ is such that

$$X_x(t^{n+1}) = x.$$

### 1.2 Spatial discretization

The second step solves the Stokes problem and diffusion of the scalar variables with appropriate boundary conditions. The solution of Stokes problem is based on the Uzawa or Chorin [5] algorithm. Diffusion problems are solved using a Preconditioned Conjugated Gradient method. Diffusion and Stokes problems are discretized using the Taylor-Hood mixed finite element P2-P1 or isoP2-P1 where the degrees of freedom for pressure are the vertices of the triangles in 2D or tetrahedra in 3D and those for velocity and scalar ($T$, $k$, $\epsilon$) are the vertices plus the middles of the edges.

## 2 Adaptive facilities

A-posteriori error indicators have been implemented in N3S for laminar [3] and for turbulent flows. In the case of laminar flows there are two indicators the projection indicator and the residue indicator. For turbulent flows only the projection indicator is available. A 3D indicator is currently being developed. During the adaptive process the time step value is automatically recalculated on each mesh in order to control the CFL number. For steady cases the time convergence is achieved on each mesh before the adaptation step however in the case of transient problems, the number of time steps completed between two adaptations has to be determined.

Several strategies have been implemented : firstly a uniform meshing refinement where all elements are divided is useful to test the indicators. Secondly refinement at predefined thresholds can be carried out. Another strategy is to define a maximum number of adaptations for a final maximum number of elements which must not be exceeded during successive refinements. Finally a percentage of elements which are going to be refined can be defined.

The coupling between N3S and the refinement software is external. A UNIX program controls the process.

## 3 Error indicators

### 3.1 Laminar flows

#### 3.1.1 *Projection indicator*

This indicator was first introduced in elasticity [9] and then transposed to fluid dynamics [7]. It was then implemented with appropriate adjustements at Electricité de France in N3S. It is based on the fact that the discrete constraint tensor $\sigma_h = -p_h I + 2\mu\gamma(u_h)$ is discontinuous at the boundary of the element because of the discrete deformation rate tensor $\gamma(u_h)$. The projection $\gamma(u_h)^*$ of this term on the velocity discretization space $V = Vect(\varphi_k)$ has been computed. The projection problem can be written :
find $\gamma(u_h)^* \in V$, such that

$$\forall \tau \in V, \int_{\Omega_h} 2(\mu + \mu_T)(\gamma(u_h) - \gamma(u_h)^*) : \tau \, d\omega = 0$$

In the laminar case assuming the viscosity is constant, the problem is equivalent to a $L^2$ projection of $\gamma(u_h)$. The pressure is continuous so its discontinuous gradient is projected. Finally homogeneity considerations lead to the expression of the indicator

$$I_K = (I_{K,u}^2 + I_{K,p}^2)^{\frac{1}{2}}$$

where

$$I_{K,u} = \{\int_K 2(\mu + \mu_T)(\gamma(u_h)^* - \gamma(u_h)) : (\gamma(u_h)^* - \gamma(u_h))\, d\omega\}^{\frac{1}{2}} \tag{3.1}$$

$$I_{K,p} = \{\int_K \frac{1}{2\mu}(\nabla p_h^* - \nabla p_h).(\nabla p_h^* - \nabla p_h)\, d\omega\}^{\frac{1}{2}} \tag{3.2}$$

3.1.2 *Residue indicator for Navier-Stokes*

Theoretical proof [2] has been provided for a residue indicator [8] in the case of Poisson and Stokes problems. First local indicators are limited upward by the exact local error and furthermore the associated global error estimator is an upper limit of the exact global error. It is not easy to apply these results in the case of laminar isothermal Navier-Stokes problem, considering the time discretization method used. Indeed, the characteristic method generates time and spatial error (even in steady cases). In order to take this spatial error into account and to eradicate a part of the time discretization error, the following indicator (here in the isoP2-P1 case) has been built :

$$I_K = \begin{pmatrix} h_K^2 \| \dfrac{u_h^{n+1} - \mathcal{I}_h \tilde{u}_h^{n+1}}{\Delta t} + \nabla P_h^{n+1} - f_{mh}^{n+1} \|_{L^2(K)^2}^2 \\ + \ \|\nabla . u_h^{n+1}\|_{L^2(K)}^2 \\ + \displaystyle\sum_{F \in S'(K)} h_F \| [\nu \frac{\partial u_h^{n+1}}{\partial n}] \|_{L^2(F)^2}^2 \end{pmatrix}^{\frac{1}{2}} \tag{3.3}$$

where $f_{mh}$ is the orthogonal projection of $f$. The quantity $[\nu \frac{\partial u_h^{n+1}}{\partial n}]$ represents the jump of the normal derivative of velocity across the edge F of $K$. As mentioned above $\tilde{u}_h^{n+1}$ is computed using the characteristic method and is not piecewise polynomial on the triangulation.

Our conclusions according to many tests is that first the Navier-Stokes residue indicator behaves as the theoretical error, and it is more regular than the projection indicator which may underestimate the error.

### 3.2 Turbulent flows

In order to take into account the turbulent character of the flow, the quantities $k$, $\epsilon$ and $\mu_T$ are introduced in the error indicator expression. The eddy viscosity $\mu_T$ is a nonlinear expression of $k$ and $\epsilon$. The coefficient has to be removed from the above expressions and to be treated separatly as a variable itself. It would also be projected. Each term has to be normalized to be sure they are together comparable in dimension and in order of magnitude. So the following indicators have been defined :

- For the velocity

$$I_{K,u} = \frac{\{\int_K (\gamma(u_h)^* - \gamma(u_h)) : (\gamma(u_h)^* - \gamma(u_h))\, d\omega\}^{\frac{1}{2}}}{\{\int_{\Omega_h} \gamma(u_h) : \gamma(u_h)\}^{\frac{1}{2}}}$$

- For $C$ in $(k, \epsilon, \mu_T)$

$$I_{K,C} = \frac{\{\int_K ((\nabla C_h)^* - \nabla C_h).((\nabla C_h)^* - \nabla C_h)\, d\omega\}^{\frac{1}{2}}}{\{\int_{\Omega_h} \nabla C_h . \nabla C_h\}^{\frac{1}{2}}}$$

The whole error indicator for all the quantities is written :

$$I_K = (I_{K,u}^2 + I_{K,k}^2 + I_{K,\epsilon}^2 + I_{K,\mu_T}^2)^{\frac{1}{2}}$$

This time the error indicator is relative. The proceeded test is :

$$refinement\ where\ \frac{I_K}{4} \geq \alpha \tag{3.4}$$

$$derefinement\ where\ \frac{I_K}{4} \leq \alpha * 0.8 \tag{3.5}$$

where $\alpha$ is the predefined threshold. 4 is the number of quantities taken into account in the calculation of the indicator. This division avoids obtaining relative error greater than the unit. It is like the average value of all contributions.

## 4 Refinement module for 2D or 3D geometry

A refinement tool is developed and has been designed to be used in the environment of three major 3D finite element computer codes : N3S but solid mechanics and electromagnetic fields also. This tool is divided into different parts : on the one hand the interfaces and on the other hand algorithm program. This program contains for instance only refinement facilities but derefinement will soon be developed and will be useful for transient phenomena. The originality of the algorithm is to process faces and edges and not tetrahedra directly. So it runs on meshes containing tetrahedra, triangles and edges which is useful in mechanics or electromagnetic fields. With the same ideas as [1] for 2D geometries tetrahedra are broken down into eight sub-elements. Mesh conformity is obtained dividing tetrahedra into two or into four parts.

In order to allocated the right boundary conditions to the new entities, nodes and elements (tetrahedra, faces and edges) are sorted by families. The new nodes belong to a family which is determined by the edge-family or the face on which they are located. The solution is generated over the new mesh by interpolations on the new nodes.

In both 2D and 3D cases, the algorithm is efficient. The CPU time which is

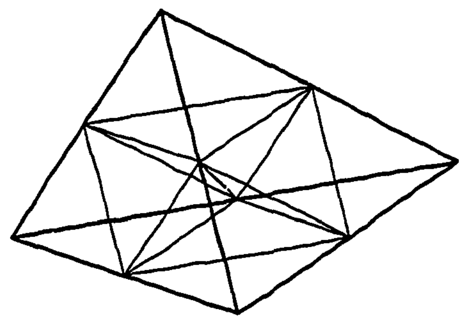


FIG. 1. CUTTING UP OF A TETRAHEDRON INTO EIGHT SUB-ELEMENTS

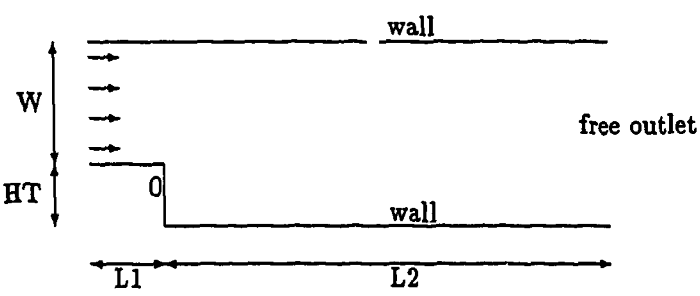
required to analyze the error indicators and to build a new mesh is lower than the average CPU time of a time step.

For example, in a 16000 tetradra mesh, the N3S time step is 8 s. To reach the new mesh (59600 tetrahedra), the core of the process lasts 1.3 s. With interfaces, the whole CPU time is 2 s. However, care has to be taken with the nature of exchange file between the adaptive software and the main code. With a formatted file for the mesh, the I/O elapsed time is about 20 s, which is ten times larger than the process itself !

## 5 Applications, a 2D example

The method is tested in the case of the backward-facing step [6].

- Width of the pipe $W = 0.0762\ m$
- Height of the backward-facing step $HT = 0.0381\ m$
- Length before the step $L1 = 0.190\ m$
- Length after the step $L2 = 0.762\ m$
- Entrance velocity $U_\infty = 18.2\ ms^{-1}$
- Entrance turbulent energy $k_\infty = 6.6248\ m^2s^{-2}$
- Entrance turbulent dissipation $\epsilon_\infty = 1205.7\ m^2s^{-3}$
- Reynolds number $Re = 95250$



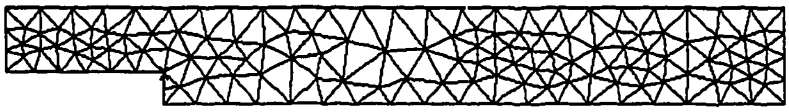


FIG. 2. *initial mesh*

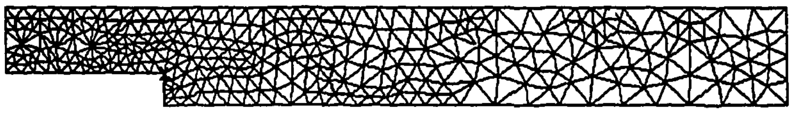


FIG. 3. *mesh after one adaptation*

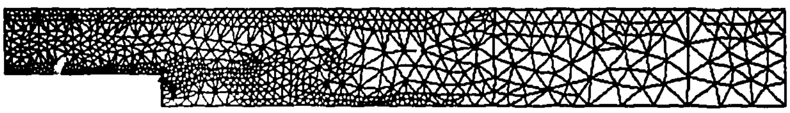


FIG. 4. *mesh after two adaptations*

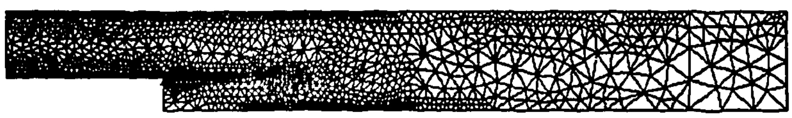


FIG. 5. *mesh after three adaptations*

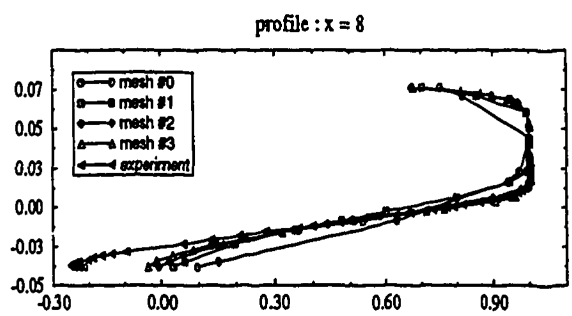



FIG. 6. *comparison of the different velocity profiles*

The adaptive method has been able to adjust the mesh refinement to the complex flow structure : near the inlet but also in the recirculation area, whereas the exit has been marginally affected by the refinement. The bubble length has been improved during the adaptive processing. The error has been evenly spread and reduced. This stands for a good behaviour of the error indicator. However, the error level seems to be stabilized on the last mesh, as also indicated by comparisons between experimental velocity profiles and numerical ones.

Table 1 **The adaptive processing**

| Mesh | Number of elements | Time step value | Number of time steps | Relative error estimation | Relative error on the length bubble |
|---|---|---|---|---|---|
| 0 | 242 | 0.5E-2 | 40 | 46 | 40 |
| 1 | 592 | 0.22E-3 | 906 | 36 | 29 |
| 2 | 1265 | 0.11E-3 | 1795 | 35 | 25 |
| 3 | 2860 | 0.56E-4 | 3533 | 35 | 21 |

## 6 Conclusion

An adaptive finite element method based on cutting of the elements has been presented for $k-\epsilon$ model of turbulence and applied to the backward-facing step case. A technique using a least-squared projection of the solution gradients has been described to compute error estimates. The 3D adaptive process is currently tested and the error indicators are going to be extended to turbulent 3D configurations.

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