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A Problem of Optimization for the Specific Cost of Installed Electric Power in Nuclear Plants

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خلاصة

يقوم هذا البحث بتحليل العوامل المؤثرة في الاتزان الحراري للمحطات النووية ذات مولدات البخار بغرض التوصل إلى أعلى كفاءة تشغيل عند الاحمال المختلفة وكذلك عند ظروف تشغيل متباينة. والتقدير الامثل لهذه العوامل يؤدي إلى زيادة الطاقة المولدة وبالتالي خفض التكلفة النوعية. ويعتبر التحليل أن درجة حرارة سطح الوقود داخل قلب المفاعل هي المحددة للتشغيل الآمن. وقد حسبت كفاءات الوحدات المختلفة للمحطة النووية عند أقصى قدرة متولدة باستخدام علاقات أمكن استنتاجها.

Abstract

The optimization problem analyzed in this paper is related to the thermal cycle parameters in nuclear power stations having steam generators. The optimization maximizes the electric power generation and hence minimizes the specific cost of installed power with respect to the average operating saturation temperature in the station thermal cycle. The analysis considers the maximum fuel cladding temperature as a limiting factor in the optimization process as it is related to the safe operation of the reactor.

Introduction :

The cost of installed power is an important economic indication by which we can compare nuclear with fossil fuel power plants. In nuclear power plants the capital investment can be put as the sum of two components :

a - A component which is a function of installed electric power P_e, such as the reactor building, reactor, steam generator, turbine, alternator, etc. This component can be denoted by A (P_e).

b - A component which is independent of the installed power, such as service buildings, workshops, auxiliary power systems, etc. this component can be denoted by B.

The capital investment K can therofere, be put as :

$$K = A(P_e) + B \quad (1)$$

The specific cost $\mathcal{K}$ of installed electrical power is given by :

$$\mathcal{K} = \frac{K}{p_e} = \frac{A(P_e)}{p_e} + \frac{B}{p_e} \quad (2)$$

It can be seen from equation (2) that the optimization of P_e with repsect to the thermodynamic operating conditions in the nuclear plant impiles an optimization of the specific cost of installed electrical power in the nuclear station. Therefore, in this paper analysis is made to maximize the electric power Pe with respect to the average operating saturation temperature as the optimizing parameter in the steam cycle.

Optimization Analysis of Electric Power in the Nuclear Station

Fig.1 shows the different components in the nuclear power plant.

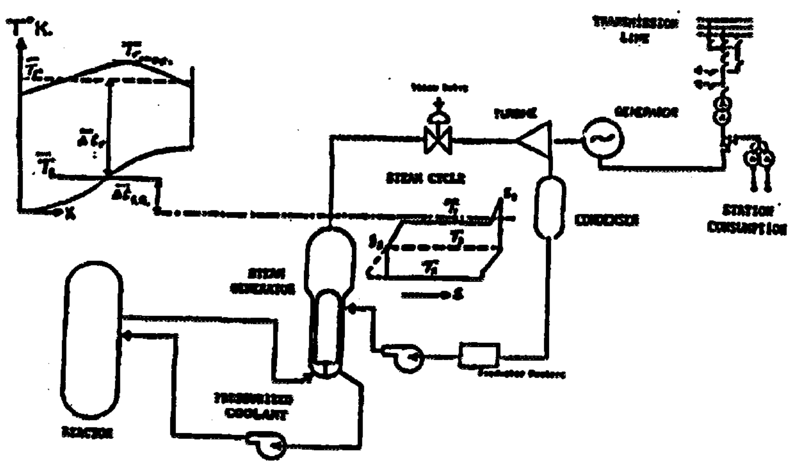

Fig. 1 : Schematic diagram and distribution of temperature in the nuclear power plant

The nuclear reactor which may be pressurized light water, pressurized heavy water, or gas cooled, generates thermal energy by which steam is generated in the steam generator. It is clear that a cascade of efficiencies promotes the thermal power generated in the reactor fuel until it appears as a net electrical power available at the transmission lines. Therefore, we have :

$$P_e = P_t \eta_{sg} \eta_t \eta_m \eta_g = P_r \eta_r \eta_{sg} \eta_t \eta_m \eta_g \quad (3)$$

Where : P_t = Thermal power carried by the reactor coolant.

P_r = Thermal power generated in the reactor fuel.

η_r = Thermal efficiency of reactor.

η_{sg} = Efficiency of steam generator

η_t = Efficiency of thermal cycle.

η_m = Mechanical efficiency of turbo-generator set.

η_g = Efficiency of electric generator.

Fig.1: Illustrates also the distribution of temperature in the nuclear power station. In the steam cycle part the Temperature-Entropy (T-S) diagram is shown. Steam is generated having enthalpy i_t and specific entropy S_t. The water at the condenser outlet is heated in the feedwater heater up to a temperature T_3°K. It is assumed that reversible regeneration of feedwater heating takes place so that the temperautre T_2 corresponding to the condenser pressure justifies a Carnot cycle process in the steam cycle.

The pressurized coolant is heated in the reactor up to an average temperature T_c. Fig.1 shows also the distribution of the coolant and fuel clad temperature with its maxium value $T_{r.max}$ along the fuel lenght.

In order to carry out the optimization analysis it is required to express the electric power P_e in terms of some thermal paramters such as the average operating saturation temperature, maximum temperature of fuel clad, and the temperature corresponding to back pressure in the steam cycle. This analysis can be pursued as follows :

The thermal cycle efficiency based on the above mentioned assumption is given by :

$$\eta_t = 1 - \frac{T_2}{T_1} \quad (4)$$

Where : T_2 = Temperature in degrees Kelvin corresponding to pressure in condenser, and

$\overline{T_1}$ = Average operating saturation temperature in degrees Kelvin in the thermal cycle. This temperature is given by :

$$\overline{T_1} = \frac{i_1 - i_3}{s_1 - s_3} \qquad (5)$$

Where : s_1, s_3 are the specific entropies indicated in Fig.1.

i_1, i_3 are the corresponding enthalpies

The heat balance in the reactor can be expressed as follows :

$$P_t = \eta_r P_r = \alpha_r A_r \Delta\overline{t_r} = C\ F_c \Delta T_c \qquad (6)$$

The heat balance in the steam generator can be given by :

$$\alpha_r A_r \Delta\overline{t_r} = \frac{\alpha_{sg} A_{sg} \Delta\overline{t_{sg}}}{\eta_{sg}} \qquad (7)$$

Where : α_r = Heat transfer coefficient from fuel surface to coolant.

α_{sg} = Heat transfert coefficient in steam generator.

A_r = Surface area of reactor fuel.

A_{sg} = Surface area of steam generator.

$\Delta\overline{t_r}$ = $(\overline{T_r} - \overline{T_c})$, represents the average temperature drop in the reactor as indicated in Fig.1.

$\Delta\overline{t_{sg}}$ = $(\overline{T_c} - \overline{T_1})$, represents the average temperature drop in the steam generator as indicated in Fig. 1.

F_c = Mass flow of reactor coolant.

C = Average specific heat of coolant.

ΔT_c = Increase of coolant temperature in reactor.

From (6) we have :

$$\frac{\Delta T_c}{\Delta\overline{t_r}} = \frac{\alpha_r A_r}{C\,F_c} = \theta \qquad (8)$$

θ can be considered constant assuming that; the primary coolant flow F_c is constant, and the average specific heat of coolant C does not change with temperature.

From (7) we have:

$$\frac{\Delta \bar{t}_r}{\Delta \bar{t}_r + \Delta \bar{t}_{sg}} = \frac{\Delta \bar{t}_r}{\bar{T}_r - \bar{T}_1} = \frac{\alpha_{sg} A_{sg}}{\alpha_{sg} A_{sg} + \eta_{sg} \alpha_r A_r} = \gamma \qquad (9)$$

γ can also be considered constant assuming that α_r, α_{sg} do not change with temperature.

Therefore, from (9) we have :

$$\gamma\,(\bar{T}_r - \bar{T}_1) = \Delta \bar{t}_r \qquad (10)$$

In case of a cosine function distribution of the heat generated along the fuel element, the relation between the maximum temperature of fuel clad $T_{r.max}$ and the average coolant temperature $\bar{T}_c$ is given by [3].

$$T_{r.max} - \bar{T}_c = \frac{\Delta T_c}{2} \sec\ \tan^{-1} \frac{\pi C\ F_c}{\alpha_r A_r} = \frac{\Delta T_c}{2} \sec\ \tan^{-1} \frac{\pi}{\theta}$$

From which :

$$T_{r.max} - \bar{T}_c = \frac{\Delta T_c}{2} \sqrt{1 + \frac{\pi^2}{\theta^2}} \qquad (11)$$

Therefore, from equations (8) and (11) we have :

$$T_{r.max} - \bar{T}_c = \frac{\Delta \bar{t}_r}{2} \sqrt{\theta^2 + \pi^2} \qquad (12)$$

From equations (10), (12), and knowing that $\Delta \bar{t}_r = (\bar{T}_r - \bar{T}_c)$, we have :

$$\Delta \bar{t}_r = \mu (T_{r.max} - \bar{T}_1) \qquad (13)$$

where : $$\mu = \frac{\gamma}{(1-\gamma) + \frac{\gamma}{2}\sqrt{\theta^2 + \pi^2}} \qquad (14)$$

From equations (6) and (13) we have :

$$P_t = \mu \alpha_r A_r\ (T_{r.max} - \bar{T}_1) \qquad (15)$$

From equations (3), (4), (15) we have

$$P_e = \mu \alpha_r A_r \eta_{sg} \eta_m \eta_g (T_{r.max} - \bar{T}_1) \; (1 - \frac{T_2}{\bar{T}_1}) \qquad (16)$$

Putting :

$$\acute{P}_e = \frac{P_e}{\Gamma} \qquad (17)$$

Where :

$$\Gamma = \mu \alpha_r A_r \eta_{sg} \eta_m \eta_g \qquad (18)$$

we have from equations (16, (17), (18):

$$\acute{P}_e = (T_{r.max} - \bar{T}_1)\;(1 - \frac{T_2}{\bar{T}_1}) \qquad (19)$$

Now maximizing $\acute{P}_e$ as given by equation (19) with respect to $\bar{T}_1$, we have for $d\acute{P}_e / d\bar{T}_1 = 0$:

$$\bar{T}_{1\,opt} = \sqrt{T_2 T_{r.max}} \qquad (20)$$

Substituting the optimum parameter $\bar{T}_{1opt}$ given by (20) in (19), we get the following expression for $P_{e.opt}$:

$$\acute{P}_{e.opt} = (\sqrt{T_{r.max}} - \sqrt{T_2}\,)^2 \qquad (21)$$

From equation (21) it can be seen that the higher the allowable value of $T_{r.max}$, the higher will be the value of $\acute{P}_{e.\,opt}$ and hence the lower will be the specific cost K of the installed electric power in the nuclear station. However, the optimization process must consider the limitation on the maximum fuel cladding temperature which is highly associated with the safe operation of the nuclear reactor.

From equations (4) and (20) we have the following expression for the thermal cycle efficiency under optimum operating conditions:

$$\eta_{t.opt} = [1 - \frac{\sqrt{T_2}}{\sqrt{T_{r.max}}}\,]^2 \qquad (22)$$

and from equations (21) and (22) we have:

$$\acute{P}_{e.opt} = T_{r.max}\, \eta^2_{t.opt} \qquad (23)$$

Table I gives the optimum average operating saturation temperature in the thermal cycle $\bar{T}_{1opt}$ as given by equation (20), for different values of

T_2 and $T_{r.max}$. Table II gives $\acute{P}_{e.opt}$ as given by equation (21), for different values of T_2 and $T_{r.max}$. Table III gives the optimum thermal cycle efficiency $\eta_{t.opt}$ as given by equation (22), also for different values of T_2 and $T_{r.max}$. The optimum parameters $\bar{t}_{lopt}$ (°C), $\acute{P}_{e.opt}$, $\eta_{t.opt}$ are depicted in the curves of Fig. 2.

Fig. 3 shows the contours of constant $\acute{P}e$ as given by equation (19) for $t_2 = 32$ °C. It can be seen from Fig. 3 that the t_{1opt} dotted line intersects the ($\acute{P}e$ = constant) contours at a minimal value of $t_{r.\ max}$.

At these points of intersection the optimum condition of operation given by equation (20) is justified.

An example is given to illustrate the application of the optimizing process analyzed in this paper.

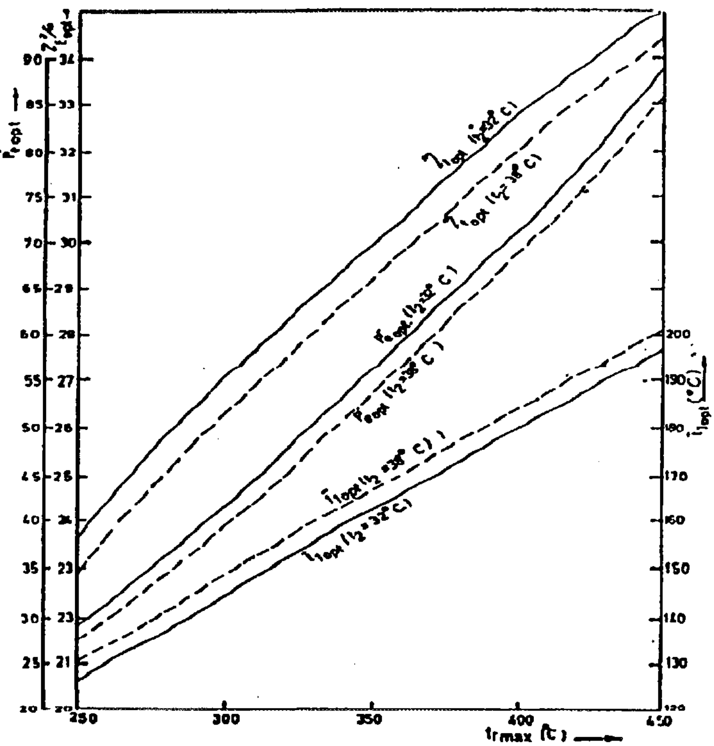

Fig. 2 : Optimum average saturation temperature ($\bar{t}_{l.opt}$):
Optimum thermal efficiency ($\eta_{t.opt}$), Optimum electric power ($\acute{P}_{e.opt}$) depicted as function of maximum fuel clad temperature ($t_{r.\ max}$)

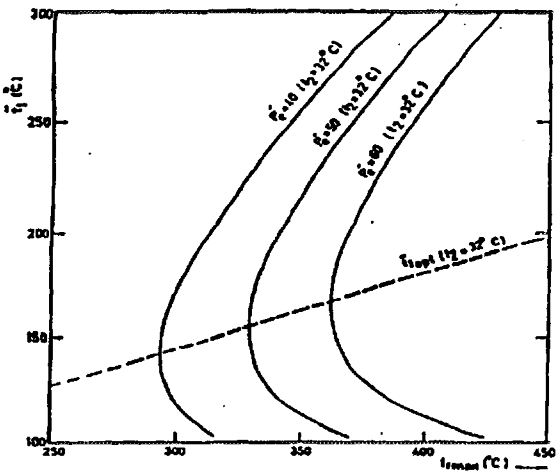

Fig. 3 : Contours of $\acute{P}_e$ = constant.

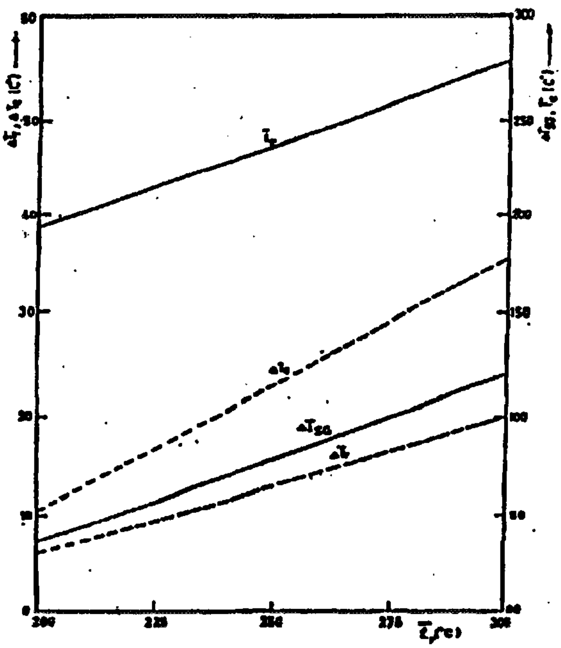

Fig. 4 : Depicted relations required in the given example

Table I : Optimum Average Operating Saturation Temperature of Steam Cycle

T2 \ Tr.max	523.2°K (250 °C)	573.2°K (300 °C)	623.2°K (350 °C)	673.2°K (400 °C)	723.2°K (450 °C)
301.2°K (28 °C)	396.97 (123.77)	415.5 (124.31)	433.25 (160.05)	450.3 (177.1)	466.72 (193.5)
303.2°K (30 °C)	398.29 (125.09)	416.88 (143.68)	434.69 (161.49)	451.8 (178.6)	468.27 (195.07)
305.2°K (32 °C)	399.6 (126.4)	418.26 (145.06)	436.12 (162.92)	453.28 (180.08)	469.8 (196.8)
307.2°K (34 °C)	400.9 (127.7)	419.63 (146.43)	437.55 (146.35)	454.76 (181.56)	471.35 (198.15)
309.2°K (36 °C)	402.2 (129.01)	421.0 (147.79)	438.97 (165.77)	456.24 (183.04)	472.87 (199.67)
311.2°K (38 °C)	403.51 (130.31)	422.35 (149.15)	440.38 (167.18)	457.71 (184.51)	474.2 (201.2)

Table II : Values of $P'_{e.opt}$ Under Optimum Operating Conditions

T2 \ Tr.max	523.2°K (250 °C)	573.2°K (300 °C)	623.2°K (350 °C)	673.2°K (400 °C)	723.2°K (450 °C)
301.2°K (28 °C)	30.45	43.38	57.89	73.84	90.96
303.2°K (30 °C)	29.82	42.63	57.03	72.81	89.86
305.2°K (32 °C)	29.2	41.88	56.16	71.84	88.79
307.2°K (34 °C)	28.58	41.4	55.31	70.88	87.71
309.2°K (36 °C)	27.96	40.42	54.46	69.92	86.64
311.2°K (38 °C)	27.38	39.67	53.63	68.98	85.59

Table III : Steam Cycle Efficiency $\eta_{t.opt}$ Under Optimum Operating Conditions

Tr.max / T2	523.2°K (250 °C)	573.2°K (300 °C)	623.2°K (350 °C)	673.2°K (400 °C)	723.2°K (450 °C)
301.2°K (28 °C)	0.2412	0.2757	0.3048	0.3311	0.3546
303.2°K (30 °C)	0.2387	0.2727	0.3025	0.3289	0.3525
305.2°K (32 °C)	0.2362	0.2703	0.3002	0.3267	0.3507
307.2°K (34 °C)	0.2337	0.2679	0.2979	0.3245	0.3482
309.2°K (36 °C)	0.2312	0.2655	0.2956	0.3228	0.3461
311.2°K (38 °C)	0.2287	0.2631	0.2933	0.32	0.344

Example:

In a nuclear power station having pressurized light water reactor the following data is given:

Gross generated electric power.	= 922 MWe
Maximum temperature of fuel clad.	= 348°C
Temperature in condenser corresponding to back pressure 0.05 kg/cm^2	= 32.5°C
Surface area of reactor fuel.	= 4.62×10^7 cm^2
Heat transfer coefficient from fuel surface to coolant	= 3 Watt / cm^2 °C
Mass flow of reactor coolant	= 56300 Ton/ hour
Average specific heat of coolant	= 5.016 Joules/gm. °C

$$\eta_{sg}\,\eta_m\,\eta_g = 0.91$$

It is required to find out the following relation satisfying the operating conditions for optimum specific cost of installed electric power in the nuclear station:

$\Delta \bar{t}_r$, ΔT_c, $\bar{T}_c$, $\Delta \bar{t}_{sg}$ as function of the average fuel clad temperature $\bar{T}_r$ (°K).

Solution

$T_{r.max}$ = 348 + 273.2 = 621.2 °K
T_2 = 32.5 + 273.2 = 305.2 °K
From equation (20) we have:

$$\bar{T}_{1.opt} = \sqrt{T_{r.max} T_2} = \sqrt{621.2x305.7} = 435.77°K ,$$

$\bar{t}_{1opt}$ = 435.77 - 273.2 = 162.57 °C
From equations (17) and (21) we have the following expression for installed electric power satisfying the required optimum specific cost.

$$P_{e.opt} = \Gamma (\sqrt{T_{r.max}} - \sqrt{T_2})^2 \qquad (24)$$

Substituting the above given data in (24) we have:
Γ = = $1.66x10^7$
Substituting then in equation (18) we have:
μ = 0.1332
From equation (8) we have:
$\theta = (\alpha_r A_r)/(C F_c) = (3x4.62x10^7x3600) / (5.016x5.63 x 10^{10}) = 1.767$
Substituting the values of μ and θ in (14) we get:
γ = 0.1489
The required relations can therefore, be obtained:
From equations (10) and (20) we have:

$$\Delta \bar{t}_r = \gamma(\bar{T}_r - \sqrt{T_2 T_{r.max}}) = 0.1489 \bar{T}_r - 64.88$$

From (8) we have:
$\Delta T_c = \theta \Delta \bar{t}_r = 0.263 \bar{T}_r - 114.63,$

$$\bar{T}_c = \bar{T}_r - \Delta \bar{t}_r = (1 - \gamma) \bar{T}_R + \gamma \sqrt{T_2 T_{r.max}}$$

$$\bar{T}_c = 0.8511 \bar{T}_r + 64.88$$

From equation (9) we have:

$$\Delta \bar{t}_{sg} = \frac{1-\gamma}{\gamma} \Delta \bar{t}_r = 5.716 \, \Delta \bar{t}_r$$

Hence : $\Delta \bar{t}_{sg} = 0.851 \, \bar{\bar{T}}_r - 370.85$

The required relations satisfying the operating conditions for optimum specific cost of installed electric power are depicted in the curves of Fig. 4.

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