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REDUCTION OF ANGULAR SPREAD AT NONADIABATIC ELECTRON MOTION IN MAGNETICALLY INSULATED DIODE

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Behaviour of the electron pitch-angle is investigated by analytical and numerical methods for the case of a magnetically insulated diode with a ribbon geometry. It has been shown that at the boundary of the adiabaticity of the electron motion the angle can be reduced in many times by a choice of a special nonhomogeneity of the magnetic field. Analytic expressions for the final pitch-angle of the beam electrons are given.

1. Introduction. High-power electron beams with a high current density and a small angular spread are a base for various scientific researches and practical applications. However a generation of such beams is not a problem solved now. Using of relativistic diodes with a magnetic insulation is investigated by us as one of the promising ways to generate them. The main goal of the investigations is to obtain a long duration of the beam pulse (up to tens of microseconds) at appropriate values of the density and angular spread. Theoretical estimation made according to [1] for the influence of a nonzero angle between the electric and magnetic fields near the emitting cathode for the typical diode parameters, has given the electron pitch-angle on the level of 0.01. But experimental measurements under similar conditions [2] and our numerical simulations [3] have shown that the pitch-angle of the electrons exceeds this level more than in ten times. In order to describe the influence of various factors on the electron pitch-angle we have created a theoretical model considering the motion of the beam electrons in the diode with a ribbon geometry for the case of nonhomogeneous electric and magnetic fields.

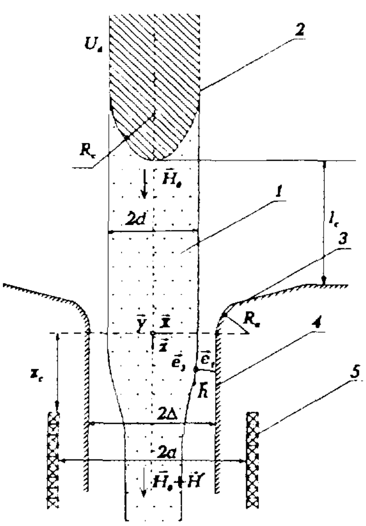

Fig. 1. The schematic of the diode geometry

2. Analytical model. The schematic of the ribbon magnetically insulated diode that is homogeneous along the y-axis, is shown in the Fig. 1. An electron flow (1) emitted by the cathode (2), is accelerated along the guiding magnetic field. Then it passes through the anode slit (3) and propagates in the slit vacuum channel (4). To simplify equations describing the electron motion in this system, let us suppose a few assumptions. Firstly, self magnetic field of the beam $\vec{H}_b$ is small enough in comparison with external magnetic field $\vec{H} = (H_x, 0, H_z)$. Secondly, the values of the electron pitch-angles are much less than 1. Thirdly, the shortest spatial scale L of the magnetic and electric field variations in the diode and in the channel is larger than Larmour radius of the electron ρ_L calculated on its total velocity in this region. Let's consider that a beginning of a local frame is placed in the centre of the electron Larmour circle, $\vec{h}$ is an unit vector tangent to the magnetic field line, $\vec{e}_2$ is an unit vector parallel to the y-axis, and the unit vector $\vec{e}_1 = [\vec{e}_2 \times \vec{h}]$ is perpendicular to both of them. A system of equations for the electron motion in approximation of a small value of the parameter ρ_L / L can be easily transformed to the equation of classic oscillator with a time dependent frequency and a driving force $\dot{p}_{dr}$. This equation is :

$$\ddot{r} + \omega^2{}_H(t) r = -\dot{p}_{dr}, \qquad (1)$$

where $r = p_1/\omega_H(t)$, $\dot{r} = p_2 - p_{dr}$; p_1, p_2, $p_{\|}$ are the components of the momentum vector along the unit vectors $\bar{e}_1$, $\bar{e}_2$ and $\bar{h}$ respectively, $\omega_H(t)$ is a cyclotron frequency of the electron, and $p_{dr} = p_{\|}v_{\|}\left(\bar{e}_1\left(\bar{h}\nabla\right)\bar{h}\right)/\omega_H(t) - \gamma mcE_{\perp}/H - \gamma mv_{\|}H_b/H$ is a momentum of the electron drift motion. The first part of this expression for the drift momentum is an inertial drift, the second one is a $\bar{E}\times\bar{B}$ drift and the last one is a result of the addition of the beam magnetic field to the external one. In quasiclassic approximation a full solution of the equation (1) is the following:

$$p_{\perp}(t) = ip_1(t) + p_2(t) = p_{dr} - e^{i\varphi(t)}\cdot\int_{t_0}^{t}\sqrt{\frac{\omega_H(t)}{\omega_H(t')}}\dot{p}_{dr}e^{-i\varphi(t')}dt' + i\sqrt{\frac{\omega_H(t)}{\omega_0}}p_0e^{i\varphi(t)}, \qquad (2)$$

where $\varphi(t) = \int_{t_0}^{t}\omega_H(t')dt' + \varphi_0$ is a phase of rotation, $p_0 = 4\pi mcj_c\sin(\varepsilon)/(cH_c^2)$ is a transverse momentum of the electron due to the angle between $\bar{E}$ and $\bar{H}$ fields near the cathode surface (see [1]). The electron has this momentum p_0 in that moment of time t_0 when its motion becomes magnetised. Since at the electron motion the variation of the vector $\bar{h}$ is directed perpendicularly to $\bar{h}$, one can modify the expression for p_{dr} by using $\left(\bar{e}_1\left(\bar{h}\nabla\right)\bar{h}\right) \approx \dot{h}_x \backslash v_{\|}$ that is correct with accuracy of the first order on the parameter ρ_L/L. Here it is assumed that $h_z \approx 1$ because the angle between the magnetic field line and z-axis is small enough. To transform the part of p_{dr} that is connected with the $\bar{E}\times\bar{B}$ drift, one can split up the transverse electric field in two parts: $E_{\perp} = E_v + E_b$. The first one E_v is created by the charges induced on the surface of the anode slit and the walls of the channel. This component of the electric field is sharply changing near the anode slit. The second one E_b is provided by the self space charge of the beam far from the cathode where the electron velocity becomes close to the speed of light c and as a result, it is almost invariable. It should be pointed that the strengths of the fields E_b and H_b are determined by the linear charge and the linear current of the ribbon beam respectively, so they are practically invaried along the trajectory of the electron. After transformations of the integral in the expression (2) we have obtained the final transverse momentum of the electron in that part of the channel which is far from the anode slit, and on this reason the electric and magnetic fields are invariable there. The expression for this momentum is the following:

$$p_{\perp}(t) = -\frac{c(E_b - \beta_{\|}H_b)}{\omega_H(t)} + e^{i\varphi(t)}\int_{t_0}^{+\infty}\sqrt{\frac{\omega_H(t)}{\omega_H(t')}}e^{-i\varphi(t')}\times$$
$$\times\left\{icE_v + \frac{eH_x(t')}{\gamma mc}p_{\|} - \frac{c(E_b - \beta_{\|}H_b)}{\omega_H^2(t)}\cdot\dot{\omega}_H(t)\right\}\cdot dt' + i\sqrt{\frac{\omega_H(t)}{\omega_0}}p_0e^{i\varphi(t)} \qquad (3)$$

Obtaining this expression we have considered that $p_{\|}$ and γ are invariable in regions of nonhomogeneities of the electric and magnetic fields near the anode slit.

Analysis of the expression (3) shows that the main addition to the transverse momentum of the electron is collected on these parts of the electron trajectory where the electric field E_v or the magnetic field H_x as well as the cyclotron frequency ω_H are changing sharply. This effect is substantially increased on that part of the trajectory where the guiding magnetic field has a small value. The another significant conclusion from expression (3) will be obtained in the following part of the paper.

Let us assume that the magnetic field in the diode gap is homogeneous and equal to H_0. Then the field rises along the z-axis at the entrance of the channel to the value $H_0+H^*=H_0(1+\alpha)$ due to the placement of the additional semi-infinite plane coil (5) outside the channel. The components of the field are given for this case by following expressions:

$$H_x(x,z)=\frac{\alpha H_0}{4\pi}\ln\frac{(z-z_c)^2+(a+x)^2}{(z-z_c)^2+(a-x)^2},\quad H_z(x,z)=H_0+\frac{\alpha H_0}{2\pi}\left[\pi+\mathrm{arctg}\left(\frac{z-z_c}{a-x}\right)+\mathrm{arctg}\left(\frac{z-z_c}{a+x}\right)\right] \tag{4}$$

where z_c is the coordinate of the beginning of the winding and 2a is the gap between plates of the coil.

In the described case the electron beam should be like a plane layer with a linear current I'. The highest value of the charge density of the beam is near the cathode surface due to a small velocity of the electrons there. For the case of $\gamma >> 1$ the electron velocity becomes close to the speed of light c when it goes from the cathode surface on a small distance. On this reason the charge density is practically independent on z coordinate in the main part of the diode gap. Taking into account the pointed peculiarity we replace the space charge of the beam by a homogeneously charged cylinder with a radius equalled to the cathode one R_c and by a homogeneously charged layer with the density $\rho=I'/(2dv_{\|})$. As a result of these approximations we have obtained the following expressions for the varied part of the electric field: $E_v=E_{v1}+E_{v2}$,

$$E_{v_1}=\begin{cases}E_0(x)\left[\frac{R_a+\Delta+x}{z^2+(R_a+\Delta+x)^2}-\frac{R_a+\Delta-x}{(R_a+\Delta-x)^2+z^2}-\frac{2x}{(R_c+l_c-z)^2}\right], & \text{at } z<0\\ E_0(x)/\mathrm{ch}\left(\frac{z\pi}{2\Delta}\right), & \text{at } z\geq 0\end{cases} \tag{5}$$

$$E_{v_2}=\frac{\pi I' R_a z}{v_{\|}}\left[\frac{1}{(R_a+\Delta-x)^2+z^2}-\frac{1}{(R_a+\Delta+x)^2+z^2}\right],$$

where $E_0(x)=\dfrac{U_d}{2\ln\left(\frac{l_c}{R_c}+1\right)+\ln\left(\frac{l_c}{R_a}+1\right)+\ln\left(\frac{l_c+R_a}{(R_a+\Delta)}\right)}\cdot\dfrac{2x}{(R_a+\Delta)^2-x^2}$

Then we have obtained the final result for the transverse momentum by putting the expressions (4) and (5) in the expression (3):

$$p_\perp(t)=p_{dr}+(p_C+p_E e^{-i\varphi_a}+p_H e^{-i\varphi_c})\cdot e^{i\varphi(t)},\ \text{where } p_C=p_0\sqrt{1+\alpha},$$

$$p_E=\sqrt{1+\alpha}\left\{eE_0(x_a)A_1(x_a)+2\pi I' e A_2(x_a)\right\}\ \text{and}\ p_H=\sqrt{\frac{1+\alpha}{1+\alpha/2}}\cdot A_3(x_c) \tag{6}$$

Variables A_1, A_2 and A_3 are given in Appendix. φ_a and φ_c are the phases of the electron rotation at which it passes through the regions with the maximal value of E_v and H_x. Position of these regions are corresponded to the position of the anode slit and the beginning of the additional coil (5) (see Fig.1). x_a and x_c are the coordinates of the center of the electron Larmour circle at its passing through the mentioned regions respectively. Expression (6) shows that the resulting transverse momentum of the electron is equal to a sum of a constant vector and three vectors shifted on the angle and rotating with the same velocity. The component p_C is connected with the angle between $\vec{E}$ and $\vec{H}$ fields near the cathode, the second component p_E is determined by the nonhomogeneity of the electric field and the third component p_H is due to the magnetic one. Further we show that the transverse momentum can be essentially decreased by appropriate shift of phase φ_c in respect to φ_a.

3. Calculations. The obtained formula for the transverse momentum has been checked up by comparison of its results with ones of the numerical simulation of self-consistent problem. For example, we have taken the geometry of the diode used in the experiments on the U-2 accelerator [3] with following parameters: R_a=2.5cm, R_c=3cm, l_c=10cm, H_0=2kOe, H*=6kOe, a=4.3cm, Δ=2.5cm, d=1.1cm, U_d=1MV, I'=0.3kA/cm. Using this data we have obtained the values of the three mentioned vectors $p_E=0.06p_\parallel$, $p_H=0.04p_\parallel$ and $p_C=0.01p_\parallel$ for the beam boundary. The dependencies of the electron angle Θ on x coordinate both for analytical model (lines) and numerical simulation (squares) are represented in the Fig. 2 for two cases. The upper line and unfilled squares show the case when the components p_E and p_H are added at the phase difference $\varphi_a-\varphi_c\approx 2\pi$ (z_c=12cm). The electron angle increases with an increase of X and becomes close to 0.1 at the beam boundary. From another hand, by appropriate choosing of the coil location we can obtain this phase difference close to π. For this case (lower line and filled squares) the resulting angle becomes in many times smaller and has value not more than 0.02. The oscillations of squares near the lines can be explained by the behaviour of the momentum component p_C, which has varied phase across the beam in respect to components p_E and p_H.

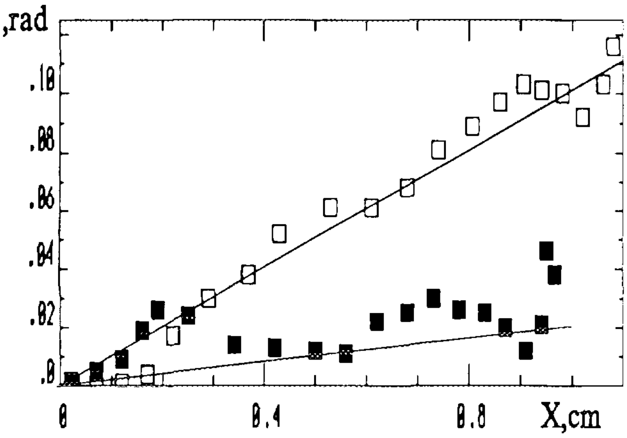

Fig.2. The electron pitch-angles in the beam cross section, derived from analytical and numerical calculations.

So, both the analytical model and numerical simulations have shown that the angular spread of the electron beam generated by the magnetically insulated diode can be reduced in many times by a choice of the special nonhomogeneity of the guiding magnetic field.

[1] Ryutov D.D. Proc. of 9-th Intern. Conf. on High-Power Particle Beams, Washington, 1992, p.1009-1014.

[2] Sloan M.L., Davis H.A. Phys. Fluids, 1982, v.25, N 12, p. 2337-2343.

[3] Arzhannikov A.V., Astrelin V.T., Kapitonov V.A. et al. Proc. of 8-th Intern. Conf. on High-Power Particle Beams, Novosibirsk, 1990, p.256-263.

Appendix

$$A_1 = i\pi\frac{(R_a+\Delta)^2-x^2}{2xv_\parallel}e^{-\frac{R_a+\Delta}{\rho_L}}\mathrm{sh}(\frac{x}{\rho_L})+\frac{i\Delta}{v_\parallel \mathrm{ch}(\frac{\Delta}{\rho_L})},\quad A_2=\frac{\pi R_a}{v_\parallel^2}e^{-\frac{R_a+\Delta}{\rho_L}}\mathrm{sh}(\frac{x}{\rho_L}),\quad \rho_L=\frac{cp}{eH_0},$$

$$A_3=\left[p_\parallel \mathrm{sh}(\frac{x}{\rho_L^*})-\gamma mc\left(\frac{E_b-\beta_\parallel H_b}{H_0+H^*/2}\right)\mathrm{ch}(\frac{x}{\rho_L^*})\right]e^{-\frac{a}{\rho_L^*}},\quad \rho_L^*=\frac{cp}{e(H_0+H^*/2)},\quad E_b=\frac{2\pi I'x}{v_\parallel d}\quad H_b=\frac{2\pi I'x}{d}$$