



# CHAOTIC DYNAMICS OF ELECTRON BEAM WITH VIRTUAL CATHODE IN THE BOUNDED SYSTEM

V.G.Anfinogentov

*College of Applied Science, 83 Astrakhanskaya str., Saratov 410026, RUSSIA*

## Abstract

The electron beam with virtual cathode in the bounded system with feedback is studied with the help of PIC simulation. Different types of nonlinear behaviour are shown. The typical structures are recognized and influence of feedback on the structure formation are investigated. Relations between nonlinear oscillations and structure formation and interaction are discussed.

## Introduction

The microwave electron devices with supercritical current received a widespread attention in the last decade [1]. Dynamics of such devices determined by the formation of the region with space charge potential close to the accelerating voltage. This region is usually called a virtual cathode (VC). The practical significance of the devices with the VC is connected with problems of inertial confinement fusion and superhigh power radiation [2].

Experimental data and theoretical models reveals a strong nonlinearity of the regimes with the VC in the electron beam, including an appearance of deterministic chaos [1]. Thus the problem of governing the characteristics of chaotic oscillations is very important for application to such devices.

One of the most common methods of controlling an active system is to incorporate a feedback. The aim of our work is to investigate the feedback influence on the processes in the electron beam with the VC, especially in the chaotic regimes. Simple triode which consists of two diode regions with a common grid was investigated. In each of the diodes the beam propagates between two infinitely wide grounded grids [3]. The space between the grids is filled by a background of immobile positive ions. Electron velocity and charge density are kept constant at the input of the system. The only bifurcation parameter of this system is

$$\alpha = \omega_p L / v_0. \quad (1)$$

Here  $\omega_p$  is the plasma frequency corresponding to the charge density on the input grid,  $L$  is the distance between the grids and  $v_0$  is the initial velocity of the beam.

In the case of external feedback a microwave signal from the second diode is fed into the first one through a transmission line with a delay time  $d$ . This model may be treated as a simplest model of a virtod [4]. In the second case (internal feedback) the electrons transmitted through the VC in the second diode are reflected by the output grid and they interact with the nonreflected ones. The bifurcation parameter is the potential of the output grid  $f_2$ . Such a system may be treated as a simplest model of the reflecting triod [1].

## Nonlinear dynamics

The influence of the feedback on the dynamics of the electron beam with the VC was investigated with help of PIC-simulation [5]. The bifurcation diagrams are presented on

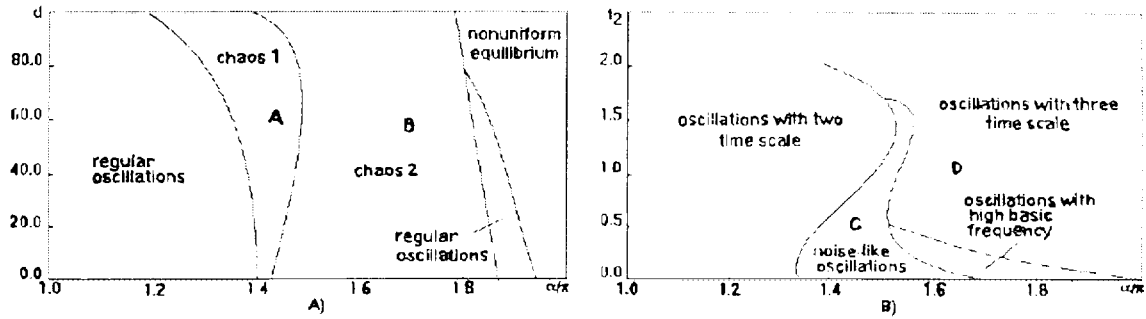


Figure 1: Bifurcation diagrams for external (a) and internal (b) feedback.

plane  $(\alpha, d)$  for the external feedback (Fig.1a) and on the plane  $(\alpha, f_2)$  for the internal one (Fig.1b).

The external feedback leads to the regular oscillations of the VC for small values of  $\alpha$  as well as for  $\alpha$  near  $2\pi$ . The oscillations of the VC becomes chaotic when  $\alpha$  increases. The strange attractor may appear either on the base of one unstable cycle which coincides with the limit cycle for  $\alpha < 1.4\pi$  (Fig.2a), or on the base of set of unstable periodical orbits (Fig.2b). Transition from regular oscillations to chaotic ones take place through a sudden destruction of the cycle in the phase space whereas the transitions between different chaotic regimes occur through intermittency.

The internal feedback leads to the strongly irregular behaviour for any values of  $\alpha$ . For small values of  $f_2$  (which means weak reflection) there are two different time scales of the VC oscillations. Two sharp peaks which correspond to these scales can be easily recognized on the power spectrum for small values of  $\alpha$ . The increasing of  $\alpha$  leads to the noise-like oscillations with the basic frequency growing with the growth of  $\alpha$ . The uniform structure of the attractor (Fig.2c) means that a large number of degrees of freedom is excited. For large values of  $f_2$  high-dimensional irregular oscillations are observed again, and a third sharp peak appears in the power spectra when  $\alpha$  increases (Fig.2d).

### Structures

The internal structure of the electron beam is investigated with the help of the time histories of space charge density. The method of proper orthogonal decomposition is applied [6]. The typical kinds of spatial space charge distribution (further named as modes) are obtained, and time histories of their amplitudes as well as relative energies were calculated (Fig.3).

In the regular regime with the external feedback three modes appear, and the first one

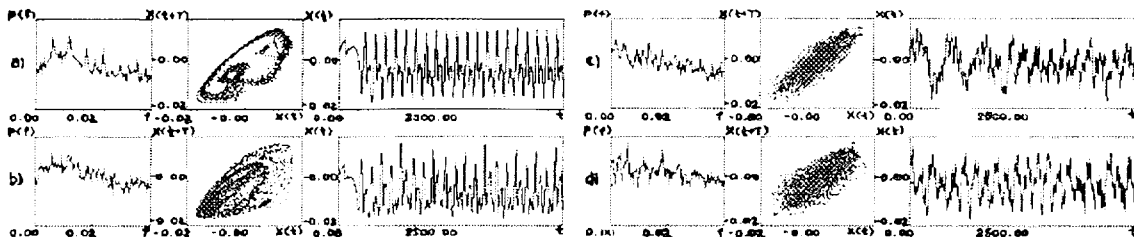


Figure 2: Phase portraits, power spectrum and time histories for different types of chaotic oscillations at external (a,b) and internal (c,d) feedback.

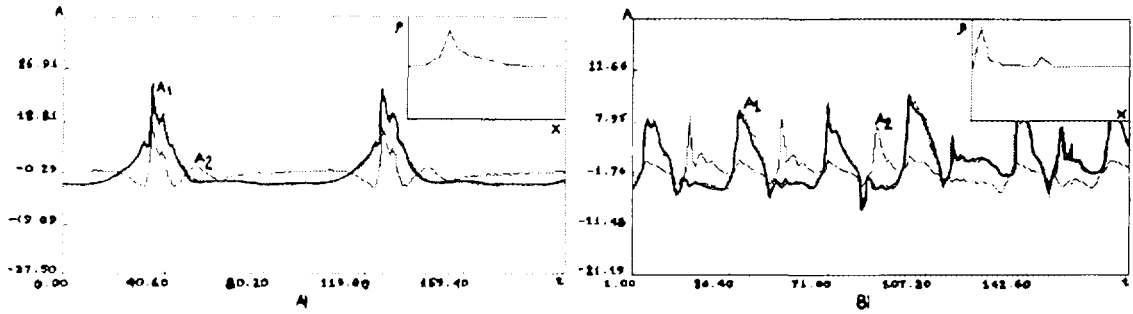


Figure 3: Time histories of mode amplitudes for external (a) and internal (b) feedback.

contains a main part of the energy. The space charge distribution in this mode coincides with that one at the VC appearance (the inner picture on Fig.3a), and the dynamics of its amplitude governs the behaviour of the system (see Fig.3a).

Transition to the chaotic oscillations is accompanied by the excitation of new modes and more smooth energy distribution among them. The basic mode again corresponds to the appearance of the VC, but the following ones have more complicated distributions corresponding to simultaneous existence of more than one bunches in the interaction space. The amplitudes of different modes are almost equal and they are excited simultaneously.

In the case of external feedback the typical distribution of the charge density is demonstrated in the inner picture in Fig.3b. Not only the first sharp peak which corresponds to the VC but also the second one is clearly visible. This peak corresponds to the second structure in the interaction space. The energy of oscillations is distributed smoothly among the modes (the highest mode contains less than 45 % of energy in all regimes). When there exist two time scales, two modes are excited and they exchange the energy during the oscillations, as it is shown on the Fig.3b. Transition to the noise-like oscillations is accompanied by the excitation of a large number of the modes with almost equal amplitudes.

Incorporation of the feedback can lead to changing of the number of the excited modes (external feedback) as well as to changing of its origin (internal feedback). On the other hand chaotic oscillations are accompanied by excitation of a large number of modes with close amplitudes. As well as in the Pierce diode without feedback [7], this excitation occurs simultaneously.

### Physical processes

The typical modes may be interpreted as a set of electron bunches (patterns) in the interaction space. This description is based on the strongly nonuniform character of the space charge distribution. There are the following kinds of the structures with different types of instability. The VC (Fig.5a) appears as a result of electrostatic instability, and

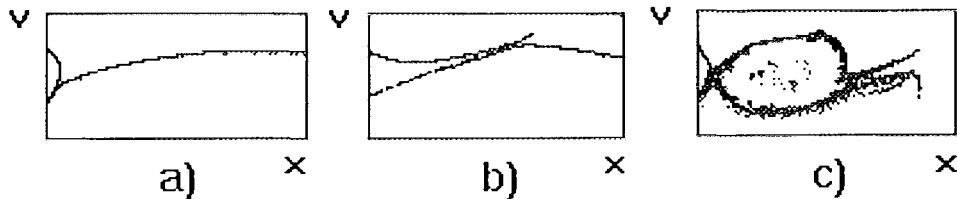


Figure 4: Phase space for different types of structures: (a) - virtual cathode, (b) - bunch, (c) - vortex.

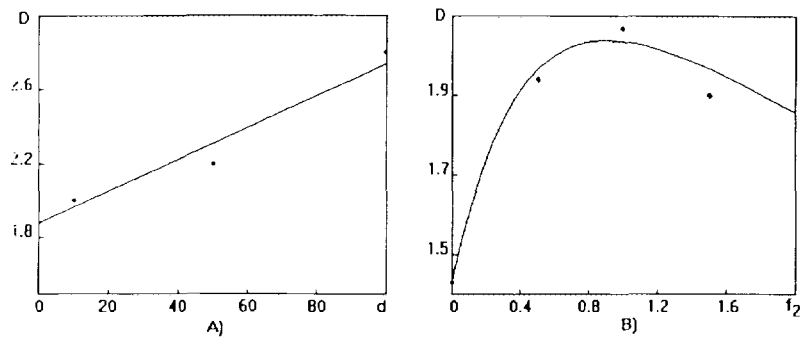


Figure 5: Dependences of attractor dimensions on the parameter of (a) external and (b) internal feedbacks.

it determines the behaviour of the system and the main time scale. The bunches (Fig.5b) in the flow which transmits through the VC are formed on the base of kinematic instability for the external feedback. The vortex in the phase space (Fig.5c) is formed after growing of the two-stream instability for the internal feedback.

There are three different types of the electron trajectories corresponding to these structures: reflection from the VC, deceleration in the VC region followed by drifting to the output, and repeated rotation in the vortex. Each motion provides its own time scale.

Transition to the chaotic regimes is caused by increasing of the number of structures and by strengthening of interaction between them. Increasing of  $d$  leads to the increasing of the number of bunches for the external feedback. The complication in this case is confirmed by the enhancement of the attractor dimension (Fig.5a).

In the case of the internal feedback when the potential  $f_2$  is sufficiently large, and, hence, there is a considerable reflected current, a rather uniform vortex emerges in the phase space. Interaction between the structures strengthens and the oscillations become more chaotic. Maximal value of attractor dimension corresponds to just this values of  $f_2$  (Fig.5b). The further increasing of  $f_2$  makes the vortex to close upon itself and thus leads to breaking of the interaction and to decreasing of attractor dimension.

## Conclusion

Incorporating of the external or the internal feedback leads to the significant transformation of nonlinear behavior of the electron beam. This process, determined by the structures formation and interaction, may cause simplification as well as complication of oscillations in comparison with the beam without a feedback.

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