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EXCHANGE NONLOCAL EFFECTS IN THE NUCLEAR HEAVY-ION ELASTIC SCATTERING

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An interesting feature of the presence of exchange nonlocal effects in the nucleus-nucleus collision description is the modification of the nuclear barrier. This results, on the one hand, in an enhancement of the nuclear fusion cross sections due to the modification introduced in the basic quantum mechanical tunnelling calculation and, on the other hand, leads one to the study of the consequences of those effects in the nuclear elastic scattering. In this paper, we discuss the manifestations of the presence of those effects through an approximated model Schrödinger equation describing a nuclear system colliding at energies around the barrier. As an aplication, the elastic channel and, concomitantly, the fusion processes are studied for the $^{16}O+^{60}Ni$ system at energies close to the barrier.

I. INTRODUCTION

In a series of papers, it has been pointed out that the introduction of kinematical nonlocal effects, i.e., those of quantum exchange nature, in the description of nuclear colliding systems can be responsible for an important modification in the nuclear barrier transmission function [1-3]. In fact, it was shown that in the effective reduced mass approximation those exchange nonlocal effects modify the barrier, in such a form to redefine the static barrier curvature so that the tunnelling, and consequently the fusion process at subbarrier energies, is enhanced. It was emphasized that this redefinition constitutes, together with the standard tunnelling calculation, the starting quantum mechanical background upon which the many-body nuclear collision description must introduce the additional degrees of freedom related to the relevant coupled channels to the process [2]. The comparison between the contributions for the fusion process coming from the exchange nonlocal terms and from the coupled channels ones was discussed in a schematic model [3] in which the nonlocal potential form was the one proposed by Perey and Buck [5] and the treatment of the coupled channels was that proposed by Dasso [6]. The results of this calculation, although schematic, have pointed out to a very peculiar difference between the coupled channels effects and those stemming from the nonlocal effects, namely, while the coupled channels generate a family of barriers of different heights, the exchange nonlocal effects only redefine the barrier curvature, as already mentioned. It is clear that one expects both effects to occur in a realistic description of the process, or even that in a full coupled channels calculation the barrier curvature may also to be redefined, at least in part, due to the exchange nonlocal effects.

These considerations about the presence of exchange nonlocal effects in the description of the nuclear fusion leads us to the related problem of verifying how these effects manifest themselves in the nuclear elastic scattering. In this form, we have drawn attention to the description of the elastic scattering by adapting a model Schrödinger equation in such a form to embody the exchange nonlocal effects through an approximation to the full integro-differential equation. In fact, we have made the usual adiabatic approximation which, in our case, reduced the original problem to an effective reduced mass approach. The approximated Schrödinger equation can then be solved and the relevant information about the nuclear collision extracted in a direct way.

In section 2 we present a brief discussion about the introduction of the exchange nonlocal effects in the Hamiltonian energy kernel and how one can introduce the relevant approximations. The Schrödinger-like equation is then introduced in section 3, in which we discuss how the approximated form of the Hamiltonian can give rise to a differential equation that can be numerically solved. The results from an application carried out to the $^{16}O + {}^{60}Ni$ system are discussed in section 4, while in section 5 we present our conclusions.

II. A MODEL TREATMENT OF EXCHANGE NONLOCAL EFFECTS

A particularly convenient way of treating an Hamiltonian associated to a nuclear system consisting of two colliding nuclei, which is expressed in terms of global collective coordinates $(\vec{q}, \vec{p})$, referring to the relative motion, and a coordinate $\vec{\xi}$, characterizing a particular intrinsic degree of freedom of the nuclei that is coupled to the relative motion, is through the Weyl-Wigner transformation of a nonlocal energy kernel [2] which is expressed as

$$H\left(\vec{p}, \vec{q}, \vec{\xi}\right) = \delta_{\vec{\kappa},\vec{\xi}} \int \exp\left(\frac{i}{\hbar}\vec{p}\cdot\vec{v}\right) H\left(\vec{q}, \vec{v}, \vec{\kappa}\right) d\vec{v} \tag{1}$$

where

$$H\left(\vec{q}, \vec{v}, \vec{\kappa}\right) = \langle \vec{q} - \vec{v}/2, \vec{\kappa} \left| \hat{H} \right| \vec{q} + \vec{v}/2, \vec{\kappa} \rangle. \tag{2}$$

Instead of first writting a full microscopic Hamiltonian operator $\hat{H}$, we will start our approach by modelling the description of the nuclear collision through the introduction of the nonlocal energy kernel which must be written in terms of all the corresponding variables associated to the whole process. At the end of the construction of the nuclear collision model we will be able to obtain the Hamiltonian operator by just performing a Weyl-Wigner inverse transformation [4]. This way of treating the system Hamiltonian is particularly convenient because we want also to deal with exchange nonlocal effects of the type described by a Perey-Buck-like nucleus-nucleus interacting potential [5]. In the model Hamiltonian this term is introduced so as to simulate nonlocal effects originating from quantum correlations, i.e., mainly those stemming from exchange effects, so that $H(\vec{q},\vec{v},\vec{\kappa})$ is given in an explicit form

$$H(\vec{q},\vec{v},\vec{\kappa}) = -\frac{\hbar^2}{2\mu}\delta''(\vec{v}) + V_{NL}(\vec{q},\vec{v}) + V_L(\vec{q},\vec{v})\delta(\vec{v}) + \\ + V_{cpl}(\vec{q},\vec{v},\vec{\kappa})\delta(\vec{v}) + H_0(\vec{\kappa})\delta(\vec{v}). \tag{3}$$

Here μ is the reduced mass of the relative motion, $V_{NL}(\vec{q},\vec{v})$ is the nonlocal potential written as a Perey-Buck-like interaction

$$V_{NL}(\vec{q},\vec{v}) = \frac{1}{b^3\,\pi^{3/2}}V(\vec{q})\exp\left[-\left(\frac{\vec{v}}{b}\right)^2\right], \tag{4}$$

where b measures the nonlocal range. From this expression one can verify that the considered quantum exchange effects between the two colliding nuclei will be relevant only for separation distances of the order of the nonlocality range. In expression (3), V_L is a local potential, $V_{cpl}(\vec{q},\vec{v},\vec{\kappa})$ is the interaction coupling the relative motion and the intrinsic degree of freedom and $H_0(\vec{\kappa})$, the intrinsic Hamiltonian of the system, is associated with the eigenvalue problem

$$\hat{H}_0 \mid \vec{\kappa}\rangle = \varepsilon_\kappa \mid \vec{\kappa}\rangle. \tag{5}$$

The eigenvectors $\mid \vec{\kappa}\rangle$ characterize the spectrum of the selected intrinsic degree of freedom.

As has been already discussed previously in connection with nuclear fusion reactions [2,3], one can write the Weyl-Wigner mapped expression for the Hamiltonian describing a nuclear colliding system as an integral of the kernel of a nonlocal operator through a Fourier transform

$$H(\vec{p},\vec{q}) = \int e^{\left(\frac{i}{\hbar}\right)\vec{p}\cdot\vec{v}} H(\vec{q},\vec{v})\, d\vec{v} \tag{6}$$

thus giving

$$H\left(\vec{p},\vec{q},\vec{\xi}\right) = \frac{\vec{p}^{\,2}}{2\mu} + \sum_{n=0}^{\infty}\frac{(-i)^n}{\hbar^n}\vec{p}^{\,n}V^{(n)}(\vec{q}) + V_L(\vec{q}) + V_{cpl}\left(\vec{q},\vec{\xi}\right) + H_0\left(\vec{\xi}\right), \tag{7}$$

where $V^{(n)}(\vec{q})$ is the nth-moment of the nonlocal potential with respect to $\vec{v}$. Due to the conservative character of the Hamiltonian description, only even values of n will be present in the power series, furthermore we will keep only the first two terms which will give the dominant contributions to our Hamiltonian. This assumption corresponds to an adiabatic approximation and is valid for $pb/\hbar < 1$. Therefore, we have, up to $n = 2$, the model Hamiltonian

$$H\left(\vec{p},\vec{q},\vec{\xi}\right) \simeq \frac{\vec{p}^{\,2}}{2\mu\left(\vec{q};b\right)} + V^{(0)}\left(\vec{q}\right) + V_L\left(\vec{q}\right) + V_{cpl}\left(\vec{q},\vec{\xi}\right) + H_0\left(\vec{\xi}\right). \tag{8}$$

Hereafter $V^{(0)}(\vec{q})$, the zeroth-moment of the nonlocal potential, will be identified as a standard nucleus-nucleus attractive potential $V_N(\vec{q})$. Furthermore

$$\begin{aligned}\mu\left(\vec{q};b\right) &= \mu/\left(1-\frac{\mu}{\hbar^2}V^{(2)}\left(\vec{q}\right)\right)\\ &= \mu/\left(1-\frac{\mu b^2}{2\hbar^2}V^{(0)}\left(\vec{q}\right)\right)\end{aligned} \tag{9}$$

is an effective reduced mass now depending on the form of the potential $V^{(0)}(\vec{q})$ as well as on the nonlocality parameter b. We observe the asymptotic behavior of the mass $\mu(\vec{q};b) \to \mu$ in the local limit $b \to 0$ or when $V^{(0)}(\vec{q}) \to 0$.

Now, in order to deal with the heavy-ion elastic scattering we have to adapt the general approach just presented so as to properly treat that channel. To begin with, we must reduce the general Hamiltonian in such a form as not to explicitly present the term which couples the relative motion and intrinsic degrees of freedom and, consequently, the intrinsic collective Hamiltonian and its corresponding spectrum will be ignored in what follows. With these assumptions we end up with a model Hamiltonian which embodies the exchange nonlocal effects through the effective reduced mass, and, since this Hamiltonian is obtained by making an adiabatic approximation, it has therefore a validity range which restricts its use to system energies around the Coulomb barrier.

The model Hamiltonian used in the present approach for the heavy-ion elastic scattering is then written as

$$H(\vec{p},\vec{q}) \cong \frac{\vec{p}^{\,2}}{2\mu(\vec{q};b)} + V_N(\vec{q}) + V_L(\vec{q}), \tag{10}$$

where $\mu(\vec{q};b)$ is the effective reduced mass of the system which also depends on the non-local range. As is known from the Weyl-Wigner mapping techniques [4], it is possible to find the Hamiltonian operator $\hat{H}(\hat{p},\hat{q})$ by taking the Weyl-Wigner inverse transform of the Hamiltonian kernel, Eq. (2), which can be cast in the form

$$\hat{H}(\hat{p},\hat{q}) = \int \delta(\vec{q}-\hat{q})\,\delta(\vec{p}-\hat{p})\,h_0(\vec{p},\vec{q})\,d\vec{p}\,d\vec{q}, \tag{11}$$

with

$$h_0(\vec{p},\vec{q}) = \exp\left(\frac{\hbar}{2i}\frac{\partial}{\partial\vec{p}}\frac{\partial}{\partial\vec{q}}\right) H(\vec{p},\vec{q}). \tag{12}$$

This expression shows that we can find the operator $\hat{H}(\hat{p},\hat{q})$ from its Weyl transform, $H(\vec{p},\vec{q})$, by first calculating $h_0(\vec{p},\vec{q})$ and then substituting the variables $\vec{p}$ and $\vec{q}$ by their respective operators $\hat{p}$ and $\hat{q}$. To follow the Weyl-Wigner prescription it is necessary to always keep the coordinate operator to the left of the momentum operator. Substituting Eq. (10) in Eq. (12) leads to

$$\begin{aligned} H(\hat{p},\hat{q}) &= \frac{1}{2\mu(\hat{q};b)}\hat{p}^{\,2} + \frac{\hbar}{2i}\left(\frac{d}{d\vec{q}}\frac{1}{\mu(\hat{q};b)}\right)\hat{p} - \frac{\hbar^2}{8}\left(\frac{d^2}{d\vec{q}^{\,2}}\frac{1}{\mu(\hat{q};b)}\right) \\ &\quad + V_N(\hat{q}) + V_L(\hat{q}) \quad . \end{aligned} \tag{13}$$

In the usual representation for the momentum operator, $\hat{p} = -i\hbar\,(d/d\vec{q})$, the Hamiltonian can still be written as

$$\begin{aligned} H(\hat{p},\hat{q}) &= -\frac{\hbar^2}{2}\frac{1}{\mu(\hat{q};b)}\nabla^2 - \frac{\hbar b^2}{2}\left[\nabla\frac{1}{\mu(\hat{q};b)}\right]\cdot\nabla - \frac{\hbar^2}{8}\left[\nabla^2\frac{1}{\mu(\hat{q};b)}\right] + \\ &\quad + V_N(\hat{q}) + V_L(\hat{q}). \end{aligned} \tag{14}$$

It is important to stress that in the local limit, i.e., when $b \to 0$, or in nucleus-nucleus interaction free regions, expression (14) reduces to the usual Hamiltonian operator.

The Schrödinger differential equation associated to the eigenvalues of the Hamiltonian operator can be directly given

$$H\Psi_{NL} = E\Psi_{NL},$$

where Ψ_{NL} is the nonlocal wave function. Now, it has been numerically verified [5,15] that the nonlocal model results can be reproduced with a great accuracy by a calculation where use is made of the local wave function

$$\Psi_{NL} \approx \Psi_L,$$

then:

$$\left\{-\frac{\hbar^2}{2}\frac{1}{\mu(\hat{q};b)}\nabla^2 - \frac{\hbar^2}{2}\left[\nabla\frac{1}{\mu(\hat{q};b)}\right]\cdot\nabla - \frac{\hbar^2}{8}\left[\nabla^2\frac{1}{\mu(\hat{q};b)}\right] + V_N(\hat{q}) + V_L(\hat{q})\right\}\Psi_L(\vec{q}) = E\Psi_L(\vec{q})\,. \qquad (15)$$

This is the wave equation in an effective reduced mass approximation which embodies now the exchange nonlocal effects through a redefinition of the mass. Similar equations have been obtained in the past for the description of a nucleon in the nuclear medium [7]. Taking into account that the main purposes of the present studies are just to have an estimation of local effects, we did not consider the second term in the wave equation. We point this approach, which has been already adopted in other different calculations of local equivalent potentials [8,9], corresponds to consider in equation (15) an unsymmetrized kinetic energy operator. With these considerations, the wave equation (15) is reduced to a Schrödinger-like equation which can be treated in a standard way.

The local part of the potential was assumed to be the Coulomb potential plus an imaginary term to simulate the absorption by the reaction channels,

$$V_L(\hat{q}) = V_{Coul}(\hat{q}) + iV_I(\hat{q}) \quad .$$

In the usual partial wave expansion, the approximated form of Eq. (15) is written as

$$\frac{d^2}{dr^2}v(r) + \frac{2\mu(r;b)}{\hbar^2}\left[E - V_N(r) - V_{Coul}(r) - iV_I(r) - \frac{b^2}{16}\nabla_r^2 V_N(r) - \frac{l(l+1)\hbar^2}{2\mu(r;b)r^2}\right]v(r) = 0, \tag{16}$$

where $v(r)$ is the radial part of the wave function. After a straightforward calculation Eq. (16) can be rewritten as

$$\frac{d^2}{dr^2}v(r) + \frac{2\mu}{\hbar^2}\left[E - V_{Coul}(r) - \frac{l(l+1)\hbar^2}{2\mu r^2} - U_{eq}(r,E;b)\right]v(r) = 0 \ , \tag{17}$$

which is the equivalent radial Schrödinger equation and $U_{eq}(r,E;b)$ is the equivalent local potential to the nonlocal one, whose expression reads:

$$U_{eq}(r,E;b) = V_{eq}(r,E;b) + i\,W(r;b) \ , \tag{18}$$

with

$$V_{eq}(r,E;b) = V_N(r) + [E - V_B(r)]\left(1 - \frac{\mu(r;b)}{\mu}\right) + \frac{b^2}{16}\frac{\mu(r;b)}{\mu}\nabla_r^2 V_N(r) \tag{19}$$

and

$$W(r;b) = \frac{\mu(r;b)}{\mu}V_I(r) \ ; \tag{20}$$

$$V_B(r) = V_{Coul}(r) + V_N(r) \ . \tag{21}$$

As expected, the nonlocal character of the real part of the complex nuclear potential (originally independent of the energy) gives rise to an explicit energy dependence in the equivalent local potential. It is remarkable that this same form of energy dependence for the real optical potential was found in a study of nucleon-nucleus interaction under the assumption of nuclear matter [10].

The adiabatic approximation expressed by the relation $Kb < 1$, which also gives a measure of the vality of the semiclassical method to study the effects of nonlocal potentials [11], can be write in the form $2\ \mu\ b^2/\hbar^2 \mid E - V_B(r) \mid < 1$, where $V_B(r)$ is the effective local ($b=0$) potential in according to equation (21). The maximum of the function $V_B(r)$ defines the height and radius of the local barrier, $V_B(R_B)$ and R_B respectively. The elastic scattering process is defined mainly at interaction distances $r \sim R_B$ what means to consider, in the adiabatic approximation, only energies close to the barrier height $E \sim V_B(R_B)$. We remark that the external region ($r \gg R_B$) is dominated by the long-range local Coulomb potential and the inner region ($r \ll R_B$) is not important to define the elastic and fusion cross sections if the penetrating waves are absorved, which is the case of heavy-ion collisions at energies around the barrier.

IV. APPLICATION

A numerical solution of Eq. (17) allows us to obtain the angular distribution for the elastic scattering, as well as the fusion cross sections. As an application we choose the $^{16}O + ^{60}Ni$ system at energies close to the barrier. In recent studies of this system [12], by measuring the elastic and inelastic (2_1^+) target excitation cross sections at sub-barrier energies ($E_{\rm Lab} \leq 38$ MeV), it was possible to explain the data using the coupled channel approach in a two channel model (0_1^+, 2_1^+). The coupled channel calculations have also shown that the effects of other reaction channels, with much smaller cross sections compared to the 2_1^+ target excitation, are negligible for the elastic cross sections. It was possible to obtain the bare nucleus-nucleus potential — in the sense that no coupled channel affects are included — with a well defined form in the surface region ($r \geq 9$ fm) assuming the nuclear coupling potential as energy independent in the small energy range considered ($\Delta E_{\rm Lab} = 3$ MeV). The resulting nuclear nucleus-nucleus potential [12] has a Woods-Saxon shape with parameters $V_0 = -360$ MeV, $r_0 = 1.06$ fm and $a = 0.58$ fm. In the present application, we have considered these parameters for the nuclear potential $V_N(r)$ and an imaginary potential $W(r;b)$, also of Woods-Saxon shape, confined to the interior of the nucleus in such a way to allow us to identify the reaction cross section with the fusion cross section. In the calculations we considered $W_0 = -30$ MeV, $r_W = 0.8$ fm, $a_W = 0.2$ fm and we adopted $b = 1.0$ fm, which is an approximation to the value already discussed in connection to exchange nonlocal effects only [13,5,3].

Initially, we evaluated the equivalent local barrier, $V_{ef}(r,E;b) = V_{Coul}(r) + V_{eq}(r,E;b)$. The Figure 1(a) shows the $V_{ef}(r,E;b)$ function at interaction distances around the barrier radius (R_B) for three bombarding energies, $E_{\rm Lab}$ = 35 (dashes), 39 (dots) and 42 MeV (dash-dots). In the Figure it is also included the local effective potential $V_{ef}(r,E;b=0) = V_B(r)$ (solid line). For the region $r > R_B$, the equivalent local potentials are equal the local one due the asymptotic behavior of the effective reduced mass, i.e., $\mu(r \to \infty; b) \to \mu$. In the barrier region, $r \sim R_B$, the lower the bombarding energy the lower will be the maximum height of the local equivalent barrier. This behavior is related to the $[E - V_B(r)]$ dependence of the equivalent local potential, eq. (19). It is remarkable that for $E_{\rm Lab}$ = 39 MeV ($E_{\rm c.m.}$ = 30.8 MeV $\approx V_B(R_B)$) the heights for the equivalent barrier and local one are approximately the same showing, thus, that in this interacting region the expression for $V_{eq}(r,E;b)$ is reduced to $V_N(r) + [E - V_B(r)]\,(1 - \mu(r;b)/\mu)$. Therefore, the barrier height of the equivalent potential assumes values greater or smaller as compared to the local one, $V_B(R_B)$, for energies $E > V_B(R_B)$ or $E < V_B(R_B)$, respectively. In the inner region $r \ll R_B$, Figure 1(b), the adiabatic approximation is not justified since the wave number K is very large, as consequence the equivalent potential is even eventually repulsive for energies some MeV above the local barrier. However, for energies closer to the local barrier height there is still an inner pocket in the effective potential, which allows the absorption of the penetrating waves and , therefore, at these energies it is possible to calculate the elastic scattering and fusion cross sections using the present approach.

In the Figure 2 the calculated angular distributions are shown for the elastic scattering considering the local (solid line) and the equivalent local (dashed line) potentials at $E_{\rm Lab}$ = 35 MeV ($E_{\rm c.m.}$ = 27.6 MeV). We point out that in the effective mass approach the predicted elastic scattering cross sections present an hindrance at backward angles ($\Theta_{\rm c.m.} > 130$) as compared to the local case. Nevertheless, the bump ($100 < \Theta_{\rm c.m.} < 130$) is more pronounced in the equivalent local case. Also, in the equivalent local case occurs a shift of flux from the elastic to the reaction channel, here identified with the fusion process. This can be also observed in Figure 3 where are shown the corresponding calculated phase shift Φ_l (Figure 3(a)) and modulus $|S_l|$ (Figure 3(b)) of the S matrix as a function of the orbital angular momentum. For low partial waves the behavior of Φ_l and $|S_l|$ shows that the equivalent local potential is more atractive and more absorptive as compared to the local one.

Finally, we comment our results for the fusion when we take into account the exchange nonlocal effects. The local Coulomb barrier height ($l = 0$) is defined as

$$V_B \equiv V_B(R_B) = V_{Coul}(R_B) + V_N(R_B) \quad .$$

In the present case $V_N(R_B)$ is replaced by $V_{eq}(R_B, E; b)$, thus:

$$V_B^{eq} = V_{Coul}(R_B) + V_{eq}(R_B, E; b) \quad .$$

Using the Wong approach [14], we can write the following expression for the fusion cross section:

$$\sigma_f^{eq}(E) = \frac{\hbar\omega_B \, R_B^2}{2\,E} \ln\left\{1 + \exp\left[\frac{2\pi\,(E - V_B^{eq})}{\hbar\omega_B}\right]\right\} \quad .$$

We already verified that in the adiabatic approximation the equivalent local potential is described approximatelly by

$$V_{eq}(R_B, E; b) \approx V_N(R_B) + [E - V_B]\left(1 - \frac{\mu(R_B; b)}{\mu}\right) \quad ,$$

thus

$$E - V_B^{eq} \approx (E - V_B)\,\frac{\mu(R_B; b)}{\mu} \quad ,$$

where, from eq. (9),

$$\frac{\mu(R_B; b)}{\mu} = \frac{1}{1 - \frac{\mu\, b^2}{2\,\hbar^2}\, V_N(R_B)} \quad .$$

Defining

$$\hbar\tilde{\omega}_B = \hbar\omega_B \frac{\mu}{\mu(R_B;b)}$$

with $\hbar\tilde{\omega}_B \geq \hbar\omega_B$ for $b \geq 0$, we have:

$$\sigma_f^{eq}(E) = \frac{\hbar\tilde{\omega}_B\, R_B^2}{2\,E}\,\frac{\mu(R_B;b)}{\mu}\,\ln\left\{1+\exp\left[\frac{2\pi(E-V_B)}{\hbar\tilde{\omega}_B}\right]\right\} \quad .$$

For energies $E < V_B$, this expression is dominated by a term which decreases exponentially with $[2\pi(E-V_B)/\hbar\tilde{\omega}_B]$ and the redefinition of the barrier curvature $\hbar\tilde{\omega}_B$ prevails, resulting in an enhancement of the fusion cross section $\sigma_f^{eq}(E)$ in relation to the local one. We remark that this result is in accordance with previous works [2,3] for the fusion process on the framework of barrier tunnelling with the effective mass approximation. For energies $E > V_B$, $\sigma_f^{eq}(E) = \mu(R_B;b)/\mu\ \sigma_c(E)$, where $\sigma_c(E)$ is the geometrical classical fusion cross section proportional to E^{-1}; therefore, in this energy range $\sigma_f^{eq}(E)$ decreases with respect to $\sigma_c(E)$. This behavior of the equivalent local calculations in relation to the local one can be understood if we remember that the effective mass acts on the barrier curvature $\hbar\omega_B$ as well as on the centrifugal potential. For energies below the barrier, the fusion cross section is associated to low partial waves and the curvature redefinition predominates. For energies above the barrier the mass redefinition corresponds to an increasing of the repulsive character of the centrifugal potential which implies in a reduction of the fusion cross section. In Figure 4 the fusion cross section is shown for energies around the local barrier. It is directly seen that the results from the equivalent local calculation (dashes) are larger than the local one (solid line) for energies below or close to the local barrier and are smaller for energies above this one.

V. CONCLUSIONS

Starting from a Weyl transform of an Hamiltonian operator describing a nuclear colliding system, which also contains a nucleus-nucleus interaction of nonlocal exchange character, we adopted the effective mass approximation to describe the dominant contribution of those nonlocal effects and we end up with a differential wave equation. This wave equation was rewritten under the assumption that the nonlocal wave function is approximately equal to the local wave function and, in this form, the final wave equation is a Schrödinger-like equation with a local equivalent potential which must exhibit the main features of the starting nonlocal potential. In order to describe all the interesting nuclear processes we have considered at the total nuclear potential to include an optical term, although supposing that the only nonlocal contribution arises from the real part of the potential, being the imaginary part introduced to simulate the absorption. The energy independent nonlocal potential gives rise to an equivalent local potential which has an explicit energy dependence.

The numerical results for the elastic scattering cross section ($^{16}O + {}^{60}Ni$ system) show that, due to the nonlocal effects, there is a shift of flux from the elastic to the fusion channel for low partial waves and energies below the barrier. The enhancement of the tunnelling, as compared to the local calculations, confirms results obtained by a previous nonlocal fusion model [2,3]. For energies above the barrier the equivalent local calculations give, due to the enhancement of the centrifugal potential, results which are smaller than the local ones.

The effective mass approximation used in this work is valid only for energies around the local barrier height. It is worth stressing that the full calculation, with no restriction for the energy, must start from a Schrödinger integro-differential equation, constructed out of the Hamiltonian with the complete nonlocal term without any additional assumption. That equation is much more complicated than the present approximated version and it must be numerically solved so as to give the results of interest. For heavy-ion collisions, the solution to that equation must be obtained with accurate numerical methods to avoid convergence problems [15].

VI. ACKNOWLEDGEMENT

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VIII. FIGURE CAPTIONS

Figure 1: The behavior of the effective equivalent local potential for the $^{16}O+^{60}Ni$ system in the external (a) and internal (b) interaction region at the bombarding energies: 35 MeV (dashed lines), 39 MeV (pointed lines) and 42 MeV (dash-pointed lines). For comparison purposes, the solid lines in the Figure represent the results of calculations considering the local potential.

Figure 2: The predicted elastic scattering angular distributions at $E_{\rm Lab} = 35$ MeV for the $^{16}O + ^{60}Ni$ system considering the local potential (solid line) and equivalent local one (dashed line) in the effective mass approach.

Figure 3: The calculated phase shift (Φ_l) and modulus ($|S_l|$) of the S matrix as function of the orbital angular momentum for the elastic scattering process for $^{16}O + ^{60}Ni$ at the subbarrier energy of 35 MeV, considering the local (solid lines) and the equivalent local (dashed lines) potentials.

Figure 4: The predicted fusion cross section excitation functions at barrier energies for the $^{16}O + ^{60}Ni$ system, considering the local (solid lines) and the equivalent local (dashed lines) potentials.

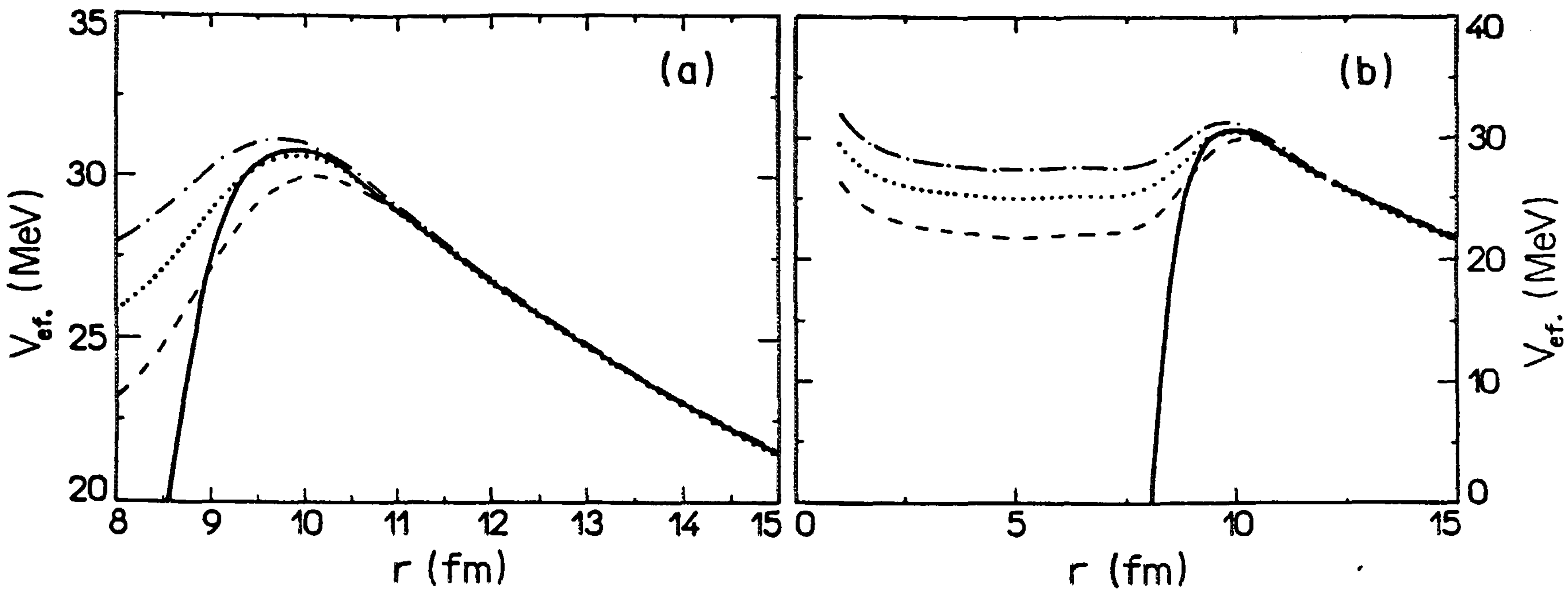

Fig. 1