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THE TOROID MOMENT OF MAJORANA NEUTRINO

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I. INTRODUCTION

The properties of neutrinos are of great interest in the physical literature motivated by different puzzles in neutrino physics [1]. A direction of investigations of neutrinos is the study of their electromagnetic properties. The usual electromagnetic characteristics: magnetic, electric dipole moments and neutrino charge radius have been widely discussed everywhere [2]. Here we will investigate a more exotic kind of electromagnetic characteristic of Majorana neutrinos, the toroid moment.

As early as in 1939, Pauli remarked that Majorana neutrinos have neither magnetic dipole moment nor electric dipole moment [3]. However as was pointed out a long time ago in Ref. [4], for the T-invariant interaction with non-conservation of P and C symmetries, there is a diverse dipole moment of a particle with 1/2-spin: the anapole moment. Subsequentely there was pointed out a more convenient characteristic, the toroid dipole moment, which is the first term of the third multipole family, toroid moments [5]. [1] This type of static multipole moments does not produce any external fields in vacuum and only reduces a free-field (gauge-invariant) transverse-longitudinal potential [6] responsible for topological effects like Aharonov – Bohm ones. In brief, the experts of neutrino physics show that the toroid dipole moment [7], is a single electromagnetic characteristic of Majarano neutrinos valid for any values of the spin [8]. The Majarano neutrinos possesing the toroid moment provide very interesting results in different media [9,10]. Indeed, as was pointed out in Ref. [10], the toroid dipole moment moving in a medium with a sufficiently high dielectric constant and magnetic permeability, $(v/c)\sqrt{\varepsilon\mu} > 1$, can generate the Vavilov-Cherenkov radiation. This radiation may be detected in experiment and gives a new possibility for investigating neutrino properties.

A calculation of toroid dipole moment (anapole) has been started in [11] and by diploma student of JINR A. A. Chepkasov in 1976. Then a number of articles about the problems of renormalizability, gauge non-invariance and consequently observability of anapole moment and neutrino charge radius was published [11–13]. However, as was pointed out in [14], these quantities are finite and well-defined in the Standard Model as being its axial-vector and vector contact interactions with an external electromagnetic field, respectively. Since a great interest to neutrino properties at present and development of experimental setups for detection of neutrinos, we reanalysed the previous results and present here the calculation of the toroid moment of Majarano neutrino for a general class of modern gauge theories of electroweak interactions.

II. ONE-LOOP RESULT

In the case of Majorana neutrinos, a toroid moment can be defined in the one-loop approximation of the Standard Model of electroweak interactions from the Feynman

[1]The toroid dipole is well-defined in the classic limit that is not a case of anapole which coinsides with it only on mass-shell of external massive particles.

graphs shown in Fig. 1. The electromagnetic vertex of a neutrino, $\Gamma_\mu(q)$, has anapole or toroid parametrization:

$$\Gamma_\mu(q) = \left\{G(q^2)[q^2\gamma_\mu - \hat{q}q_\mu]\gamma_5\right\}_{\text{anapole}} = \left\{G(q^2)[i\varepsilon_{\mu\nu\lambda\sigma}P_\nu q_\lambda\gamma_\sigma\gamma_5]\gamma_5\right\}_{\text{toroid}}. \quad (1)$$

They coincide identically only on mass-shell and different off mass-shell [5], which follows from the indentity:

$$\overline{u}_f(\mathbf{p}')\left\{(m_f - m_i)\sigma_{\mu\nu}q^\nu + \left[q^2\gamma_\mu - \hat{q}q_\mu\right] + i\varepsilon_{\mu\nu\lambda\sigma}P_\nu q_\lambda\gamma_\sigma\gamma_5\right\}\gamma_5 u_i(\mathbf{p}) = 0, \quad (2)$$

where $\varepsilon_{\mu\nu\lambda\sigma}$ is an antisymmetric tensor, $P_\nu = p_\nu + p'_\nu$ and $q_\nu = p_\nu - p'_\nu$. As can see from (2) the transition toroid moment is equal to diagonal one plus part, which is proportional to the neutrino mass difference, therefore for calculation of diagonal toroid moment and estimation of transition one it is enough only anapole parametrization.

For illustration, we write down some details of the calculation of two graphs with $\ell\ell W$ states, see Fig. 2. It is easy to verify that contributions of particle and antiparticle currents are equal to each other and we will consider one of them multiplying the amplitude by factor 2. Using the Feynman rules, summarized in Appendix A, we can write the amplitude in the following form

$$\mathcal{M} = 2\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}(2\pi)^4\delta^4(p_1 - p_2 - (k_1 - k_2))\overline{u}(p_1)\left[i\Gamma_\lambda^{(\ell N)}\right] i\Delta_F(k_1)$$
$$\times(-ie\gamma_\mu)\, i\Delta_F(k_2)\left[i\overline{\Gamma}_\nu^{(\ell N)}\right] u(p_2)\left[i\Delta_W^{\lambda\nu}(p_1 - k_1)\right]\mathcal{A}^\mu(k_1 - k_2). \quad (3)$$

Here $\mathcal{A}^\mu(k_1 - k_2)$ is an external electromagnetic field, a source of virtual photons. [2] For convinience of our calculations, we pass to the t-channel where the momenta of particles transform as

$$p_1 \to p_-, \quad p_2 \to -p_+, \quad k_1 \to k_- \equiv k_1, \quad k_2 \to -k_+ \equiv -k_2,$$

$$q = (k_1 + k_2) = (p_- + p_+),$$

and using the transformation

$$\frac{1}{k^2 - m^2} \to (-2\pi i)\delta\left(k^2 - m^2\right)\Theta(k_0), \quad (4)$$

valid when we take into account the unitary condition for the S-matrix [15,16], we can write the imaginary part of the amplitude as

---

[2]In the case of real photons, $q^2 = 0$ and $\varepsilon \cdot q = 0$, the toroid (anapole) interaction disappears.

$$\mathrm{Im}\mathcal{M} = e\int d\tau \frac{\overline{u}(p_-)\Gamma_\lambda^{(\ell N)}\left(\widehat{k}_1 + m_\ell\right)\gamma_\mu\left(\widehat{k}_2 - m_\ell\right)\overline{\Gamma}_\nu^{(\ell N)} g^{\lambda\nu} v(p_+)}{(p_- - k_1)^2 - M_W^2}\mathcal{A}^\mu(q). \tag{5}$$

Here we have denoted the two-body phase-space factor as

$$d\tau = \frac{1}{(2\pi)^2} d^4k_1 d^4k_2 \delta^4(p_- + p_+ - k_1 - k_2)\delta\left(k_1^2 - m_\ell^2\right)\Theta(k_{10})\delta\left(k_2^2 - m_\ell^2\right)\Theta(k_{20}).$$

Now, keeping only terms with $\gamma_\mu\gamma_5$ and performing the two-particle phase space integration, following Refs. [11], we obtain

$$\begin{aligned}
\mathrm{Im}G(t) = {} & \frac{1}{16\pi}\left[\left(A_L^{(\ell N)}\right)_{1\ell}\left(A_L^{(\ell N)*}\right)_{\ell 2} - \left(A_R^{(\ell N)}\right)_{1\ell}\left(A_R^{(\ell N)*}\right)_{\ell 2}\right]\frac{1}{\sqrt{t(t-4m_1^2)}} \\
& \times\left\{t\ln c - \frac{1}{2}\left(t - 4m_\ell^2\right)\left(\ln c + 2\frac{a}{b} - \frac{a^2}{b^2}\ln c\right) - t\sqrt{\frac{t-4m_\ell^2}{t-4m_1^2}}\left(2 - \frac{a}{b}\ln c\right)\right. \\
& - \frac{1}{2}\left(t - 4m_\ell^2\right)\left(\frac{m_1^2 - m_2^2}{t - 4m_1^2}\right)\left(\ln c + 6\frac{a}{b} - 3\frac{a^2}{b^2}\ln c\right) \\
& \left. + 2m_1(m_1 - m_2)\sqrt{\frac{t-4m_\ell^2}{t-4m_1^2}}\left(2 - \frac{a}{b}\ln c\right)\right\},
\end{aligned} \tag{6}$$

$$a = M_W^2 + \frac{1}{2}\left(t - 2m_\ell^2 - m_1^2 - m_2^2\right), \quad b = \frac{1}{2}\sqrt{t - 4m_1^2}\sqrt{t - 4m_\ell^2},$$

$$c = |(a+b)/(a-b)|, \quad t = q^2.$$

The real part of the toroid form factor can be derived by using the dispersion relation with one subtraction [15]. For $t < 0$, with the new variable $x = t/2M_W^2$ and in $m_\nu = m_1 = m_2 = 0$ approximation we find

$$G(t) = \frac{1}{32\pi^2 M_W^2}\left(\left|A_L^{(\ell N)}\right|^2 - \left|A_R^{(\ell N)}\right|^2\right)\int_{2\gamma}^{\infty}\frac{F(x,\gamma)dx}{x(x+\alpha)}, \tag{7}$$

where $\alpha = -t/2M_W^2 > 0$ and the integrand reads

$$\begin{aligned}
F = {} & \left(\frac{\gamma - 1}{x} - 3\right)\sqrt{1 - \frac{2\gamma}{x}} + \left[2\left(1 + \frac{1}{2x}\right)^2 - \frac{\gamma}{x} - \frac{\gamma}{x^2} + \frac{\gamma^2}{2x^2}\right] \\
& \times\ln\left(\frac{1 + x - \gamma + \sqrt{x(x-\gamma)}}{1 + x - \gamma - \sqrt{x(x-\gamma)}}\right),
\end{aligned}$$

with $\gamma = m_\ell^2/M_W^2$. Finally, using the definition of matrices $A_{L,R}^{(x)}$ and $B_{L,R}^{(x)}$ in the Standard Model (A.3) and performing the elementary integrations for these two graphs and other ones (making appropriate expansions in $m_f^2/m_{W,Z}^2$, $f = e, \mu, \tau, u, d, s, c, b, t$) we obtain for $|t| = 0$:

$$
\begin{aligned}
G(0) &= \frac{\sqrt{2}G_F}{16\pi^2}\Bigg\{\sum_{\ell=e,\mu,\tau}(C_{\ell\ell W} + C_{\ell\ell\phi} + C_{WW\ell} + C_{W\phi\ell} + C_{\phi W\ell} + C_{\phi\phi\ell}) \\
&\qquad\qquad + C_{WWZ} + C_{W\phi Z} + C_{\phi\phi Z} + \sum_f C_{ffZ}\Bigg\} \\
C_{\ell\ell W} &= |\mathcal{K}_{\ell i}|^2\left\{\frac{22}{9} - \frac{4}{3}\ln\frac{4m_\ell^2}{M_W^2} + \frac{m_\ell^2}{M_W^2}\left(\frac{7}{3} + \frac{2}{3}\ln 2 - \frac{3}{2}\ln 3\right) + \mathcal{O}(\gamma^2)\right\}, \\
C_{\ell\ell\phi} &= |\mathcal{K}_{\ell i}|^2\frac{m_\ell^2}{M_W^2}\left(\frac{2}{9} - \frac{2}{3}\ln\frac{4m_\ell^2}{M_W^2} + \mathcal{O}(\gamma^2)\right), \\
C_{WW\ell} &= |\mathcal{K}_{\ell i}|^2\left(-\frac{10}{9}\right), \\
C_{W\phi\ell} &= C_{\phi W\ell} = |\mathcal{K}_{\ell i}|^2\left(-\frac{m_\ell^2}{6M_W^2}\right), \\
C_{\phi\phi\ell} &= |\mathcal{K}_{\ell i}|^2\left(-\frac{m_\ell^2}{9M_W^2}\right), \\
C_{ffZ} &= \frac{\Omega_{ii}}{2}\sum_{f\neq t}(g_L^f + g_R^f)\left\{\frac{5}{9} - \frac{4}{3}\ln\frac{4m_f^2}{M_Z^2} + \frac{m_f^2}{M_Z^2}\left(\frac{20}{3}\ln 2 - 3\ln 3 - \frac{28}{3}\right) + \mathcal{O}(\gamma^2)\right\} \\
&\quad + \frac{\Omega_{ii}}{2}(g_L^t + g_R^t)\cdot(-0.139), \quad m_{\text{top}} = 180\ \text{GeV}, \\
C_{WWZ} &= -\frac{\Omega_{ii}}{3}\left[\frac{77}{3} + 4d - \sqrt{d}(27 + 4d)\arctan\left(d^{-1/2}\right)\right], \\
C_{W\phi Z} &= -\frac{\Omega_{ii}}{2}\sin^2\theta_W\left[\frac{2}{3} + d - \sqrt{d}(1 + d)\arctan\left(d^{-1/2}\right)\right], \\
C_{\phi\phi Z} &= -\frac{\Omega_{ii}}{3}(1 - 2\sin^2\theta_W)\left[\frac{1}{3} - d + d^{3/2}\arctan\left(d^{-1/2}\right)\right],
\end{aligned}
\tag{8}
$$

where $d = 4\cos^2\theta_W - 1$, $\mathcal{K}_{\ell i}$ and Ω_{ii} are elements of the mixing matrices K and Ω, see Appendix A.

III. REMARKS ON SOME PROPERTIES OF TOROID MOMENT

Let us disscuss some general properties of toroid interaction of neutrinos. In the non-relativistic limit, the energy of interaction with external electromagnetic fields is defined, in general, by three characteristics: magnetic ($\boldsymbol{\mu}$), electric ($\mathbf{d}$) and toroid ($\mathbf{T}$) dipole moments:

$$
\mathcal{H}_{\text{int}} \sim -(\boldsymbol{\mu}\cdot\mathbf{H}) - (\mathbf{d}\cdot\mathbf{E}) - \left[\mathbf{T}\cdot\left(\text{rot}\mathbf{H}\ \text{or}\ \dot{\mathbf{E}}\right)\right]. \tag{9}
$$

In the case of Majorano neutrinos, imposing the restriction of CPT-invarince and using C-, P-, T-properties of $\mathcal{H}_{\text{int}}$, which we have combined in Table 1, we see that magnetic and electric dipole moments are absent in the static limit ($m_i = m_f = m_\nu$). This means that Majarano neutrino can possesses only one electromagnetic characteristic, the toroid (anapole) moment if masses of initial and final neutrino eigenstates are equal to each other. The electromagnetic matrix element connecting two different mass eigenstates of the Majarano neutrino can be described, in principle, by all three moments [7]. Adding all contributions from (8) and using the standard definitions of dipole moments [5,9,14] we define the toroid moment of Majarano neutrino as being its axial-vector contact interaction with an external electromagnetic field: [3]

$$T_\mu = \frac{eG(0)}{2m_\nu}\overline{u}(0)\gamma_\mu\gamma_5 u(0), \quad \mathbf{T} = eG(0)\varphi^\dagger\boldsymbol{\sigma}\varphi,$$

$$G(0) = 1.28 \times 10^{-33}(|K_{ei}|^2 + 0.55|K_{\mu i}|^2 + 0.31|K_{\tau i}|^2 - 0.21\Omega_{ii}) \quad (\mathrm{cm}^2),$$

where φ is the Pauli spinor. It has the density $g(\mathbf{r}) = [\mathbf{r}(\mathbf{Jr}) - 2r^2\mathbf{J}]/10$ in the coordinate space and the following interaction with an external electromagnetic field [17]:

$$\begin{aligned}\mathcal{H}_{\mathrm{int}} &= J_\mu^{\mathrm{EM}}(x)\mathcal{A}^\mu(x) = eG(q^2)\overline{N}(x)\left[q^2\gamma_\mu\mathcal{A}^\mu(x) - \gamma_\mu q^\mu q_\nu\mathcal{A}^\nu(x)\right]\gamma_5 N(x)\\ &= eG(q^2)\overline{N}(x)\gamma_\mu\gamma_5 N(x)\left[\frac{\partial^2\mathcal{A}^\mu(x)}{\partial x^\nu\partial x_\nu} - \frac{\partial^2\mathcal{A}^\nu(x)}{\partial x^\nu\partial x_\mu}\right]\\ &= eG(q^2)\overline{N}(x)\gamma_\mu\gamma_5 N(x)\frac{\partial F^{\mu\nu}(x)}{\partial x^\nu}.\end{aligned}$$

Here $F^{\mu\nu}(x)$ is the tensor of the electromagnetic field that produces the external current

$$j^\mu(x) = -\partial_\nu F^{\mu\nu}(x).$$

In the non relativistic limit we obtain

$$\mathcal{H}_{\mathrm{int}} = -eG(0)\varphi^\dagger\boldsymbol{\sigma}\varphi\mathbf{j} \equiv -eG(0)\varphi^\dagger\boldsymbol{\sigma}\varphi(\mathrm{rot}\mathbf{H} - \dot{\mathbf{E}}).$$

How can we think of the toroid moment of the Majorana neutrino? The answer to this question was given in the original idea of Zel'dovich [4]: a conventional solenoid folded into a torus having poloidal currents is a classical example of a toroid dipole. For such a solenoid there is neither azimuthal component of the poloidal currents nor electric fields around torus that are responsible for the absence of electromagnetic fields outside the torus and the presence of a non zero magnetic field inside the torus. That is why the fields outside the toroid dipole are zero in vacuum. However when the toroid dipole moves in a medium, the medium should be regarded as permitting

[3] Here we have used the normalization $\overline{u}(\mathbf{p})u(\mathbf{p}) = 2m_\nu$ and chiral representation of gamma matrices.

the dipole itself and the fields outside the dipole to appear producing, for instance [10], the Vavilov-Cherenkov radiation.

In conclusion, we have calculated the diagonal toroid dipole moment of Majarono neutrino without specifying the gauge group under consideration. In the Standard Model, it has a finite value and does not depend on the neutrino mass, i.e. it is different from zero in the case of massless neutrinos. The toroid moment of Dirac or Majorano neutrinos should be taken into account in various situations: it gives an extra contribution to the total cross section of scattering of neutrinos by a spinless nucleus and it has a finite value in different media [9]; it is responsible for the Vavilov-Cherenkov and transition radiations when neutrinos move through an uniform medium [10] and it may play a sensible role in neutrino oscillations, since the toroid interaction with an external source of electromagnetic fields (in media it can be the electron current) should be added to the Hamiltonian of evolution of neutrinos. For instance, the evolution equation for three neutrino flavors (in the presence of non zero rot**H** or $\dot{\mathbf{E}}$) can be written as

$$i\frac{d\vec{\nu}_\ell}{dt} = K\left[\frac{1}{2E}\mathrm{diag}\left(m_1^2, m_2^2, m_3^2\right) + \mathcal{T}\right]K^\dagger\vec{\nu}_\ell,$$

where the matrix $\mathcal{T}$ is, in general, a 3×3 matrix whose elements are different from zero (they are defined by the diagonal and transition toroid moments). This problem is an analog of the well-known Wolfenstein equation for propagation of neutrinos through the medium [18].

Note added. After this manuscript was completed an article by Boyarkin and Rein [19] appeared. These authors studied the transition in a system of Majorana neutrinos with anapole and transition magnetic moments propagating in matter and twisting non potential magnetic field investigated within the asymmetric left-right model. It was shown that resonance conversion of neutrinos appeares not only in response to influence of matter but also by the availability of electromagnetic moments. Since our calculations showed that toroid (anapole) moment is non zero quantity in the Standard Model, consequently the result of Ref. [19] should be appreciated more carefully in relation to neutrino oscillations and explanation of solar neutrino deficit.

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TABLE I. C-, P-, T-properties of spin, electromagnetic fields and their interactions.

	σ	**H**	**E**	rot**H**, $\dot{\mathbf{E}}$	$\sigma\cdot\mathbf{H}$	$\sigma\cdot\mathbf{E}$	$\sigma\cdot$ rot**H**, $\sigma\cdot\dot{\mathbf{E}}$
C	+	−	−	−	−	−	−
P	+	+	−	−	+	−	−
T	−	−	+	−	+	−	+

APPENDIX A. FEYNMAN RULES

Here we give a short list of the Feynman rules used in our calculations. The weak interactions of Majorana neutrinos N and charged leptons ℓ with gauge bosons $W^{\pm}$, Z^0, non physical scalars $\phi^{\pm}$, ϕ and Higgs particles ϕ^0 may be described by five Lagrangians [20]:

$$
\begin{aligned}
\mathcal{L}_{\rm int}^{NW^{\pm}} &= \overline{N}\Gamma_{\mu}^{(\ell N)}\ell W^{+\mu} + \overline{\ell}\,\overline{\Gamma}_{\mu}^{(\ell N)} N W^{-\mu},\\
\mathcal{L}_{\rm int}^{NZ^0} &= (\overline{N}\Gamma_{\mu}^{(N)}N + \overline{\ell}\Gamma_{\mu}^{(\ell)}\ell)Z^{\mu},\\
\mathcal{L}_{\rm int}^{N\phi^{\pm}} &= \overline{N}\Gamma^{(\ell N)}\ell\phi^{+} + \overline{\ell}\,\overline{\Gamma}^{(\ell N)}N\phi^{-},\\
\mathcal{L}_{\rm int}^{N\phi} &= (\overline{N}\Gamma^{(N\phi)}N + \overline{\ell}\Gamma^{(\ell\phi)}\ell)\phi,\\
\mathcal{L}_{\rm int}^{N\phi^0} &= (\overline{N}\Gamma^{(N)}N + \overline{\ell}\Gamma^{(\ell)}\ell)\phi^0.
\end{aligned}
$$

The respective Feynman rules are:

- $i\Gamma_{\mu}^{(\ell N)}$ for outgoing W^- or incoming W^+,
- $i\overline{\Gamma}_{\mu}^{(\ell N)}$ for outgoing W^+ or incoming W^-,
- $i\Gamma^{(\ell N)}$ for outgoing ϕ^- or incoming ϕ^+,
- $i\overline{\Gamma}^{(\ell N)}$ for outgoing ϕ^+ or incoming ϕ^-,
- $i(\Gamma_{\mu}^{(N)} + \Gamma_{\mu}^{(NC)})$ for Z^0,
- $i(\Gamma^{(N)} + \Gamma^{(NC)})$ for ϕ^0,
- $i(\Gamma^{(N\phi)} + \Gamma^{(N\phi C)})$ for ϕ.

Here $\Gamma_{\mu}^{(NC)} \equiv C\left[\Gamma_{\mu}^{(N)}\right]^T C^{-1}$ and $\Gamma^{(NC)} \equiv C\left[\Gamma^{(N)}\right]^T C^{-1}$ (similarly, for $\Gamma^{(N\phi C)}$).

As was pointed out in Ref. [20], in real calculations we should take the following rule for the Dirac-Majorana transition in a Feynman graph: for an incoming (outgoing) Dirac particle the outgoing (incoming) Majorana neutrino must be treated as a particle, and vice versa, for antiparticles.

Introducing the notation $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$ we can write the general forms for all verticies

$$
\begin{aligned}
\Gamma_{\mu}^{(x)} &= \gamma_{\mu}\left(P_L A_L^{(x)} + P_R A_R^{(x)}\right), \quad x = \ell, N, \ell N\,,\\
\overline{\Gamma}_{\mu}^{(\ell N)} &\equiv \gamma_0\left[\Gamma_{\mu}^{(\ell N)}\right]^{\dagger}\gamma_0 = \gamma_{\mu}\left(P_L A_L^{(\ell N)*} + P_R A_R^{(\ell N)*}\right),
\end{aligned}
\tag{A.1}
$$

and

$$
\begin{aligned}
\Gamma^{(x)} &= P_L B_L^{(x)} + P_R B_R^{(x)}, \quad x = N, N\phi, \ell N,\\
\overline{\Gamma}^{(\ell N)} &\equiv \gamma_0\left[\Gamma^{(\ell N)}\right]^{\dagger}\gamma_0 = P_R B_L^{(\ell N)*} + P_L B_R^{(\ell N)*}.
\end{aligned}
\tag{A.2}
$$

The other Feynman rules used in our calculations are well known and are taken from [21]. Equations (A.1–A.2) have a general form and must be specified for a given gauge group. Below, we present these matrices in the Standard Model.

We will use the following definitions of charged and neutral currents:

$$J_\mu^- = \frac{1}{2}\bar{\ell}\gamma_\mu(1-\gamma_5)KN, \quad J_\mu^0 = \frac{1}{2}g_L^\nu \overline{N}\gamma_\mu(1-\gamma_5)\Omega N,$$

where K, in general, is a rectangular matrix (for the Standard Model with 3 flavor neutrinos from SU(2) doublets and k-singlets it has $3\times(3+k)$-dimension), an analog of the Kobayashi-Maskawa matrix in the quark sector and $\Omega = K^\dagger K \neq 1$ [22]. In this manner, we define the matrices $A_{L,R}^{(x)}$ and $B_{L,R}^{(x)}$ as:

$$\begin{aligned}
A_L^{(\ell N)} &= \frac{g}{\sqrt{2}}K, \quad A_R^{(\ell N)} = 0, \quad \ell = e,\mu,\tau, \\
A_L^{(\ell)} &= \frac{g g_L^\ell}{\cos\theta_W}, \quad A_R^{(\ell)} = \frac{g g_R^\ell}{\cos\theta_W}, \\
A_L^{(N)} &= \frac{g g_L^\nu}{\cos\theta_W}K, \quad A_R^{(N)} = 0, \\
B_L^{(\ell N)} &= -\frac{g m_\nu}{\sqrt{2}M_W}K, \quad B_R^{(\ell N)} = \frac{g m_\ell}{\sqrt{2}M_W}K, \\
B_L^{(N\phi)} &= \frac{g m_\nu}{2M_Z\cos\theta_W}\Omega, \quad B_R^{(N\phi)} = \frac{-g m_\nu}{2M_Z\cos\theta_W}\Omega,
\end{aligned} \tag{A.3}$$

where

$$\begin{aligned}
g_L^\ell &= -\frac{1}{2} + \sin^2\theta_W, \quad g_R^\ell = \sin^2\theta_W, \\
g_L^u &= \frac{1}{2} - \frac{2}{3}\sin^2\theta_W, \quad g_R^u = -\frac{2}{3}\sin^2\theta_W, \\
g_L^d &= -\frac{1}{2} + \frac{1}{3}\sin^2\theta_W, \quad g_R^d = \frac{1}{3}\sin^2\theta_W, \\
g_L^\nu &= \frac{1}{2}, \quad g_R^\nu = 0, \quad \frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}.
\end{aligned}$$

For the definition of $B_{L,R}^{(N)}$ matrices entering into the vertex of neutrino Higgs-boson interactions, we use the following general form of Lagrangian which gives, after spontaneous symmetry breaking, the Dirac mass term:

$$\begin{aligned}
\mathcal{L}_{\nu\nu\phi^0} &= 2^{1/4}\sqrt{G_F}\sum_{\alpha,\beta}\left\{\left(M_D^T\right)_{\alpha\beta}\overline{\nu_{\alpha R}}\nu_{\beta L} + \left(M_D^T\right)^*_{\beta\alpha}\overline{\nu_{\beta L}}\nu_{\alpha R}\right\}\phi^0 \\
&= 2^{1/4}\sqrt{G_F}\sum_{\alpha,\beta}\sum_{ij}\Big\{(U_R^*)_{j\alpha}\left(M_D^T\right)_{\alpha\beta}(U_L)_{\beta i}\overline{N_j}P_L N_i \\
&\qquad + (U_L^*)_{i\beta}\left(M_D^T\right)^*_{\beta\alpha}(U_R)_{\alpha j}\overline{N_i}P_R N_j\Big\}\phi^0.
\end{aligned}$$

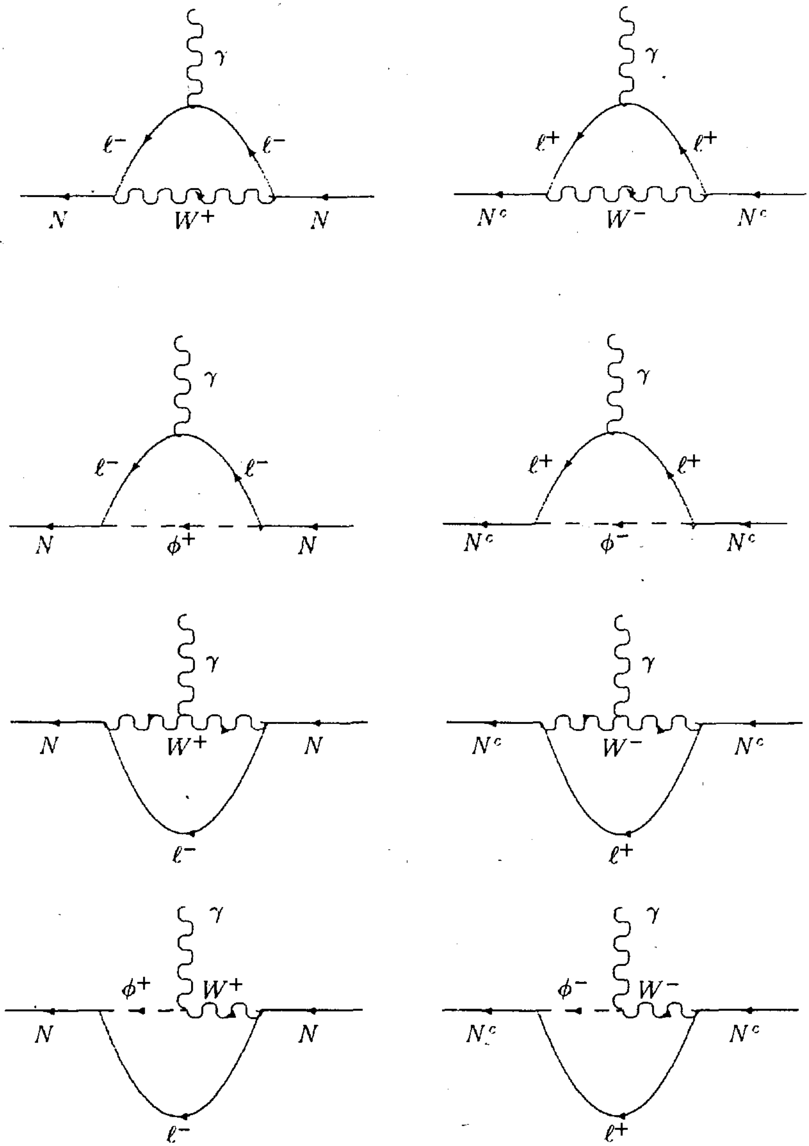
From this equation we immediately find

$$\left(B_L^{(N)}\right)_{ji} = \frac{-g}{2M_W}\sum_{\alpha,\beta}(U_R^*)_{j\alpha}\left(M_D^T\right)_{\alpha\beta}(U_L)_{\beta i},$$
$$\left(B_R^{(N)}\right)_{ij} = \frac{-g}{2M_W}\sum_{\alpha,\beta}(U_L^*)_{i\beta}\left(M_D^T\right)^*_{\beta\alpha}(U_R)_{\alpha j}.$$

The general form of $\Gamma^{(\ell\phi)}$ and $\Gamma^{(\ell)}$ are

$$i\Gamma^{(\ell\phi)} = \frac{gm_\ell}{2M_Z\cos\theta_W}\gamma_5, \quad i\Gamma^{(\ell)} = \frac{igm_\ell}{2M_W}.$$

FIGURES



γ
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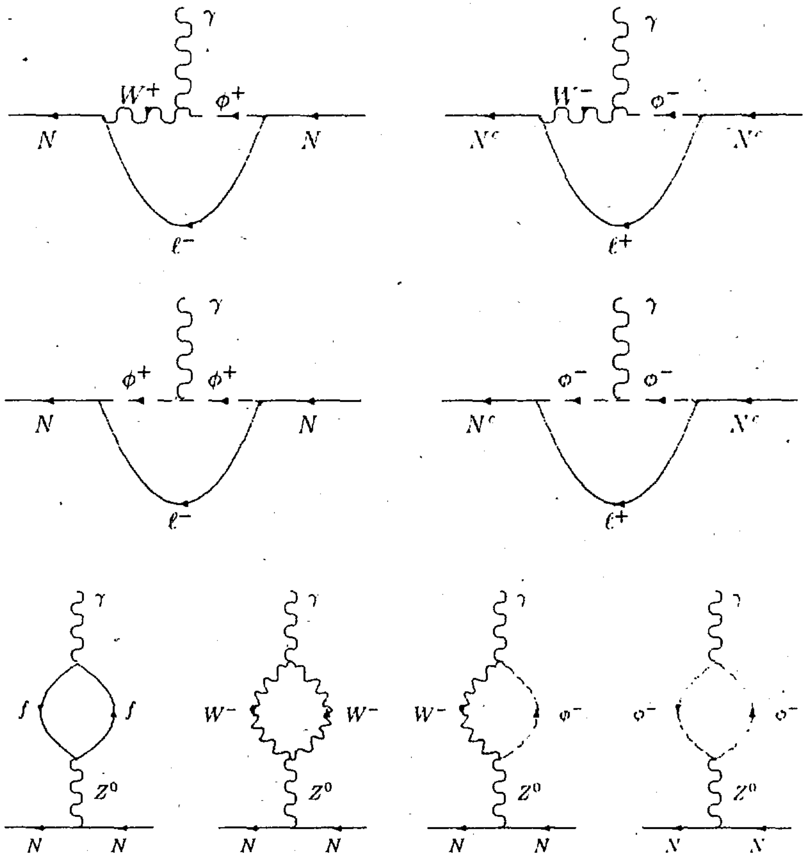

FIG. 1. Feynman diagrams which are responsible for toroid moment of Majorana neutrino.

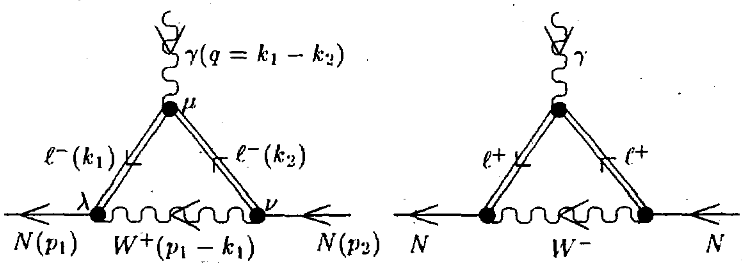

FIG. 2. Feynman graphs with $\ell\ell W$ intermediate states.

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Дубовик В.М., Кузнецов В.Е. E2-96-53
Тороидный момент майорановского нейтрино

Рассмотрен полный набор электромагнитных характеристик майорановских нейтрино. Показано, что в статическом пределе ($m_i = m_f = m_\nu$) майорановские нейтрино обладают только одной электромагнитной характеристикой — тороидным дипольным моментом (анаполем). С помощью дисперсионного метода вычислен диагональный тороидный момент (формфактор) майорановского нейтрино в однопетлевом приближении стандартной модели. Все внешние частицы находятся на массовых поверхностях, и не возникает проблем с физической интерпретацией коненного результата. Также обсуждаются различные приложения тороидного момента майорановского нейтрино.

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Dubovik V.M., Kuznetsov V.E. E2-96-53
The Toroid Moment of Majorana Neutrino

The total set of electromagnetic characteristics of Majorana neutrinos is considered. It is shown that in the static limit ($m_i = m_f = m_\nu$) the Majorana neutrinos possess only one electromagnetic characteristic, the toroid dipole moment (anapole). We have calculated the diagonal toroid moment (form factor) of the Majorana neutrino in the one-loop approximation of the Standard Model by the dispersion method. All external particles are on the mass shells and there are no problems with the physical interpretation of the final result. Different applications of the toroid moment of Majorana neutrino are also discussed.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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