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ALGORITHM
FOR RECOGNIZING TRACKS DETECTED BY DRIFT TUBES IN A MAGNETIC FIELD*

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1 Introduction

Conventional track recognition problem can be reduced to the search for a "sufficient" number of data points, which must satisfy conditions of a "sufficient" smoothness of their alignment along a straight line or a higher-order curve. The notion "sufficient" depends on the statistical efficiency of the track recognition problem for a given experiment.

In cases, when the experimental data are 2D- or 3D-coordinates registered by a track chamber, the track recognition problem is usually solved by an exhaustive sorting of all data point into subsets (track candidates). Then the smoothness of the data point alignment for each subset is to be estimated by some criterion (usually by fitting of a second order curve to some of 2-D projection of these points and then applying the χ^2– criterion).

The efficiency of the track reconstruction algorithm depends on reasonability of a clustering method applied to group data points into track candidates, i.e. on the maximum possible reducing of the search trials made by the used method over all points. As examples of such reasonable algorithms one can point out well known methods like variable slope histogramming or stringing (track following) methods [1, 2], as well as relatively new approaches like Hopfield neural networks [3, 4].

One of detector systems widely used in modern experiments of high energy physics (ATLAS, EVA/E850) are drift straw tube detectors (DSTD). Each time, when a passing particle track hits a tube, it registers two data: its own center coordinate and the drift radius, i.e. the drift distance between particle tracks and the anode wire situated in the center of this tube. Thus a track passing the DSTD provides a set of anode wire coordinates and corresponding drift radii. Unfortunately, some of these data can be lost due to the straw tube anefficiency, besides a number of noise coordinates is also registered additionally. However the main problem, which hinders applications of above mentioned conventional track recognition methods, is so-called left-right-ambiguity of drift radii. They don't contain the information about, on which side of anode wire the track was passed. Anode wire coordinates themselves are very rough indicators of particle locations. So if one would even recognize a subset of these points belonging to a concrete track and would then approximate it by a second-order curve (circle or parabola), the resulting parameter accuracy will not be satisfactory.

In this report the algorithm of track recognition in an uniform magnetic field is proposed for the DSTD system of solenoidal geometry. A problem solution is given for (x,y) plane perpendicular to the magnetic field and anodes of drift straw tubes. Our algorithm is elaborated on the basis of modifications of the Hough transform and deformable template methods. However, the main features of the proposed algorithm have the common character and are independent of the experimental setup geometry.

2 Formulation of the Problem

The DSTD system of solenoidal geometry consists of cylindrical modules formed by several layers of straw tubes arranged in honeycomb order. In the middle of every tube there is an anode wire with known XY-coordinates. All tracks of some event passing through these layers produce N signals, i.e. set $M = \{x_i, y_i; r_i, \quad i = \overline{1,N}\}$, where (x_i, y_i) are coordinates of the hitt tube centers, r_i are drift radii. Let us suppose, first, that the recognition problem is solved, i.e. from the set M a subset S was extracted of triplets $(x_i, y_i; r_i)$ produced by one of tracks and, possible, by also some of extra noise tubes. For the sake of simplicity let's keep for S the same notation, as for M, i.e. $S = \{x_i, y_i; r_i, \quad i = \overline{1,N}\}$. Geometrically the set S can be considered as the set of circles on the plain with centers (x_i, y_i) and radii r_i.

Thus **the mathematical formulation of the problem** is to draw the track line as a circle (a, b, R) tangential to the maximum number of these little circles from S. Therefore this circle in question (a, b, R) is an envelope curve.

Let us introduce, as a measure of two circle tangency on the plain, the minimum distance between crossing points of these circles with the straight line links centers of both circles. If two circles are tangential, their tangency measure is, obviously, equal to zero. Then our above formulated problem can be reformulated as the following: **to find such a circle (a, b, R) that minimizes the sum of its tangency measures with all circles from the set S.**

Let us denote by $D_i(a, b, R)$ the distance from the center of the circle $(x_i, y_i; r_i)$ to circle (a, b, R)

$$D_i(a, b; R) = R - \sqrt{(x_i - a)^2 + (y_i - b)^2}.$$

This variable can take both positive and negative values. Therefore the tangency measure square of those two circles $(x_i, y_i; r_i)$ and $(a, b; R)$ is twofold:

if $D_i(a, b; R) > 0$, then

$$d_i^- = (D_i(a, b; R) - r_i)^2,$$

otherwise

$$d_i^+ = (D_i(a, b; R) + r_i)^2.$$

As in [5] we define the two-dimensional vector $\vec{s}_i = (s_i^+, s_i^-)$ with admissible values $(1, 0), (0, 1), (0, 0)$. Let us denote by λ the measurement error of the drift radius and define a functional L depending of five parameters (a, b, R, s_i^-, s_i^+) :

$$L = \sum_{i=1}^{N} \left\{ d_i^- s_i^- + d_i^+ s_i^+ + \lambda((s_i^- + s_i^+) - 1)^2 \right\}. \tag{1}$$

It's obvious that the circle parameters $(a, b; R)$ corresponding to a track in question would define a point in the parameter space, where our fuctional L reaches its global minimum with the conditions that $\vec{s}_i = (0, 0)$ means i-th tube for the given track is the noise tube and the combination $\vec{s}_i = (1, 1)$ is forbidden, i.e.

$$s_i^+ + s_i^- \leq 1. \tag{2}$$

Thus to recognize a track one has to:

1. from the set of all measurement M extract a subset S, which as much as possible contains all data for one of tracks;

2. find the L global minimum (although it would be enough to reach its close vicinity).

To solve the first problem we modify the Hough transform method [6], which we following to [7] call as the method of sequential histogramming by parameters (SHPM). Besides of extracting of a subset S SHPM provides also starting values of the circle $(a_0, b_0; R_0)$ needed to solve the problem on the next step. The second problem is solved by the deformable template method (DTM) with the special correction of parameters of obtained tracks.

3 Sequential histogramming method

Let $\Omega = \{X_i, Y_i, i = \overline{1, N}\}$ be a set of coordinates X_i, Y_i measured in the process of registering of an event. So to Ω belong both: coordinates of track points as well as noise coordinates. A circle arch is supposed to be a good approximation of any track .

Let us consider all triplets of points of the Ω set. If these three points do not belong to a straight line, one can draw a circle through them. As a result a set of such circle parameters is obtained $W = \{a_j, b_j; R_j, j = \overline{1, C_N^3}\}$. One could imagine a 3-D histogram constructed on that W-set as a hilly surface, where hills should most likely correspond to tracks. This idea together with so-called sequential histogramming approach [7] gives us the following algorithm for finding of initial track parameters:

1. Circles are drawn through all admissible point triplets. Then the first coordinate a_j of each circle is histogrammed.

2. The value a_m is obtained corresponding to the maximum of this histogram.

3. With the fixed a_m circles are drawn through all admissible pair of points from Ω. Then the second coordinate b_j of each circle is histogrammed.

4. The value b_m is obtained corresponding to the maximum of this second histogram.

5. With the fixed coordinates of the center a_m, b_m all admissible points R_j of the set Ω are histogrammed.

6. The value R_m is obtained corresponding to the maximum of this third histogram.

The admissibility in steps 1,3,5 above means testing of corresponding values by easy cut-off criteria (for instance, each R_j is tested whether it is outside of a prescribed minimal radius R_{min}).

Then the obtained parameters $(a_m, b_m; R_m)$ are subjected to more sophisticated tests and more precise definition. If results are positive, i.e parameters $(a_m, b_m; R_m)$ are accepted as a true track, all measurements corresponding to it are eliminated from the set Ω and the whole procedure is repeated starting from the step 1. If the circle $(a_m, b_m; R_m)$ is rejected by testing, then the maximum R_m of the third R_j-histogram is eliminated and the procedure is repeated starting from the step 6. If there is no more peaks in the R_j-histogram, then the peak b_m of the second histogram is eliminated and the procedure is repeated starting from the step 4 and so on unless the procedure would find a true circle or all peaks in the second histogramm would be eliminated. In this case the peak a_m of the first histogram is eliminated and the procedure is repeated starting from the step 2. It's clear, that this method of sequential histogramming by parameters (SHPM) gives us a possibility to "capture" an area where tracks are likely situated and provides us by initial parameters of these tracks. In order to apply SHPM the results of measurements must have a format of the Ω-set, i.e. to be a set of track point coordinates. However, we have instead the set M of little circles $\{x_i, y_i; r_i, \quad i = \overline{1,N}\}$, so we have to determine on each of these circles a point associated with some of tracks. Supposing the vertex area, from which all tracks of the given event are emanated, is known, one can roughly determine such a point, as a tangent point of the tangent line drawn to each little circle $(x_i, y_i; r_i)$ from the center of the vertex area. However there are two tangents to each circle and, therefore, we have two possible track points, i.e. left-and-right (or top-and down) uncertainty. It would not restrain us in applying of the SHPM, but it should be kept in mind that the left-and-right uncertainty factor doubles the elements number of the set $\Omega = \{X_i, Y_i, i = \overline{1,2N}\}$ in a comparison with the number of elements in the original set $M = \{x_i, y_i; r_i, i = \overline{1,N}\}$.

To decrease the histogramming search domain of the Ω-set it is necessary to use the maximum of *a priori* information, for instance, do not use data from drift tubes mounted on different sides of the target, etc.

The SHPM-description given above stresses an importance of the way used to extract a histogram peak from a background. Our experience shows that it is useless to look for an universal peak-background threshold common for all events of a given experimental run, since this threshold strongly depends on the informative load of a given event. Aiming a statistical efficiency of our method we elaborated the following heuristical formula for the peak-background threshold of a particular event:

$$N_{bound} = 5\frac{H_{max}}{12} + H_{mean}, \tag{3}$$

where H_{max} – is the maximum value of the histogram, H_{mean} - its mean value.

Choosing the bin size one should find a reasonable compromise between either too small or too big size. The first could lead to the loss of a histogram peak, i.e. one of tracks, while a big size decreases the accuracy.

4 Deformable template method

After obtaining by SHPM initial values of track parameters and choosing an area where this track could lie, we proceed to look for the global minimum of the functional L (1). One of the main problems here is how to avoid local minima of L provoked by the stepwise character of the vector $\vec{s}_i = (s_i^+, s_i^-)$ behaviour. One of known way to avoid this obstacle is the standard mean field theory (MFT) approach leads to *the simulated annealing schedule* [8]. Our system is considered as a thermostat with the current temperature T [9]. Then as it was shown in [5], parameters s_i^+ и s_i^- of the functional L with fixed $(a, b; R)$ can be calculated by the following formulae, where the stepwise behaviour of the vector $\vec{s}_i$ is replaced in fact onto sigmoidal one:

$$s_i^- = \frac{1}{1 + e^{\frac{d_i^- - \lambda}{T}} + e^{\frac{d_i^- - d_i^+}{T}}}. \tag{4}$$

$$s_i^+ = \frac{1}{1 + e^{\frac{d_i^+ - \lambda}{T}} + e^{\frac{d_i^+ - d_i^-}{T}}}. \tag{5}$$

The L global minimum is calculated according to the following scheme:

1. Three temperature values are taken: high, middle and a temperature in a vicinity of zero, as well as three noise levels corresponding to them [5, 9].

2. According to the simulated annealing schedule our scheme is started from the high temperature. With initial circle values $(a_0, b_0; R_0)$ parameters s_i^+, s_i^- are calculated by formulae (4), (5).

3. For obtained s_i^+, s_i^- new circle parameters $a, b; R$ are calculated by a modification of the standard gradient descent method. This modification consists of individual updating of L parameters and of holding a condition

$$L(a_k, b_k, R_k) < L(a_{k+1}, b_{k+1}, R_{k+1}). \tag{6}$$

4. The ending rule is as follows: either

$$|L(a_k, b_k, R_k) - L(a_{k+1}, b_{k+1}, R_{k+1})| < \epsilon \tag{7}$$

   holds or the iteration number exceeds a prescribed number $k = const$.

5. If the conditions of the step 4 are not satisfied, then with the new circle parameters $(a_{k+1}, b_{k+1}, R_{k+1})$ next values of s_i^+, s_i^- are again calculated by (4),(5) and we go to the step 3.

6. After converging the process with the given temperature, it is changed (system is cooled), values of (a, b, R) achieved with the previous temperature are taken as starting values and we go to the step 2 again.

7. With each temperature value after completing step 5 the condition

$$L < L_{cut}, \tag{8}$$

is tested. If it satisfied, then our scheme is completed and the algorithm proceeds the next stage of correcting of obtained track parameters (a, b, R). Otherwise, if with the temperature in a vicinity of zero we obtain

$$L > L_{cut}, \tag{9}$$

then a diagnostic is provided that the track finding scheme is failed.

5 Procedure of the track parameter correction

Deformable template method provide us by track parameters $(a, b; R)$. However these parameters, even if they are satisfied to (8), could appear rather apart of the L global minimum. Therefore we have to elaborate an extra stage for the track parameter correction. Its idea is in improving the procedure described in section 3 for converting measured data from the set M format to the $\Omega-$ set. Determination of two points on each little circle of the set M was there done too rough and produced a left-and-right (or top-and down) uncertainty. Now having track-candidate parameters $(a, b; R)$ and concrete values of vectors $\vec{s}_i = (s_i^+, s_i^-)$ we can make this procedure more accurate. On each circle of the set $S = \{x_i, y_i; r_i, i = \overline{1, N}\}$ taking in account corresponding values of $\vec{s}_i$ a point is found nearest to the track-candidate. Then all these points are approximated by a circle and χ^2 value is calculated as a criterion of their smoothness and fitness quality.

If it is hold

$$\chi^2 < \chi^2_{cut},$$

then the approximating parameters $(a_c, b_c; R_c)$ are accepted as true. Otherwise the track-candidate is rejected.

While statistical testing of our algorithm efficiency it was found useful to apply this procedure yet before the deformable templates to track-candidate parameters obtained on the SHPM-stage. The only difference is that, if one would obtain $\chi^2 > \chi^2_{cut}$, as the result of this preliminary testing then process is not stopped, but passed to the stage of deformable templates.

6 Results

Proposed track finding algorithm of tracks detected by DSTD system in a magnetic field was tested on simulated events. 990 tracks have been modelled as circle arches

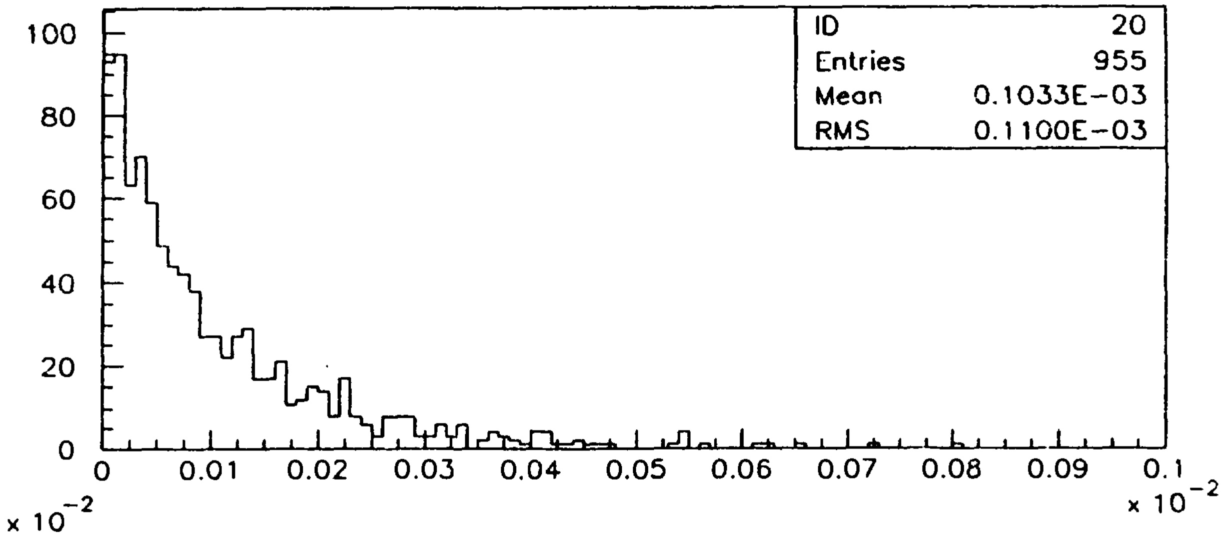

Fig. 1. Relative error DR/R distribution for all radii.

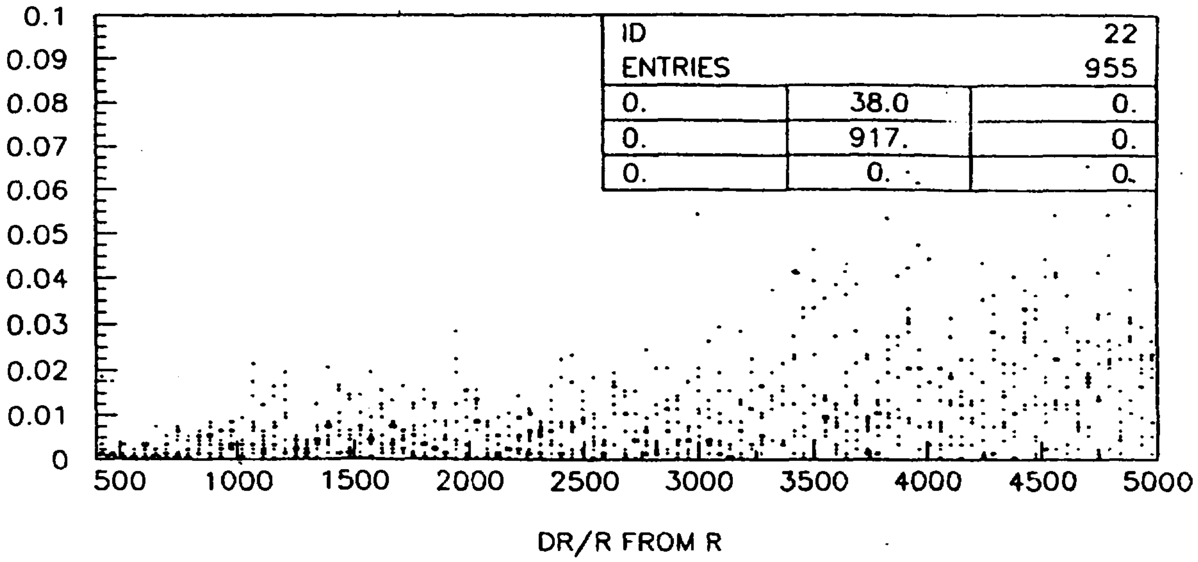

Fig. 2. Relative error DR/R distribution as a function of R.

with radii in the range from 40 cm to 5 m emanating from a target under various angles. 955 tracks from 990 have been recognized correctly that means 96,4% of the algorithm efficiency. The distribution of the relative error of radius

$$\Delta R = \frac{|R_{model} - R_{find}|}{R_{model}}. \tag{10}$$

is presented in fig.1. Fig.2 shows the distribution of ΔR as the function of radius.

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Алгоритм распознавания треков,
детектированных системой дрейфовых трубок
в магнитном поле

Предложен алгоритм распознавания треков, детектированных системой дрейфовых трубок в магнитном поле. Задача решена для системы, имеющей соленоидальную геометрию. Алгоритм разработан (1) на базе метода последовательного гистограммирования, который представляет из себя модификацию Хафф-преобразования и (2) на базе модификации метода деформированных шаблонов с последующей коррекцией параметров.

Тестирование алгоритма на моделированных событиях показало его достаточную эффективность и точность в определении момента частицы.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

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Algorithm for Recognizing Tracks
Detected by Drift Tubes in a Magnetic Field

An algorithm of track recognition in an uniform magnetic field is proposed for the drift straw tube detecting system of solenoidal geometry. The problem solution is given for (x, y) plane perpendicular to the magnetic field. Our algorithm is elaborated on the basis of (1) sequential histogramming method, which is, in fact, a modifications of the Hough transform and (2) a modification of deformable template mehtod following by a special procedure of parameter correction.

Being tested on simulated events our method shows satisfactory efficiency and accuracy in determination of particle momenta.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

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