Direct Measurements of Neutrino Masses

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1. Introduction

In this lecture I discussed direct measurements of the masses of the three known neutrinos. With 'direct' I mean the analysis of the kinematics of suitable decays. There are other types of experiments which are sensitive to effects of non-zero neutrino masses. Double beta decay has been discussed by J. Busto and oscillation experiments by L. Camilleri at this school. A general review of the status of the neutrino mass is given in [1] which contains also many references. For reviews about the tritium experiments I refer to the references [2, 3]

In the next two sections I briefly review recent measurements of the tau and the muon neutrino mass. The main part of the lecture was devoted to the tritium experiments and is presented in section 4.

2. The τ–Neutrino Mass

Tau leptons are produced in pairs at $e^+ e^-$ storage rings. Recent results have been reported by the ARGUS collaboration at DESY [4], by the CLEO collaboration at Cornell [5], and by the ALEPH collaboration at CERN [6]. Events are selected where one tau from the produced pair makes a simple decay, e.g.

$$\tau^+ \longrightarrow e^+ \, \nu_e \, \overline{\nu}_\tau \quad \text{or} \quad \mu^+ \, \nu_\mu \, \overline{\nu}_\tau$$

and the other tau decays into many pions, i.e.

$$\tau^- \longrightarrow n \, \pi^{+,-,0} \;\; \nu_\tau,$$

where n is 5 or 6. Such events are very rare when compared with decays into three pions. The idea is to convert as much decay energy into restmass as possible. In that way the sensitivity to the restmass of the ν_τ is maximized.

The measured quantities used in the analysis are the beam energy, the mass of the tau m_τ, and energies E_i and momenta $\vec{p}_i$ of the n pions. Using 4–momentum conservation

$$p_\tau = \sum_i p_i + p_\nu$$

the invariant mass of the pions can be related to the mass of the tau-neutrino.

$$\begin{aligned} m_{n\pi}^2 &= \Big(\sum_i p_i\Big)^2 = \Big(\sum_i E_i\Big)^2 - \Big(\sum_i \vec{p}_i\Big)^2 \\ &= m_\tau^2 + m_{\nu_\tau}^2 - 2 m_\tau E_\nu^0 \end{aligned}$$

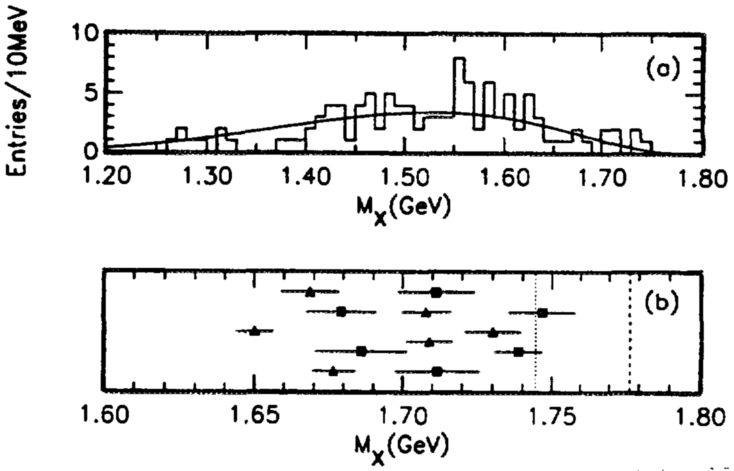

Figure 1: Measured events from CLEO. The invariant mass is denoted $M_X = m_{n\pi}$.

Table 1: Results for the tau neutrino mass from resent experiments.

Events observed	Upper limit (MeV), 95%CL	Collaboration
20	31	ARGUS [4]
113	32.6	CLEO [5]
25	24	ALEPH [6]

Here E_ν^0 is the energy of the tau-neutrino in the rest frame of the decaying tau. With $E_\nu^0 \geq m_{\nu_\tau}$ we obtain a kinematic limit

$$m_{n\pi} \leq m_\tau - m_{\nu_\tau}$$

The distribution of $m_{n\pi}$ is not known exactly. However, it is argued that the distribution should be sensitive to m_{ν_τ} only close to the kinematic limit and there the shape of the distribution is dominated by the phase space factor. The data from CLEO are shown in figure 1.

The results are summarized in table 1. One notes that CLEO has about the same upper limit as ARGUS although they observed much more events. This is because the distribution of $m_{n\pi}$ is broad and most events provide no information about m_{ν_τ}. One event close to the kinematic limit, where the expected distribution drops rapidly to zero, may easily dominate the final result. ARGUS has such events whereas CLEO does not. The background analysis of both collaborations show that the expected number of misidentified events should be much smaller than one.

The ALEPH collaboration performed a different analysis. In addition to $m_{n\pi}$ they used also the sum of the pion energies as a second variable in the analysis giving a much better upper limit. They report that their upper limit would rise to 40 MeV if only $m_{n\pi}$ would be used. There is no obvious reason why ARGUS and CLEO could not do the same analysis and it would be interesting to see the result.

3. The μ–Neutrino Mass

All recent measurements of the mass of the muon neutrino have been performed at PSI [7, 8]. The latest experiment has been presented by P. Kettle at this school and here I give only a short summary.

Studied is the pion decay at rest.

$$\pi^+ \longrightarrow \mu^+ \, \nu_\mu$$

Using 4-momentum conservation $p_\pi = p_\mu + p_\nu$ it is easy to derive a formula for the mass of the muon neutrino.

$$m_{\nu_\mu}^2 = m_\pi^2 + m_\mu^2 - 2m_\pi\sqrt{m_\mu^2 + p_\mu^2}$$

Hence the masses of the pion and muon and the 3-momentum of the muon are needed. As the formula above involves the difference of large numbers high precision measurements are necessary. The result for the muon momentum is

$$p_\mu = 27\,792\,000 \;\pm\; 110\,\mathrm{eV/c},$$

a 3.7 ppm measurement. Whereas the mass of the muon is known with sufficient precision there has been a longstanding problem with the pion mass. It is now believed to be solved [8]. The upper limit for the muon neutrino mass is given by

$$m_{\nu_\mu} < 170\,\mathrm{keV} \;\; (90\ \%\ \mathrm{CL}).$$

Considering the precision required for this result, significant improvements are certainly difficult to achieve.

4. The Electron–Neutrino Mass

The best direct limits for the mass of the electron neutrino have traditionally been obtained from studies of the beta decay of tritium.

$$^3\mathrm{H} \longrightarrow {}^3\mathrm{He}^+ \;\; e^- \;\; \overline{\nu}_e$$

For the decay of a bar nucleus, the energy distribution of the decay electrons is given by

$$N(E) := \frac{dN}{dE} \sim F(Z,W)pW\varepsilon^2\sqrt{1 - \frac{m_{\nu_e}^2}{\varepsilon^2}}\,, \quad \varepsilon \geq m_{\nu_e}$$

where $\varepsilon = E_0 - E$ is the neutrino energy, $W = E + m_e$, and $E_0 \approx 18.6$ keV is the endpoint energy (for $m_{\nu_e} = 0$). The Fermi function $F(Z,W)$ is a phasespace correction, taking into account the deceleration in the Coulomb field of He^{++} $(Z = 2)$. The complete spectrum of tritium is shown in figure 2. The spectrum is sensitive to a non-zero neutrino mass only close to the endpoint, i.e. for small neutrino energies. This is shown in the inset.

Table 2: Results for the electron neutrino mass from recent experiments. The column $m_{\nu_e}^2$ (all data) gives the results when all measured data are analysed. The upper limit (UL) is at 95% confidence level.

Experiment	Source	$m_{\nu_e}^2$ (eV^2)	$m_{\nu_e}^2$ (all data)	UL (eV)
Los Alamos [9]	T_2 gas	$-147 \pm 68 \pm 41$	-230	9.3
Zürich [10]	CHT monolayer	$-24 \pm 48 \pm 61$	same	11
Mainz [11]	frozen T_2	$-39 \pm 34 \pm 15$	-120	7.2
Livermore [12]	T_2 gas	$-130 \pm 20 \pm 15$	same	–
Troitsk [13]	T_2 gas	-22 ± 5	-60	4.35

The physical parameters of the spectrum are the neutrino mass m_{ν_e} and the endpoint energy. The latter is related to the atomic mass difference of tritium and ^{3}He which however is not known with sufficient accurary. Hence tritium data are analysed with E_0 treated as a free parameter. This should be contrasted with the tau neutrino experiments in section 2 where the endpoint (the kinematic limit) was used in the analysis. With tritium information for the neutrino mass m_{ν_e} comes only from the spectrum shape and the measured quantity is the mass squared $m^2_{\nu_e}$ and not the mass. It is accepted practice that $m^2_{\nu_e}$ is allowed to take on negative values. This may occur due to statistical fluctuations and has no physical meaning. Of course if a result is strongly negative we may suspect that there is a systematic error.

The results from recent experiments are summarized in table 2. The values in the column labeled $m^2_{\nu_e}$ are denoted best estimates in the cited references. Obviously all results are negative, some quite significantly. This is not the whole story. In the next column I have listed the results provided *all* measured data are included in the analysis. These values, when applicable, are even more negative. The obvious conclusion to be drawn is that something must be wrong either with the experiments or with the interpretation of the data, i.e. the fitted model. In the following I will briefly discuss what it could be.

The beta decay of tritium is a superallowed transition between two mirror nuclei with isospin $T = 1/2$. The ft-value is 1135 s, similar as for neutron decay. Hence one does not expect that some complicated nuclear effects are affecting the spectrum shape. Because of the small decay energy recoil effects should also be negligible.

The Fermi function can be computed exactly only for a point-like nucleus. The correction for the finite size of the nucleus is of order 10^{-4} for tritium and depends only weakly on energy. If taken into account in the analysis this correction is found to be negligible.

Radiative corrections (QED) for beta decay has been calculated to first order in the fine structure constant α. As we are only interested in the shape, it can be written in the form

$$S = 1 + \frac{\alpha}{2\pi} O\Big(\ln \frac{2\varepsilon}{m_e}\Big).$$

There is a logarithmic singularity right at the endpoint (for $m_{\nu_e} = 0$), which however is only present in the correction and not in the spectrum. Also this correction has a small effect on the fitted value of $m^2_{\nu_e}$.

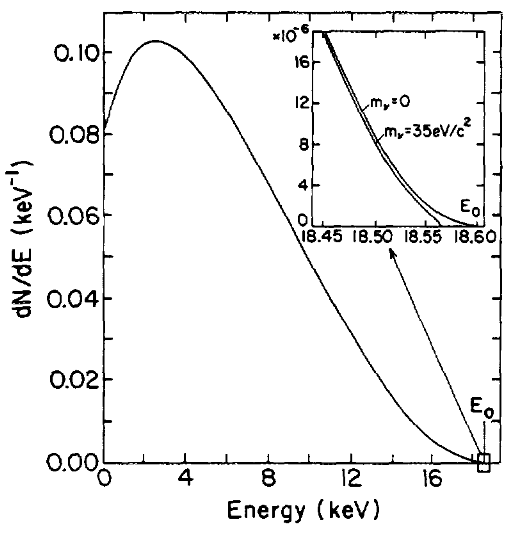

Figure 2: Beta spectrum of tritium.

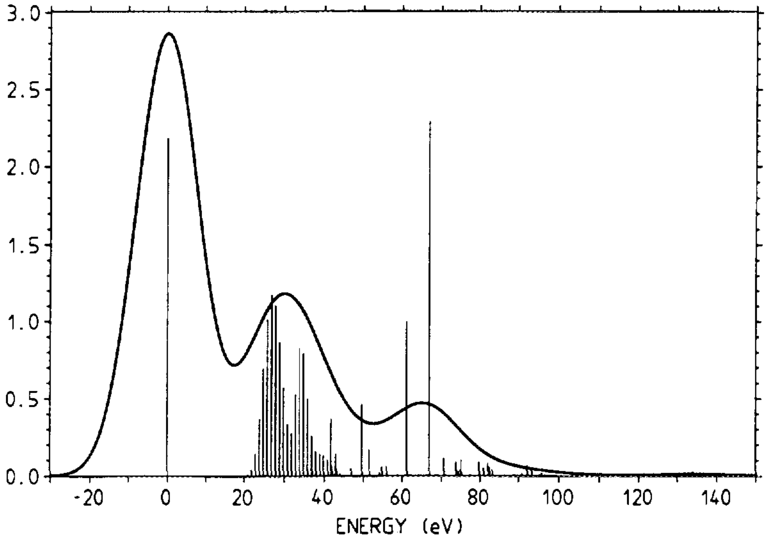

Figure 3: Transition probabilities to electronic final states in the decay of T_2. The solid line is the same distribution but convoluted with a Gaussian with 17 eV FWHM.

In the analysis of all tritium data so far, it was assumed that the electron neutrino is created in a mass eigenstate. With neutrino mixing and with a certain choice for the mass eigenvalues, one can produce noticeable distortions in the measured range of a tritium spectrum. However, all groups with strongly negative $m^2_{\nu_e}$ values say that such a model does either not fit the data or gives inconsistent results.

In experiments tritium is bound to some molecule R–T, which may be excited during the decay. Hence we have a multi-channel process

$$\text{R–T} \longrightarrow (\text{R–He}^+)_n \; e^- \; \overline{\nu}_e$$

where n denotes quantum numbers of the product molecule (which may be unbounded). It is easy to see that this process is fast compared with the orbital frequencies of the bound electrons, i.e.

$$\frac{T_{\text{escape}}}{T_{\text{orbit}}} = \frac{a_{\text{B}}/v}{2\pi a_{\text{B}}/(\alpha c)} = \frac{1}{2\pi}\frac{\alpha}{v/c} \approx 4 \times 10^{-3}$$

and the sudden approximation should be applicable. The transition probabilities to a final state n are then given by overlap matrix elements

$$W_{0n} = |\langle \Psi_0(\text{R–T}) | \Psi_n(\text{R–He}^+) \rangle|^2$$

Extensive computations have been performed using the sudden approximation. Figure 3 shows the result for the T_2 molecule [14]. The transition probability from ground state to ground state is 57.4%. It is believed that within the framework of the sudden approximation, the accuracy of the computations for T_2 is actually better than needed for the present experiments.

We should now ask for the validity of the sudden approximation. The magnitude of the leading order correction is determined by a small time parameter (also called the Sommerfeld parameter)

$$\eta = \frac{\alpha}{v/c} = 2.63 \times 10^{-2}.$$

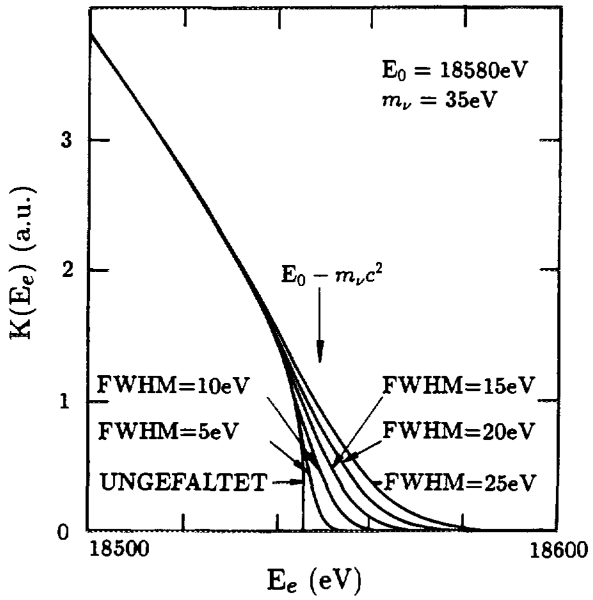

Figure 4: Kurie plot for an assumed neutrino mass m_{ν_e} = 35 eV and for several widths of the resolution function.

The same parameter determines also the size of the matrix elements and the leading order correction is thus

$$\delta W_{0n} \sim O(\eta^2) = 7 \times 10^{-4}.$$

This is a factor of 30 to 70 smaller than what would be required to 'explain' the experimental results for T_2, i.e. making $m^2_{\nu_e}$ compatible with zero. Calculations have been performed for the T atom by several workers. In this case a 'accidental' cancelation occur making the corrections even smaller. For example the correction for the ground state probability is $\delta W_{00} = -2 \times 10^{-4}$ to be compared with the sudden approximation value $W_{00} = 0.702$.

I should mention two more things. Presently there is a project [15] to calculate corrections to the sudden approximation for the T_2 molecule and one should not draw far reaching conclusions before this difficult task is completed. In a recent diploma thesis a different approach was used [16]. Assuming an initial wave function for the electron created in the decay, the time dependent Schrödinger equation for the T atom was solved numerically. An amazingly large correction $\delta W_{00} \approx -0.05$ was found. This work *must* be checked independently.

There are many experimental effects which must be properly taken into account to avoid systematic errors. Here I mention just two and refer to [2] for a more thorough discussion. Figure 3 shows the endpoint region of the tritium spectrum as a Kurie plot, defined by

$$K(E) = \sqrt{N(E)/(FpW)}.$$

Far below the endpoint, $K(E)$ should be a straight line. At the kinematic limit $E_0 - m_{\nu_e}$ and for infinite resolution, $K(E)$ drops to zero with a vertical slope if $m_{\nu_e} > 0$. This signature of a nonzero neutrino mass gets more and more diminished with increasing width of the resolution function. This is not a problem if properly taken into account. However, if the assumed resolution function is too narrow, a fit gives a smaller value of $m^2_{\nu_e}$, becoming negative if the expected value is close to zero. This is an example of a general rule. If any of the distributions needed for the analysis is erroneously taken to be too narrow, the fitted value of $m^2_{\nu_e}$ is shifted in the negative direction.

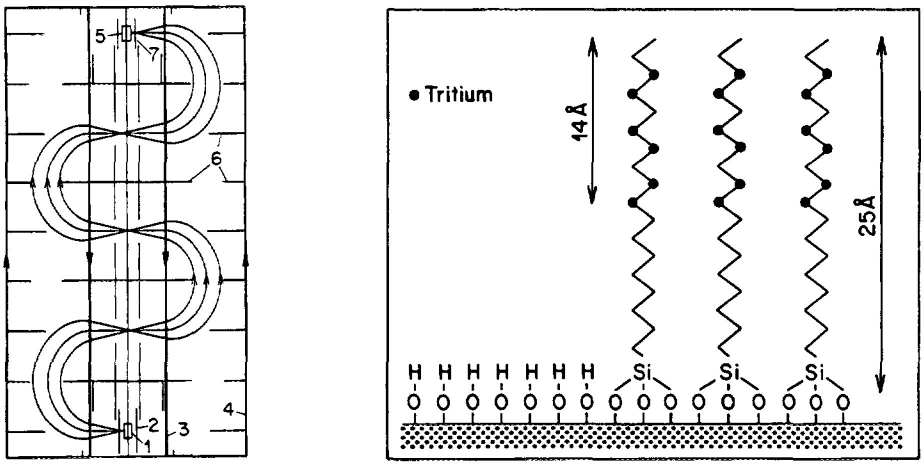

Figure 5: Left: Cross section of the Zürich spectrometer. Source 1, grids 2,7 current conductors 3,4, detector 5, baffles 6. The distance between source and detector is 2648 mm. Right: Model of the monolayer tritium source.

Tritium sources must be thin, in fact so thin that the probability for one inelastic interaction in the source is a small number. It is not sufficient to know the average energy loss, the whole distribution is needed. This is a problem which in my view has not always been appreciated. One reason is that the energy loss distribution has a very extended tail which is difficult to measure and for which theoretical extrapolations may be even more suspect.

In the following I briefly discuss the various tritium experiments. It is of course not possible here to go into any details and I refer to the original publications (see table 2).

Figure 5 (left) shows a cross-section of the spectrometer used in the Zürich experiment. It is of the Tret'yakov type. Electrons from the source are focused onto the detector in a toroidal magnetic field in four 180° bends. The tritium data were taken by stepping a high voltage applied to the source at fixed magnetic field. The resolution was 17 eV FWHM. A model of the tritium source is shown on the right side of figure 5. It was produced by chemically growing a monolayer of hydro-carbon chains on a suitable surface. There are six tritium atoms per molecule. This source is distinguished by its well defined structure and its very small thickness. Only 2% of the detected electrons had made an inelastic interaction in the source layer.

Data were recorded from 920 eV below to 180 eV above the endpoint. The results indicate a high degree of internal consistency of the data. This is shown in figure 6. Plotted are the fitted neutrino mass squared and endpoint energy as a function of a energy E_{cut} below which the data points were excluded from the fit for this test. Within a narrow band of statistical fluctuations the parameters are independent of E_{cut} as it should be. For the final result in table 2 all data were used.

The first experiment using a gaseous tritium source was performed in Los Alamos. A schematic of the set-up is shown in figure 7. The experiment in Livermore is similar. The source consists of a long tube. Tritium gas enters the tube in the middle and streams to the ends where it is pumped away by large mercury diffusion pumps. The tritium gas is recycled through a palladium foil. Decay electrons are transported in a strong longitudinal magnetic field from the source tube into a Tret'yakov type spectrometer. The energy resolution was about 22 eV for the Los Alamos and 18 eV FWHM for the Livermore experiment. The fraction of electrons making an inelastic interaction before leaving the source tube, was also similar, being 8.5% (Los Alamos) and 12% (Livermore).

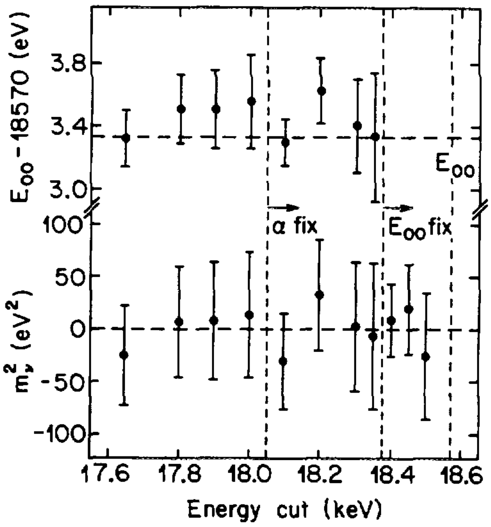

Figure 6: Fitted neutrino mass squared and endpoint energy when data points below the energy E_{cut} are excluded from the fit.

The Livermore group finds an anomalous bump in their spectrum close to the endpoint which seemingly cannot be accounted for by experimental effects. As a consequence they did not publish an upper limit for m_{ν_e}.

The groups in Mainz and Troitsk (Moscow) use instruments which they call solenoid retarding spectrometers with adiabatic magnetic collimation. The set-up from Mainz is shown in figure 8. The source is located in a strong magnetic field B_i. The electrons emitted in the forward direction spiral along the field lines into a large vacuum tank where the magnetic field drops to a small value B_f, typically $B_f/B_i = 1/3000$. The adiabaticity theorem shows that most of the transverse energy $E_{i\perp}$ at the source is converted into longitudinal energy

$$E_{f\parallel} = E_i - \frac{B_f}{B_i} E_{i\perp}.$$

At the centre of the tank, a electrostatic potential barrier is generated by a set of cylin-

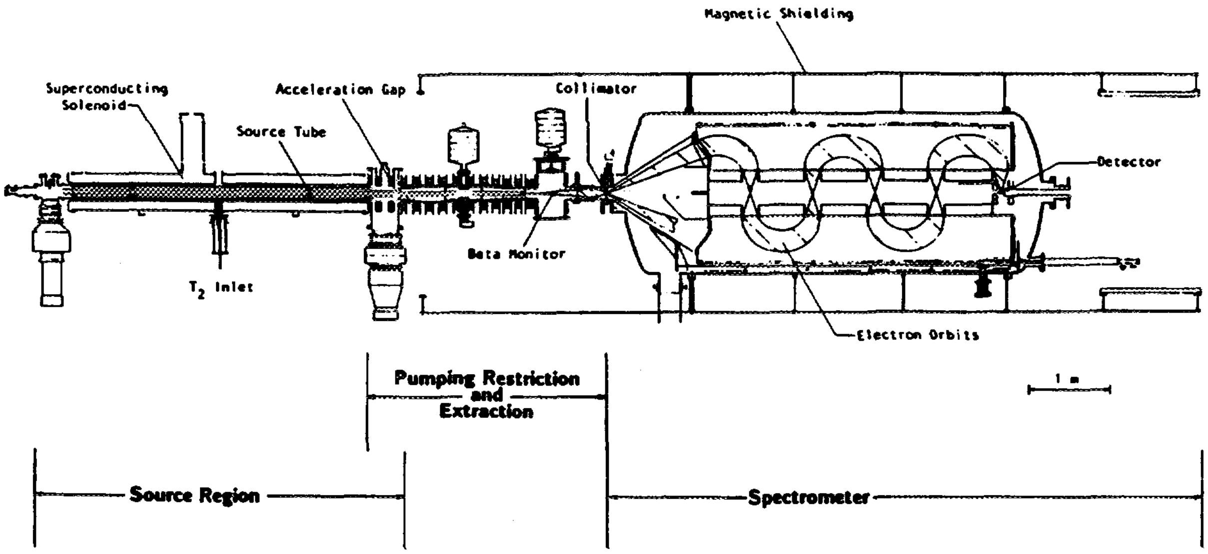

Figure 7: Overview of the Los Alamos tritium experiment. The overall length of the apparatus is 16 m.

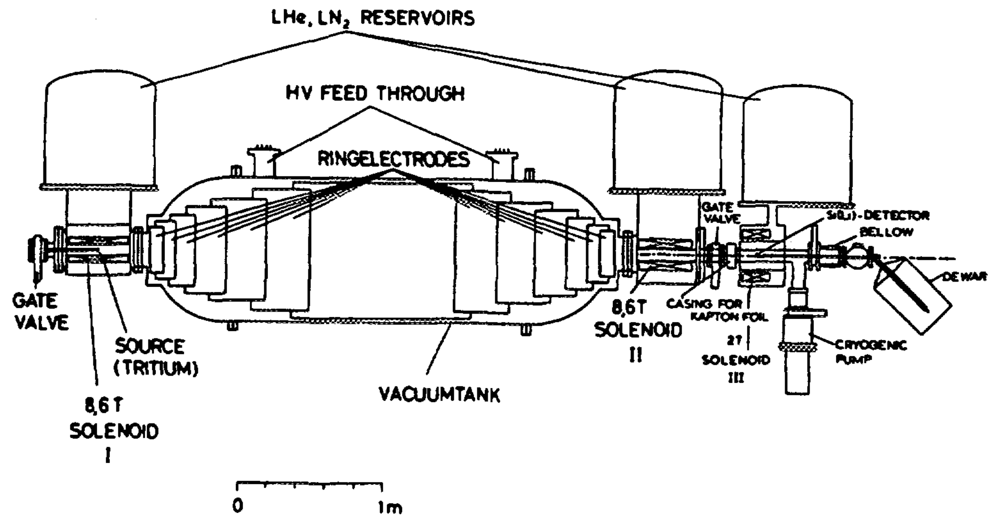

Figure 8: Schematic view of the Mainz retarding spectrometer.

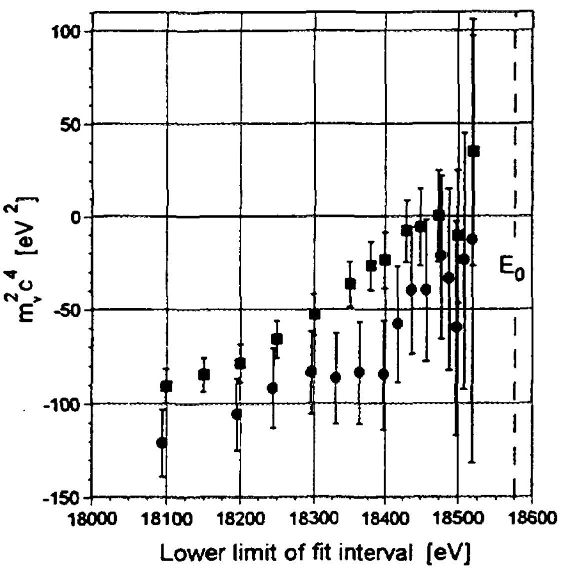

Figure 9: Fitted $m_{\nu_e}^2$ as a function of the data range for two runs (dots and squares) from the Mainz experiment.

drical high voltage electrodes. Electrons with sufficient energy pass the barrier and are reaccelerated and focused onto a detector, all other electrons returning to the source. Hence the integral of a spectrum is measured. The energy resolution, defined as the energy range over which the transmission curve drops from one to zero, was 6 eV at Mainz and 3.7 eV at Troitsk.

The source at Mainz was frozen tritium whereas a gaseous source was used at Troitsk with a set-up similar to Los Alamos. From the information given in the publications the fraction of electrons interacting in the source, can be estimated. I find 13% for Mainz and 16% to 26% for Troitsk (rather thick source).

Both groups have collected data with very high statistical power. Unfortunately, the fitted $m_{\nu_e}^2$ have highly significant negative values if all measured data are used and moreover show an unphysical dependence on the energy range. In addition a step-like distortion (corresponding to a line in a differential spectrum), about 10 eV below E_0, seems to be present in the Troitsk data. It is unclear to me whether this is something interesting or just an experimental artifact.

Conclusion

The direct measurements have so far given no indication for a nonzero (positive) mass of any of the three known neutrinos. The experiments measuring the tau and the muon neutrino are good shape. The tritium experiments are in an unfortunate situation. It is unclear to me whether the problems are experimental or theoretical or a combination of both. The electronic final states distribution have been calculated, but the results have never been tested experimentally. The most important question to be answered is about the validity of the sudden approximation.

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